ME 1020 Engineering Programming with MATLAB

Handout 09c

Homework 9c Assignment: 9.26, 9.31, 9.32, 9.36, 9.40

Topics Covered:

- Higher-Order Differential Equations
  - Euler Method
  - MATLAB ODE Solver ode45
  - ode45/Matrix Method
- Matrix Methods for Linear Equations
  - Free Response
  - Impulse Response
  - Step Response
  - Arbitrary Input Response

26. The following equation describes the motion of a mass connected to a spring, with viscous friction on the surface.

$$m\ddot{y} + c\dot{y} + ky = 0$$

Plot $y(t)$ for $y(0) = 10$, $\dot{y}(0) = 5$ if

a. $m = 3$, $c = 18$, and $k = 102$

b. $m = 3$, $c = 39$ and $k = 120$

Use the Euler Method to solve this problem.

30. The following equation describes the motion of a certain mass connected to a spring, with viscous friction on the surface

$$3\ddot{y} + 18\dot{y} + 102y = f(t)$$

where $f(t)$ is an applied force. Suppose that $f(t) = 0$ for $t < 0$ and $f(t) = 10$ for $t \geq 0$.

a. Plot $y(t)$ for $y(0) = \dot{y}(0) = 0$.

b. Plot $y(t)$ for $y(0) = 0$ and $\dot{y}(0) = 10$. Discuss the effect of the nonzero initial velocity.

31. The following equation describes the motion of a certain mass connected to a spring, with viscous friction on the surface

$$3\ddot{y} + 39\dot{y} + 120y = f(t)$$

where $f(t)$ is an applied force. Suppose that $f(t) = 0$ for $t < 0$ and $f(t) = 10$ for $t \geq 0$.

a. Plot $y(t)$ for $y(0) = \dot{y}(0) = 0$.

b. Plot $y(t)$ for $y(0) = 0$ and $\dot{y}(0) = 10$. Discuss the effect of the nonzero initial velocity.

Use the `ode45` Solver for this problem.

32. The following equation describes the motion of a certain mass connected to a spring, with no friction

$$3\ddot{y} + 75y = f(t)$$

where $f(t)$ is an applied force. Suppose the applied force is sinusoidal with a frequency of ω rad/s and an amplitude of 10 N: $f(t) = 10 \sin(\omega t)$. Suppose that the initial conditions are $y(0) = \dot{y}(0) = 0$. Plot $y(t)$ for $0 \leq t \leq 20$ s. Do this for the following three cases. Compare the results of each case.

a. $\omega = 1$ rad/s
b. $\omega = 5$ rad/s
c. $\omega = 10$ rad/s

Use the `ode45` Solver with Matrix Method for this problem.

33. Van der Pol's equation has been used to describe many oscillatory processes. It is

$$\ddot{y} - \mu(1 - y^2)\dot{y} + y = 0$$

Plot $y(t)$ for $\mu = 1$ and $0 \leq t \leq 20$, using the initial conditions $y(0) = 5$, $\dot{y}(0) = 0$.

34. The equation of motion for a pendulum whose base is accelerating horizontally with an acceleration $a(t)$ is

$$L\ddot{\theta} + g\sin\theta = a(t)\cos\theta$$

Suppose that $g = 9.81$ m/s^2, $L = 1$ m, and $\dot{\theta}(0) = 0$. Plot $\theta(t)$ for $0 \leq t \leq 10$ s for the following three cases.

a. The acceleration is constant: $a = 5$ m/s^2, and $\theta(0) = 0.5$ rad.
b. The acceleration is constant: $a = 5$ m/s^2, and $\theta(0) = 3$ rad.
c. The acceleration is linear with time: $a = 0.5t$ m/s^2, and $\theta(0) = 3$ rad.

36. The equations for an armature-controlled dc motor are the following. The motor's current is i and its rotational velocity is ω.

$$L\frac{di}{dt} = -Ri - K_e\omega + v(t) \qquad (9.6\text{–}1)$$

$$I\frac{d\omega}{dt} = K_T i - c\omega \qquad (9.6\text{–}2)$$

where L, R, and I are the motor's inductance, resistance, and inertia; K_T and K_e are the torque constant and back emf constant; c is a viscous damping constant; and $v(t)$ is the applied voltage.

Use the values $R = 0.8\ \Omega$, $L = 0.003$ H, $K_T = 0.05$ N · m/A, $K_e = 0.05$ V · s/rad, $c = 0$, and $I = 8 \times 10^{-5}$ kg · m^2.

a. Suppose the applied voltage is 20 V. Plot the motor's speed and current versus time. Choose a final time large enough to show the motor's speed becoming constant.
b. Suppose the applied voltage is trapezoidal as given below.

$$v(t) = \begin{cases} 400t & 0 \leq t < 0.05 \\ 20 & 0.05 \leq t \leq 0.2 \\ -400(t - 0.2) + 20 & 0.2 < t \leq 0.25 \\ 0 & t > 0.25 \end{cases}$$

Plot the motor's speed versus time for $0 \leq t \leq 0.3$ s. Also plot the applied voltage versus time. How well does the motor speed follow a trapezoidal profile?

<u>Use the Control System Toolbox for this problem.</u>

37. Compute and plot the unit-impulse response of the following model.

$$10\ddot{y} + 3\dot{y} + 7y = f(t)$$

38. Compute and plot the unit-step response of the following model.

$$10\ddot{y} + 6\dot{y} + 2y = f + 7\dot{f}$$

39.* Find the reduced form of the following state model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u(t)$$

40. The following state model describes the motion of a certain mass connected to a spring, with viscous friction on the surface, where $m = 1$, $c = 2$, and $k = 5$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t)$$

a. Use the `initial` function to plot the position x_1 of the mass, if the initial position is 5 and the initial velocity is 3.

b. Use the `step` function to plot the step response of the position and velocity for zero initial conditions, where the magnitude of the step input is 10. Compare your plot with that shown in Figure 9.5–1.

<u>Use the Control System Toolbox for this problem.</u>

41. Consider the following equation.

$$5\ddot{y} + 2\dot{y} + 10y = f(t)$$

a. Plot the free response for the initial conditions $y(0) = 10$, $\dot{y}(0) = -5$.

b. Plot the unit-step response (for zero initial conditions).

42. The model for the *RC* circuit shown in Figure P42 is

$$RC\frac{dv_o}{dt} + v_o = v_i$$

For $RC = 0.2$ s, plot the voltage response $v_o(t)$ for the case where the applied voltage is a single square pulse of height 10 V and duration 0.4 s, starting at $t = 0$. The initial capacitor voltage is zero.

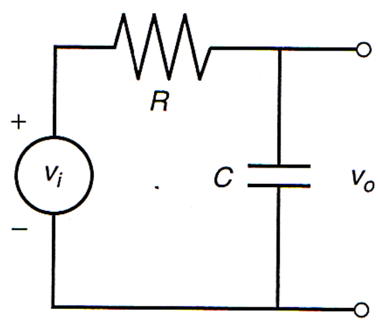

Figure P42