ME 1020 Engineering Programming with MATLAB

Handout 09a

Homework 9a Assignment: 9.2, 9.4, 9.6, 9.7, 9.9, 9.18

Section 9.1

1.* An object moves at a velocity $v(t) = 5 + 7t^2$ m/s starting from the position $x(2) = 5$ m at $t = 2$ s. Determine its position at $t = 10$ s.

Problem 9.2: The total distance traveled by an object moving at velocity $v(t)$ from the time $t = a$ to the time $t = b$ is

$$x(b) = \int_a^b v(t)\,dt + x(a)$$

Suppose an object starts at time $t = 0$ and moves with a velocity of $v(t) = \cos(\pi t)$ m. Find the object's location at $t = 1$ second if $x(0) = 2$ m.

3. An object starts with an initial velocity of 3 m/s at $t = 0$, and it accelerates with an acceleration of $a(t) = 7t$ m/s^2. Find the total distance the object travels in 4 s.

Problem 9.4:

4. The equation for the voltage $v(t)$ across a capacitor as a function of time is

$$v(t) = \frac{1}{C}\int_0^t i(t)dt + Q_0$$

where $i(t)$ is the applied current and Q_0 is the initial charge. A certain capacitor has a capacitance of $C = 10^{-2}$ F. If a current of $i(t) = 0.2[1 + \sin(200t)]$ A is applied to the capacitor, compute and plot the current and the voltage as functions of time over the period $0 \le t \le 3\pi/400$ seconds if the initial charge on the capacitor is zero. Also, determine the voltage at $t = 3\pi/400$ seconds.

5. A certain object's acceleration is given by $a(t) = 7t \sin 5t$ m/s^2. Compute its velocity at $t = 10$ s if its initial velocity is zero.

6. A certain object moves with the velocity $v(t)$ given in the table below. Calculate the object's position $x(t)$ using numerical integration. The position of the object at the initial time is $x(t = 0) = 3.0$. Plot the object's velocity and position versus time t.

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/s)	0	2	5	7	9	12	15	18	22	20	17

7. A tank having vertical sides and a bottom area of 100 ft^2 is used to store water. The tank is initially empty. To fill the tank, water is pumped into the top at the rate given in the following table. Calculate the water height $h(t)$ by numerical integration. Plot the volumetric flow rate and water height versus time.

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Flow Rate (ft^3/min)	0	80	130	150	150	160	165	170	160	140	120

8. A cone-shaped paper drinking cup (like the kind supplied at water fountains) has a radius R and a height H. If the water height in the cup is h, the water volume is given by

$$V = \frac{1}{3}\pi\left(\frac{R}{H}\right)^2 h^3$$

Suppose that the cup's dimensions are $R = 1.5$ in. and $H = 4$ in.

a. If the flow rate from the fountain into the cup is 2 in.^{3}/s, how long will it take to fill the cup to the brim?

b. If the flow rate from the fountain into the cup is given by $2(1 - e^{-2t})$ in.^{3}/s, how long will it take to fill the cup to the brim?

9. A certain object has a mass of 100 kg and is acted on by a force (in Newtons):

$$f(t) = 500(2 - 5e^{-t})$$

The mass is a rest at $t = 0$. Calculate the object's velocity $v(t)$ using numerical integration and plot the velocity as a function of time.

10.* A rocket's mass decreases as it burns fuel. The equation of motion for a rocket in vertical flight can be obtained from Newton's law, and it is

$$m(t)\frac{dv}{dt} = T - m(t)g$$

where T is the rocket's thrust and its mass as a function of time is given by $m(t) = m_0(1 - rt/b)$. The rocket's initial mass is m_0, the burn time is b, and r is the fraction of the total mass accounted for by the fuel.

Use the values $T = 48{,}000$ N, $m_0 = 2200$ kg, $r = 0.8$, $g = 9.81$ m/s^2, and $b = 40$ s. Determine the rocket's velocity at burnout.

11. The equation for the voltage $v(t)$ across a capacitor as a function of time is

$$y(t) = \frac{1}{C}\left[\int_0^t i(t)\,dt + Q_0\right]$$

where $i(t)$ is the applied current and Q_0 is the initial charge. Suppose that $C = 10^{-7}$ F and that $Q_0 = 0$. Suppose the applied current is $i(t) = 0.3 + 0.1e^{-5t}\sin(25\pi t)$ A. Plot the voltage $v(t)$ for $0 \le t \le 7$ s.

12. Compute the indefinite integral of $p(x) = 5x^2 - 9x + 8$.

13. Compute the double integral

$$A = \int_0^3 \int_1^3 (x^2 + 3xy)\,dx\,dy$$

14. Compute the double integral

$$A = \int_0^4 \int_0^{\pi} x^2 \sin y\,dx\,dy$$

15. Compute the double integral

$$A = \int_0^1 \int_y^3 x^2 (x + y)\, dx\, dy$$

Note that the region of integration lies to the right of the line $y = x$. Use this fact and a MATLAB relational operator to eliminate values for which $y > x$.

16. Compute the triple integral

$$A = \int_1^2 \int_0^1 \int_1^3 x e^{yz}\, dx\, dy\, dz$$

Section 9.2

17. Plot the estimate of the derivative dy/dx from the following data. Do this by using forward, backward, and central differences. Compare the results.

x	0	1	2	3	4	5	6	7	8	9	10
y	0	2	5	7	9	12	15	18	22	20	17

18. At a relative maximum of a curve $y(x)$, the slope dy/dx is zero. Use the following data to estimate the values of x and y that correspond to a maximum point.

x	0	1	2	3	4	5	6	7	8	9	10
y	0	2	5	7	9	10	8	7	6	8	10

19. Compare the performance of the forward, backward, and central difference methods for estimating the derivative of $y(x) = e^{-x} \sin(3x)$. Use 101 points from $x = 0$ to $x = 4$. Use a random additive error of ± 0.01.

20. Compute the expressions for dp_2/dx, $d(p_1 p_2)/dx$, and $d(p_2/p_1)/dx$ for $p_1 = 5x^2 + 7$ and $p_2 = 5x^2 - 6x + 7$.

21. Plot the contour plot and the gradient (shown by arrows) for the function

$$f(x, y) = -x^2 + 2xy + 3y^2$$

Section 9.3

22. Plot the solution of the equation

$$6\dot{y} + y = f(t)$$

if $f(t) = 0$ for $t < 0$ and $f(t) = 15$ for $t \geq 0$. The initial condition is $y(0) = 7$.