ME 1020 Engineering Programming with MATLAB

Handout 08

Homework 8 Assignment: 8.1, 8.3, 8.8, 8.10, 8.12, 8.14, 8.16

Section 8.1

1. Solve the following problems using matrix inversion. Check your solutions by computing $\mathbf{A}^{-1}\mathbf{A}$.

a.
$$\begin{aligned} 2x + y &= 5 \\ 3x - 9y &= 7 \end{aligned}$$

b.
$$\begin{aligned} -8x - 5y &= 4 \\ -2x + 7y &= 10 \end{aligned}$$

c.
$$\begin{aligned} 12x - 5y &= 11 \\ -3x + 4y + 7x_3 &= -3 \\ 6x + 2y + 3x_3 &= 22 \end{aligned}$$

d.
$$\begin{aligned} 6x - 3y + 4x_3 &= 41 \\ 12x + 5y - 7x_3 &= -26 \\ -5x + 2y + 6x_3 &= 16 \end{aligned}$$

2.* *a.* Solve the following matrix equation for the matrix $\mathbf{C}$.

$$\mathbf{A}(\mathbf{BC} + \mathbf{A}) = \mathbf{B}$$

b. Evaluate the solution obtained in part *a* for the case

$$\mathbf{A} = \begin{bmatrix} 7 & 9 \\ -2 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & -3 \\ 7 & 6 \end{bmatrix}$$

3. Use MATLAB to solve the following problems.

a.
$$\begin{aligned} -2x + y &= -5 \\ -2x + y &= 3 \end{aligned}$$

b.
$$\begin{aligned} -2x + y &= 3 \\ -8x + 4y &= 12 \end{aligned}$$

c.
$$\begin{aligned} -2x + y &= -5 \\ -2x + y &= -5.00001 \end{aligned}$$

d.
$$\begin{aligned} x_1 + 5x_2 - x_3 + 6x_4 &= 19 \\ 2x_1 - x_2 + x_3 - 2x_4 &= 7 \\ -x_1 + 4x_2 - x_3 + 3x_4 &= 30 \\ 3x_1 - 7x_2 - 2x_3 + x_4 &= -75 \end{aligned}$$

Section 8.2

4. The circuit shown in Figure P4 has five resistances and one applied voltage. Kirchhoff's voltage law applied to each loop in the circuit shown gives

$$\begin{aligned} v - R_2 i_2 - R_4 i_4 &= 0 \\ -R_2 i_2 + R_1 i_1 + R_3 i_3 &= 0 \\ -R_4 i_4 - R_3 i_3 + R_5 i_5 &= 0 \end{aligned}$$

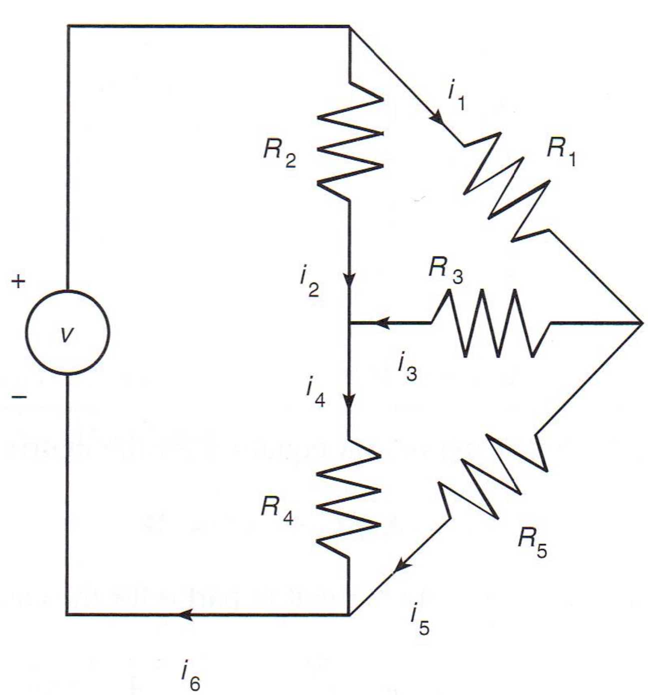

Figure P4

Conservation of charge applied at each node in the circuit gives

$$i_6 = i_1 + i_2$$
$$i_2 + i_3 = i_4$$
$$i_1 = i_3 + i_5$$
$$i_4 + i_5 = i_6$$

a. Write a MATLAB script file that uses given values of the applied voltage v and the values of the five resistances and solves for the six currents.

b. Use the program developed in part *a* to find the currents for the case where $R_1 = 1$, $R_2 = 5$, $R_3 = 2$, $R_4 = 10$, $R_5 = 5\ \text{k}\Omega$, and $v = 100$ V. ($1\ \text{k}\Omega = 1000\ \Omega$.)

5.* *a.* Use MATLAB to solve the following equations for x, y, and z as functions of the parameter c.

$$x - 5y - 2z = 11c$$
$$6x + 3y + z = 13c$$
$$7x + 3y - 5z = 10c$$

b. Plot the solutions for x, y, and z versus c on the same plot, for $-10 \leq c \leq 10$.

6. Fluid flows in pipe networks can be analyzed in a manner similar to that used for electric resistance networks. Figure P6 shows a network with three pipes. The volume flow rates in the pipes are q_1, q_2, and q_3. The pressures at the pipe ends are p_a, p_b, and p_c. The pressure at the junction is p_1. Under certain conditions, the pressure–flow rate relation in a pipe has the same form as the voltage-current relation in a resistor. Thus, for the three pipes, we have

$$q_1 = \frac{1}{R_1}(p_a - p_1)$$
$$q_2 = \frac{1}{R_2}(p_1 - p_b)$$
$$q_3 = \frac{1}{R_3}(p_1 - p_c)$$

where the R_i are the pipe resistances. From conservation of mass, $q_1 = q_2 + q_3$.

a. Set up these equations in a matrix form $\mathbf{Ax} = \mathbf{b}$ suitable for solving for the three flow rates q_1, q_2, and q_3 and the pressure p_1, given the values of pressures p_a, p_b, and p_c and the values of resistances R_1, R_2, and R_3. Find the expressions for **A** and **b.**

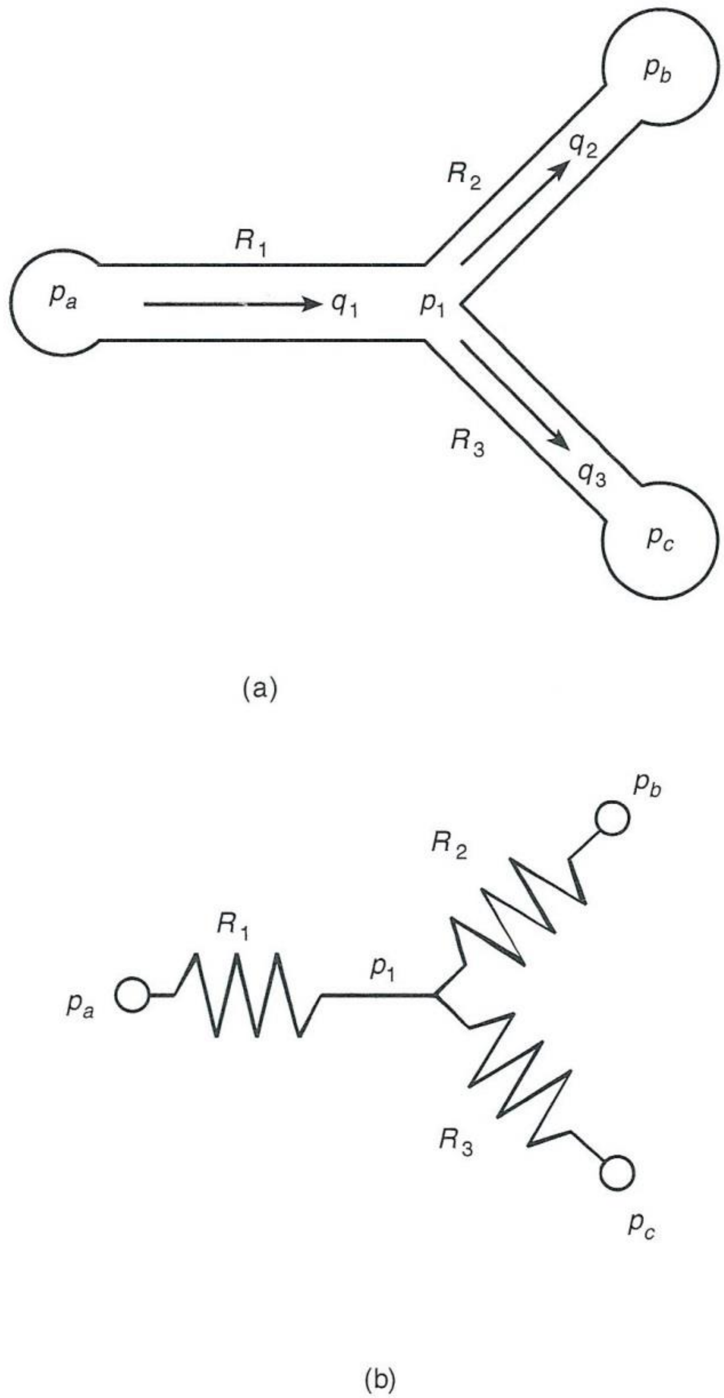

Figure P6

b. Use MATLAB to solve the matrix equations obtained in part *a* for the case where $p_a = 4320$ lb/ft^2, $p_b = 3600$ lb/ft^2, and $p_c = 2880$ lb/ft^2. These correspond to 30, 25, and 20 psi, respectively (1 psi = 1 lb/in^2, and atmospheric pressure is 14.7 psi). Use the resistance values $R_1 = 10{,}000$; $R_2 = R_3 = 14{,}000$ lb sec/ft^5. These values correspond to fuel oil flowing through pipes 2 ft long, with 2- and 1.4-in. diameters, respectively. The units of the answers are ft^3/sec for the flow rates and lb/ft^2 for pressure.

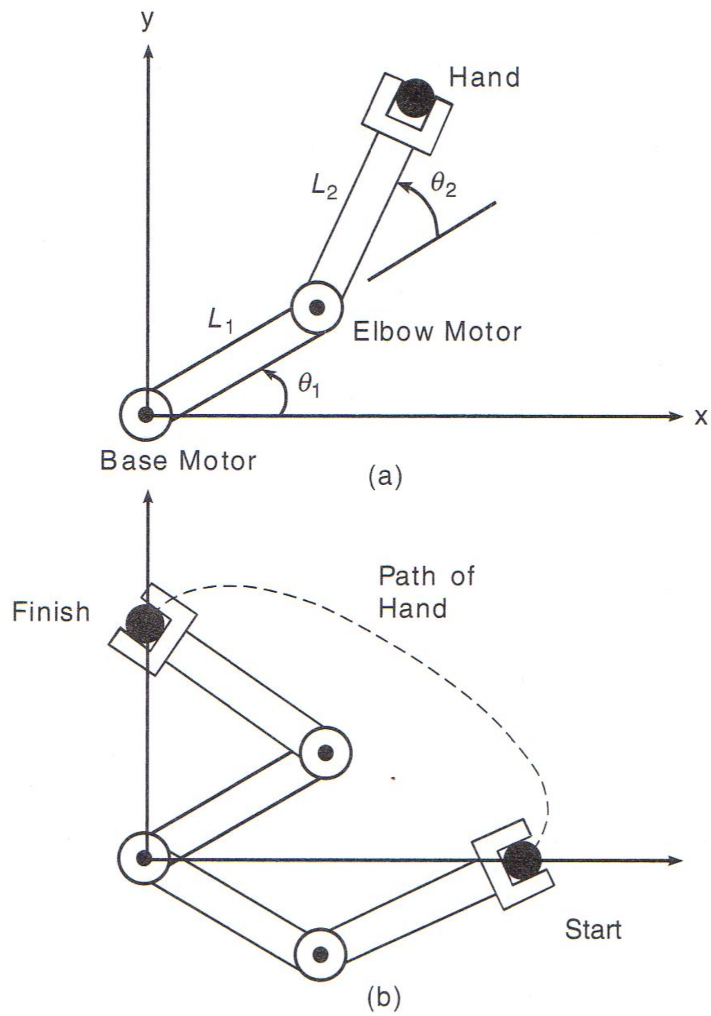

Figure P7

7. Figure P7 illustrates a robot arm that has two "links" connected by two "joints"—a shoulder or base joint and an elbow joint. There is a motor at each joint. The joint angles are θ_1 and θ_2. The (x, y) coordinates of the hand at the end of the arm are given by

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$
$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

where L_1 and L_2 are the lengths of the links.

Polynomials are used for controlling the motion of robots. If we start the arm from rest with zero velocity and acceleration, the following polynomials are used to generate commands to be sent to the joint motor controllers

$$\theta_1(t) = \theta_1(0) + a_1 t^3 + a_2 t^4 + a_3 t^5$$
$$\theta_2(t) = \theta_2(0) + b_1 t^3 + b_2 t^4 + b_3 t^5$$

where $\theta_1(0)$ and $\theta_2(0)$ are the starting values at time $t = 0$. The angles $\theta_1(t_f)$ and $\theta_2(t_f)$ are the joint angles corresponding to the desired destination of the arm at time t_f. The values of $\theta_1(0)$, $\theta_2(0)$, $\theta_1(t_f)$, and $\theta_2(t_f)$ can be found from trigonometry, if the starting and ending (x, y) coordinates of the hand are specified.

a. Set up a matrix equation to be solved for the coefficients a_1, a_2, and a_3, given values for $\theta_1(0)$, $\theta_1(t_f)$, and t_f. Obtain a similar equation for the coefficients b_1, b_2, and b_3.

b. Use MATLAB to solve for the polynomial coefficients given the values $t_f = 2$ sec, $\theta_1(0) = -19°$, $\theta_2(0) = 44°$, $\theta_1(t_f) = 43°$, and $\theta_2(t_f) = 151°$. (These values correspond to a starting hand location of $x = 6.5$, $y = 0$ ft and a destination location of $x = 0$, $y = 2$ ft for $L_1 = 4$ and $L_2 = 3$ ft.)

c. Use the results of part *b* to plot the path of the hand.

8.* Engineers must be able to predict the rate of heat loss through a building wall to determine the heating system requirements. They do this by using the concept of *thermal resistance R*, which relates the heat flow rate q through a material to the temperature difference ΔT across the material: $q = \Delta T/R$. This relation is like the voltage-current relation for an electric resistor: $i = v/R$. So the heat flow rate plays the role of electric current, and the temperature difference plays the role of the voltage difference. The SI unit for q is the *watt* (W), which is 1 joule/second (J/s).

The wall shown in Figure P8 consists of four layers: an inner layer of plaster/lathe 10 mm thick, a layer of fiberglass insulation 125 mm thick, a

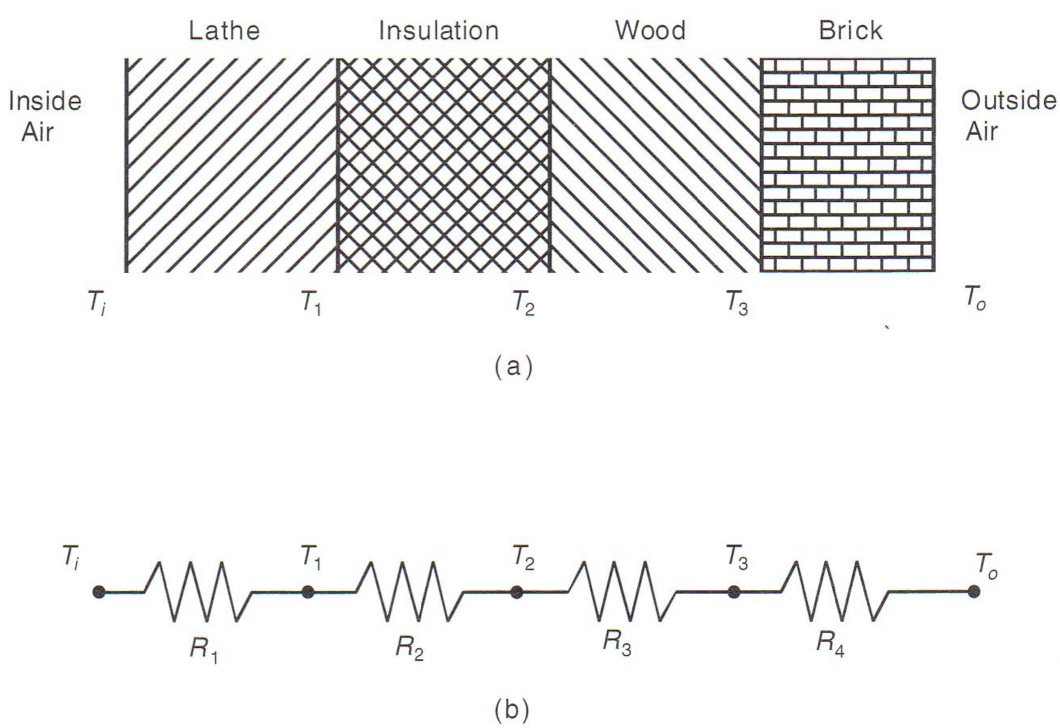

Figure P8

layer of wood 60 mm thick, and an outer layer of brick 50 mm thick. If we assume that the inner and outer temperatures T_i and T_o have remained constant for some time, then the heat energy stored in the layers is constant, and thus the heat flow rate through each layer is the same. Applying conservation of energy gives the following equations.

$$q = \frac{1}{R_1}(T_i - T_1) = \frac{1}{R_2}(T_1 - T_2) = \frac{1}{R_3}(T_2 - T_3) = \frac{1}{R_4}(T_3 - T_o)$$

The thermal resistance of a solid material is given by $R = D/k$, where D is the material thickness and k is the material's *thermal conductivity.* For the given materials, the resistances for a wall area of 1 m^2 are $R_1 = 0.036$, $R_2 = 4.01$, $R_3 = 0.408$, and $R_4 = 0.038$ K/W.

Suppose that $T_i = 20°C$ and $T_o = -10°C$. Find the other three temperatures and the heat loss rate q, in watts. Compute the heat loss rate if the wall's area is 10 m^2.

9. The concept of thermal resistance described in Problem 8 can be used to find the temperature distribution in the flat square plate shown in Figure P9(a).

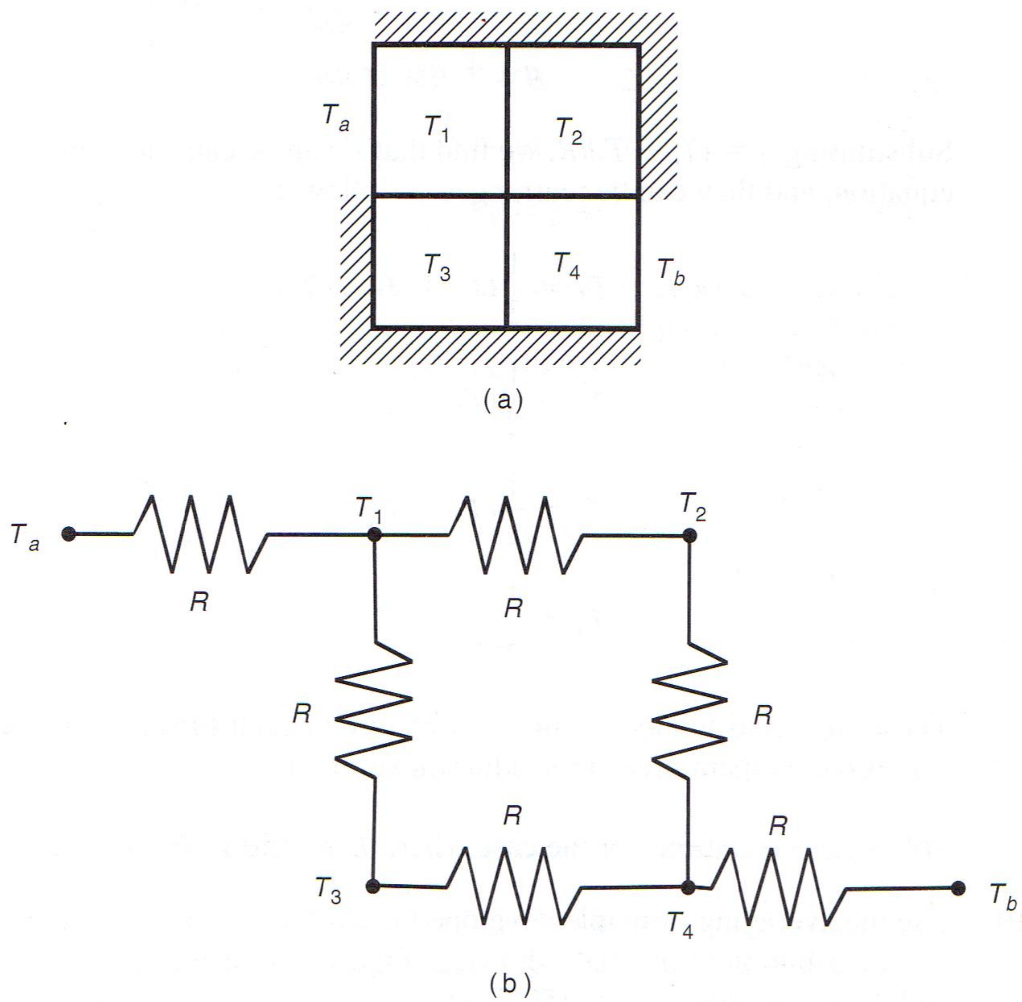

Figure P9

The plate's edges are insulated so that no heat can escape, except at two points where the edge temperature is heated to T_a and T_b, respectively. The temperature varies through the plate, so no single point can describe the plate's temperature. One way to estimate the temperature distribution is to imagine that the plate consists of four subsquares and to compute the temperature in each subsquare. Let R be the thermal resistance of the material between the centers of adjacent subsquares. Then we can think of the problem as a network of electric resistors, as shown in part (b) of the figure. Let q_{ij} be the heat flow rate between the points whose temperatures are T_i and T_j. If T_a and T_b remain constant for some time, then the heat energy stored in each subsquare is constant also, and the heat flow rate between each subsquare is constant. Under these conditions, conservation of energy says that the heat flow into a subsquare equals the heat flow out. Applying this principle to each subsquare gives the following equations.

$$\begin{aligned} q_{a1} &= q_{12} + q_{13} \\ q_{12} &= q_{24} \\ q_{13} &= q_{34} \\ q_{34} + q_{24} &= q_{4b} \end{aligned}$$

Substituting $q = (T_i - T_j)/R$, we find that R can be canceled out of every equation, and they can be rearranged as follows:

$$T_1 = \frac{1}{3}(T_a + T_2 + T_3)$$

$$T_2 = \frac{1}{2}(T_1 + T_4)$$

$$T_3 = \frac{1}{2}(T_1 + T_4)$$

$$T_4 = \frac{1}{3}(T_2 + T_3 + T_5)$$

These equations tell us that the temperature of each subsquare is the average of the temperatures in the adjacent subsquares!

Solve these equations for the case where $T_a = 150°\text{C}$ and $T_b = 20°\text{C}$.

10. Use the averaging principle developed in Problem 9 to find the temperature distribution of the plate shown in Figure P10, using the 3×3 grid and the given values $T_a = 150°\text{C}$ and $T_b = 20°\text{C}$.

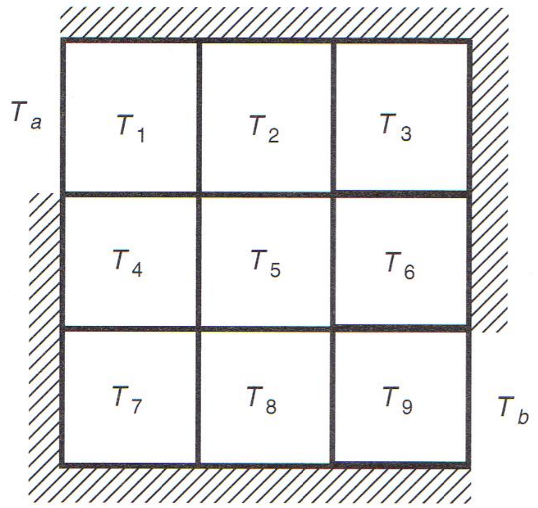

Figure P10

Section 8.3

11.* Solve the following equations:

$$7x + 9y - 9z = 22$$
$$3x + 2y - 4z = 12$$
$$x + 5y - z = -2$$

12. The following table shows how many hours in process reactors A and B are required to produce 1 ton each of chemical products 1, 2, and 3. The two reactors are available for 35 and 40 hrs per week, respectively.

Hours	Product 1	Product 2	Product 3
Reactor A	6	2	10
Reactor B	3	5	2

Let x, y, and z be the number of tons each of products 1, 2, and 3 that can be produced in one week.

a. Use the data in the table to write two equations in terms of x, y, and z. Determine whether a unique solution exists. If not, use MATLAB to find the relations between x, y, and z.

b. Note that negative values x, y, and z have no meaning here. Find the allowable ranges for x, y, and z.

c. Suppose the profits for each product are \$200, \$300, and \$100 for products 1, 2, and 3, respectively. Find the values of x, y, and z to maximize the profit.

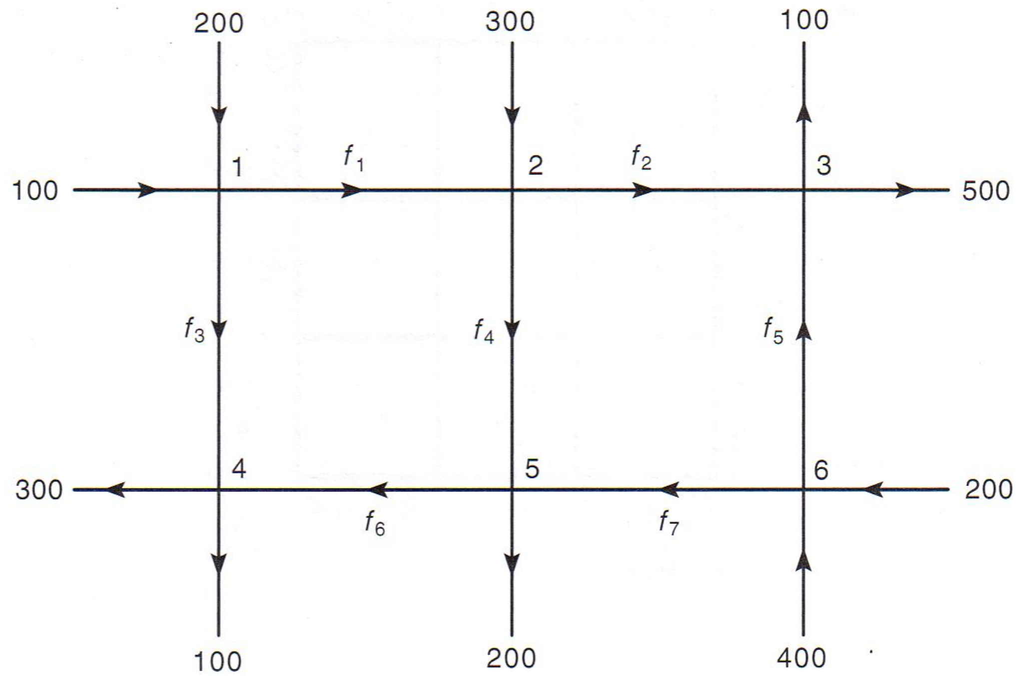

Figure P13

d. Suppose the profits for each product are \$200, \$500, and \$100 for products 1, 2, and 3, respectively. Find the values of x, y, and z to maximize the profit.

13. See Figure P13. Assume that no vehicles stop within the network. A traffic engineer wants to know if the traffic flows $f_1, f_2, \ldots, f_7$ (in vehicles per hour) can be computed given the measured flows shown in the figure. If not, then determine how many more traffic sensors need to be installed, and obtain the expressions for the other traffic flows in terms of the measured quantities.

Section 8.4

14.* Use MATLAB to solve the following problem:

$$\begin{aligned} x - 3y &= 2 \\ x + 5y &= 18 \\ 4x - 6y &= 20 \end{aligned}$$

15.* Use MATLAB to solve the following problem:

$$\begin{aligned} x - 3y &= 2 \\ x + 5y &= 18 \\ 4x - 6y &= 10 \end{aligned}$$

16. *a.* Use MATLAB to find the coefficients of the quadratic polynomial $y = ax^2 + bx + c$ that passes through the three points $(x, y) = (1, 4)$, $(4, 73)$, $(5, 120)$.

b. Use MATLAB to find the coefficients of the cubic polynomial $y = ax^3 + bx^2 + cx + d$ that passes through the three points given in part *a*.

17. Use the MATLAB program given in Table 8.5–2 to solve the following problems:

a. Problem 3d

b. Problem 11

c. Problem 14

d. Problem 15