ME 1020 Engineering Programming with MATLAB

Handout 10

Chapter 10 Homework: 10.5, 10.8, 10.10, 10.12, 10.16, 10.19, 10.23, 10.30

1. Draw a simulation diagram for the following equation.

$$\dot{y} = 5f(t) - 7y$$

2. Draw a simulation diagram for the following equation.

$$5\ddot{y} = 3\dot{y} + 7y = f(t)$$

3. Draw a simulation diagram for the following equation.

$$3\dot{y} + 5 \sin y = f(t)$$

4. Create a Simulink model to plot the solution of the following equation for $0 \le t \le 6$.

$$10\ddot{y} = 7 \sin 4t + 5 \cos 3t \quad y(0) = 3 \quad \dot{y}(0) = 2$$

5. A projectile is launched with a velocity of 100 m/s at an angle of 30° above the horizontal. Create a Simulink model to solve the projectile's equations of motion where x and y are the horizontal and vertical displacements of the projectile.

$$\ddot{x} = 0 \qquad x(0) = 0 \quad \dot{x}(0) = 100 \cos 30^\circ$$
$$\ddot{y} = -g \quad y(0) = 0 \quad \dot{y}(0) = 100 \sin 30^\circ$$

Use the model to plot the projectile's trajectory y versus x for $0 \le t \le 10$ s.

6. The following equation has no analytical solution even though it is linear.

$$\dot{x} + x = \tan t \qquad x(0) = 0$$

The approximate solution, which is less accurate for large values of t, is

$$x(t) = \frac{1}{3}t^3 - t^2 + 3t - 3 + 3e^{-t}$$

Create a Simulink model to solve this problem, and compare its solution with the approximate solution over the range $0 \le t \le 1$.

7. Construct a Simulink model to plot the solution of the following equation for $0 \le t \le 10$

$$15\dot{x} + 5x = 4u_s(t) - 4u_s(t-2) \qquad x(0) = 5$$

where $u_s(t)$ is a unit-step function (in the Block Parameters window of the Step block, set the Step time to 0, the Initial value to 0, and the Final value to 1).

8. A tank having vertical sides and a bottom area of 100 ft^2 is used to store water. To fill the tank, water is pumped into the top at the rate given in the following table. Use Simulink to solve for and plot the water height $h(t)$ for $0 \le t \le 10$ min.

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Flow Rate (ft^3/min)	0	80	130	150	150	160	165	170	160	140	120

Volume as a function of tank height:

$$V = hA$$

Take the derivative of both sides (A = constant):

$$\frac{dV}{dt} = \frac{dh}{dt}A$$

Solve for the rate of change of the height of liquid in the tank:

$$\frac{dh}{dt} = \left(\frac{1}{A}\right)\frac{dV}{dt}$$

$$\dot{h} = \left(\frac{1}{A}\right)\dot{V}$$

where $\dot{V}$ is the volumetric flow rate of liquid into the tank.

9. Construct a Simulink model to plot the solution of the following equations for $0 \le t \le 2$

$$\dot{x}_1 = -6x_1 + 4x_2$$

$$\dot{x}_2 = 5x_1 - 7x_2 + f(t)$$

where $f(t) = 3t$. Use the Ramp block in the Sources library.

10. Construct a Simulink model to plot the solution of the following equations for $0 \le t \le 3$

$$\dot{x}_1 = -6x_1 + 4x_2 + f_1(t)$$

$$\dot{x}_2 = 5x_1 - 7x_2 + f_2(t)$$

where $f_1(t)$ is a step function of height 3 starting at $t = 0$ and $f_2(t)$ is a step function of height -3 starting at $t = 1$.

Use the State-Space approach:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -6 & 4 \\ 5 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

11. Use the Saturation block to create a Simulink model to plot the solution of the following equation for $0 \le t \le 6$.

$$3\dot{y} + y = f(t) \qquad y(0) = 3$$

where

$$f(t) = \begin{cases} 8 & \text{if } 10 \sin 3t > 8 \\ -8 & \text{if } 10 \sin 3t < -8 \\ 10 \sin 3t & \text{otherwise} \end{cases}$$

12. Construct a Simulink model of the following problem.

$$5\dot{x} + \sin x = f(t) \qquad x(0) = 0$$

The forcing function is

$$f(t) = \begin{cases} -5 & \text{if } g(t) \le -5 \\ g(t) & \text{if } -5 \le g(t) \le 5 \\ 5 & \text{if } g(t) \ge 5 \end{cases}$$

where $g(t) = 10 \sin 4t$.

13. If a mass-spring system has Coulomb friction on the surface rather than viscous friction, its equation of motion is

$$m\ddot{y} = \begin{cases} -ky + f(t) - \mu mg & \text{if } \dot{y} \geq 0 \\ -ky + f(t) + \mu mg & \text{if } \dot{y} < 0 \end{cases}$$

where μ is the coefficient of friction. Develop a Simulink model for the case where $m = 1\text{kg}$, $k = 5$ N/m, $\mu = 0.4$, and $g = 9.8$ m/s^2. Run the simulation for two cases: (a) the applied force $f(t)$ is a step function with a magnitude of 10 N and (b) the applied force is sinusoidal: $f(t) = 10 \sin 2.5t$. Either the Sign block in the Math Operations library or the Coulomb and Viscous Friction block in the Discontinuities library can be used, but since there is no viscous friction in this problem, the Sign block is easier to use.

14. A certain mass, $m = 2$ kg, moves on a surface inclined at an angle $\phi = 30°$ above the horizontal. Its initial velocity is $v(0) = 3$ m/s up the incline. An external force of $f_1 = 5$ N acts on it parallel to and up the incline. The coefficient of Coulomb friction is $\mu = 0.5$. Use the Sign block and create a Simulink model to solve for the velocity of the mass until the mass comes to rest. Use the model to determine the time at which the mass comes to rest.

15. *a.* Develop a Simulink model of a thermostatic control system in which the temperature model is

$$RC\frac{dT}{dt} + T = Rq + T_a(t)$$

where T is the room air temperature in °F, T_a is the ambient (outside) air temperature in °F, time t is measured in hours, q is the input from the heating system in lb · ft/hr, R is the thermal resistance, and C is the thermal capacitance. The thermostat switches q on at the value q_{max} whenever the temperature drops below 69°F and switches q to $q = 0$ whenever the temperature is above 71°F. The value of q_{max} indicates the heat output of the heating system.

Run the simulation for the case where $T(0) = 70°F$, and $T_a(t) = 50 + 10 \sin(\pi t/12)$. Use the values $R = 5 \times 10^{-5}$ °F · hr/lb · ft and $C = 4 \times 10^4$ lb · ft/°F. Plot the temperatures T and T_a versus t on the same graph, for $0 \le t \le 24$ hr. Do this for two cases: $q_{max} = 4 \times 10^5$ and $q_{max} = 8 \times 10^5$ lb · ft/hr. Investigate the effectiveness of each case.

b. The integral of q over time is the energy used. Plot $\int q\,dt$ versus t and determine how much energy is used in 24 hr for the case where $q_{max} = 8 \times 10^5$.

16. Refer to Problem 15. Use the simulation with $q_{max} = 8 \times 10^5$ to compare the energy consumption and the thermostat cycling frequency for the two temperature bands (69°, 71°) and (68°, 72°).

17. Consider the liquid-level system shown in Figure 10.7–1. The governing equation based on conservation of mass is Equation (10.7–2). Suppose that the height h is controlled by using a relay to switch the input flow rate between the values 0 and 50 kg/s. The flow rate is switched on when the height is less than 4.5 m and is switched off when the height reaches 5.5 m. Create a Simulink model for this application using the values $A = 2$ m^2, $R = 400$ m^{-1} · s^{-1}, $\rho = 1000$ kg/m^3, and $h(0) = 1$ m. Obtain a plot of $h(t)$.

18. Use the Transfer Function block to construct a Simulink model to plot the solution of the following equation for $0 \le t \le 4$.

$$2\ddot{x} + 12\dot{x} + 10x = 5u_s(t) - 5u_s(t-2) \qquad x(0) = \dot{x}(0) = 0$$

19. Use Transfer Function blocks to construct a Simulink model to plot the solution of the following equations for $0 \le t \le 2$.

$$3\ddot{x} + 15\dot{x} + 18x = f(t) \qquad x(0) = \dot{x}(0) = 0$$

$$2\ddot{y} + 16\dot{y} + 50y = x(t) \qquad y(0) = \dot{y}(0) = 0$$

where $f(t) = 75u_s(t)$.

20. Use Transfer Function blocks to construct a Simulink model to plot the solution of the following equations for $0 \le t \le 2$

$$3\ddot{x} + 15\dot{x} + 18x = f(t) \quad x(0) = \dot{x}(0) = 0$$

$$2\ddot{y} + 16\dot{y} + 50y = x(t) \quad y(0) = \dot{y}(0) = 0$$

where $f(t) = 50u_s(t)$. At the output of the first block there is a dead zone for $-1 \le x \le 1$. This limits the input to the second block.

21. Use Transfer Function blocks to construct a Simulink model to plot the solution of the following equations for $0 \le t \le 2$

$$3\ddot{x} + 15\dot{x} + 18x = f(t) \quad x(0) = \dot{x}(0) = 0$$

$$2\ddot{y} + 16\dot{y} + 50y = x(t) \quad y(0) = \dot{y}(0) = 0$$

where $f(t) = 50u_s(t)$. At the output of the first block there is a saturation that limits x to be $|x| \le 1$. This limits the input to the second block.

22. Construct a Simulink model to plot the solution of the following equation for $0 \le t \le 4$.

$$2\ddot{x} + 12\dot{x} + 10x^2 = 8 \sin 0.8t \quad x(0) = \dot{x}(0) = 0$$

23. Create a Simulink model to plot the solution of the following equation for $0 \le t \le 3$.

$$\dot{x} + 10x^2 = 5 \sin 3t \quad x(0) = 1$$

24. Construct a Simulink model of the following problem.

$$10\dot{x} + \sin x = f(t) \quad x(0) = 0$$

The forcing function is $f(t) = \sin 2t$. The system has the dead-zone nonlinearity shown in Figure 10.5–1.

25. The following model describes a mass supported by a nonlinear, hardening spring. The units are SI. Use $g = 9.81\ \text{m/s}^2$.

$$5\ddot{y} = 5g - (900y + 1700y^3) \qquad y(0) = 0.5 \qquad \dot{y}(0) = 0$$

Create a Simulink model to plot the solution for $0 \le t \le 2$.

26. Consider the system for lifting a mast shown in Figure P26. The 70-ft-long mast weighs 500 lb. The winch applies a force $f = 380$ lb to the cable. The mast is supported initially at an angle of 30°, and the cable at A is initially horizontal. The equation of motion of the mast is

$$25\,400\,\ddot{\theta} = -17\,500\cos\theta + \frac{626\,000}{Q}\sin(1.33 + \theta)$$

where

$$Q = \sqrt{2020 + 1650\cos(1.33 + \theta)}$$

Create and run a Simulink model to solve for and plot $\theta(t)$ for $\theta(t) \le \pi/2$ rad.

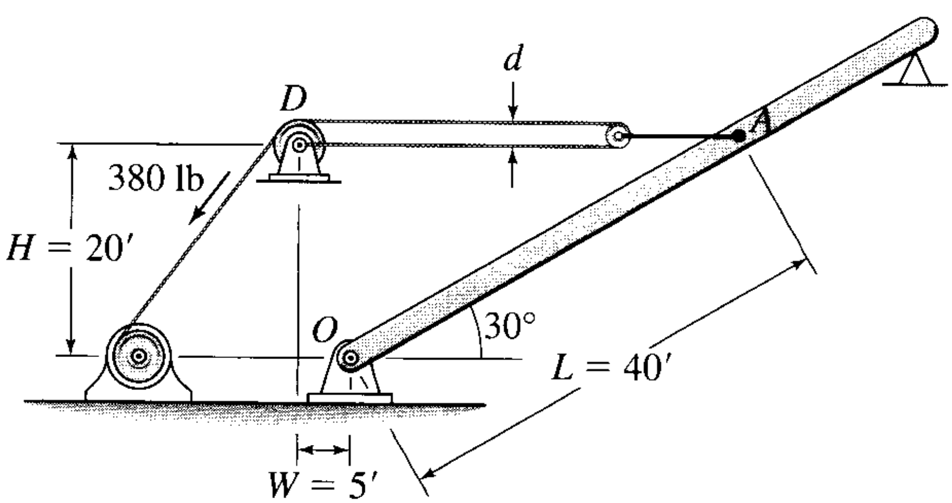


Figure P26

27. The equation describing the water height h in a spherical tank with a drain at the bottom is

$$\pi(2rh - h^2)\frac{dh}{dt} = -C_d A\sqrt{2gh}$$

Suppose that the tank's radius is $r = 3$ m and the circular drain hole of area A has a radius of 2 cm. Assume that $C_d = 0.5$ and that the initial water height is $h(0) = 5$ m. Use $g = 9.81$ m/s^2. Use Simulink to solve the nonlinear equation, and plot the water height as a function of time until $h(t) = 0$.

28. A cone-shaped paper drinking cup (like the kind used at water fountains) has a radius R and a height H. If the water height in the cup is h, the water volume is given by

$$V = \frac{1}{3}\pi\left(\frac{R}{H}\right)^2 h^3$$

Suppose that the cup's dimensions are $R = 1.5$ in. and $H = 4$ in.

a. If the flow rate from the fountain into the cup is 2 in.^{3}/sec, use Simulink to determine how long will it take to fill the cup to the brim.

b. If the flow rate from the fountain into the cup is given by $2(1 - e^{-2t})$ in.^{3}/sec, use Simulink to determine how long will it take to fill the cup to the brim.

29. Refer to Figure 10.7–2. Assume that the resistances obey the linear relation, so that the mass flow q_l through the left-hand resistance is $q_l = (p_l - p)/R_l$, with a similar linear relation for the right-hand resistance.

a. Create a Simulink subsystem block for this element.

b. Use the subsystem block to create a Simulink model of the system shown in Figure 10.7–5. Assume that the mass inflow rate is a step function.

c. Use the Simulink model to obtain plots of $h_1(t)$ and $h_2(t)$ for the following parameter values: $A_1 = 2$ m^2, $A_2 = 5$ m^2, $R_1 = 400$ m$^{-1} \cdot$ s^{-1} $R_2 = 600$ m$^{-1} \cdot$ s^{-1}, $\rho = 1000$ kg/m^3, $q_{mi} = 50$ kg/s, $h_1(0) = 1.5$ m, and $h_2(0) = 0.5$ m.

30. *a.* Use the subsystem block developed in Section 10.7 to construct a Simulink model of the system shown in Figure P30. The mass inflow rate is a step function.

b. Use the Simulink model to obtain plots of $h_1(t)$ and $h_2(t)$ for the following parameter values: $A_1 = 3$ ft^2, $A_2 = 5$ ft^2, $R_1 = 30$ ft$^{-1} \cdot$ sec^{-1}, $R_2 = 40$ ft$^{-1} \cdot$ sec^{-1}, $\rho = 1.94$ slug/ft^3, $q_{mi} = 0.5$ slug/sec, $h_1(0) = 2$ ft, and $h_2(0) = 5$ ft.

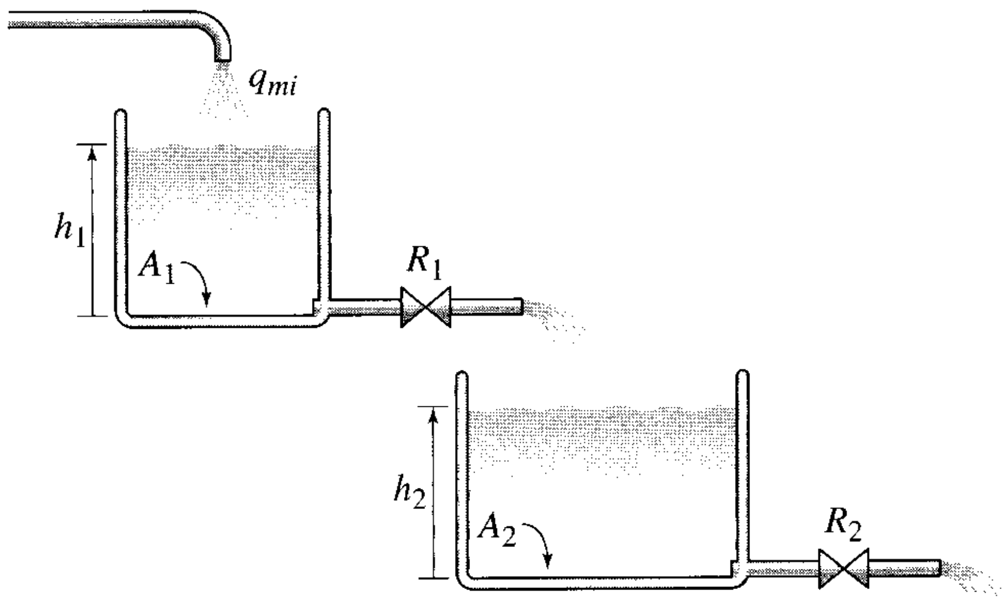


Figure P30

31. Consider Figure 10.7–7 for the case where there are three *RC* loops with the values $R_1 = R_3 = 10^4\ \Omega$, $R_2 = 5 \times 10^4\ \Omega$, $C_1 = C_3 = 10^{-6}$ F, and $C_2 = 4 \times 10^{-6}$ F.

a. Develop a subsystem block for one *RC* loop.

b. Use the subsystem block to construct a Simulink model of the entire system of three loops. Plot $v_3(t)$ over $0 \leq t \leq 3$ for $v_1(t) = 12 \sin 10t$ V.

32. Consider Figure 10.7–8 for the case where there are three masses. Use the values $m_1 = m_3 = 10$ kg, $m_2 = 30$ kg, $k_1 = k_4 = 10^4$ N/m, and $k_2 = k_3 = 2 \times 10^4$ N/m.

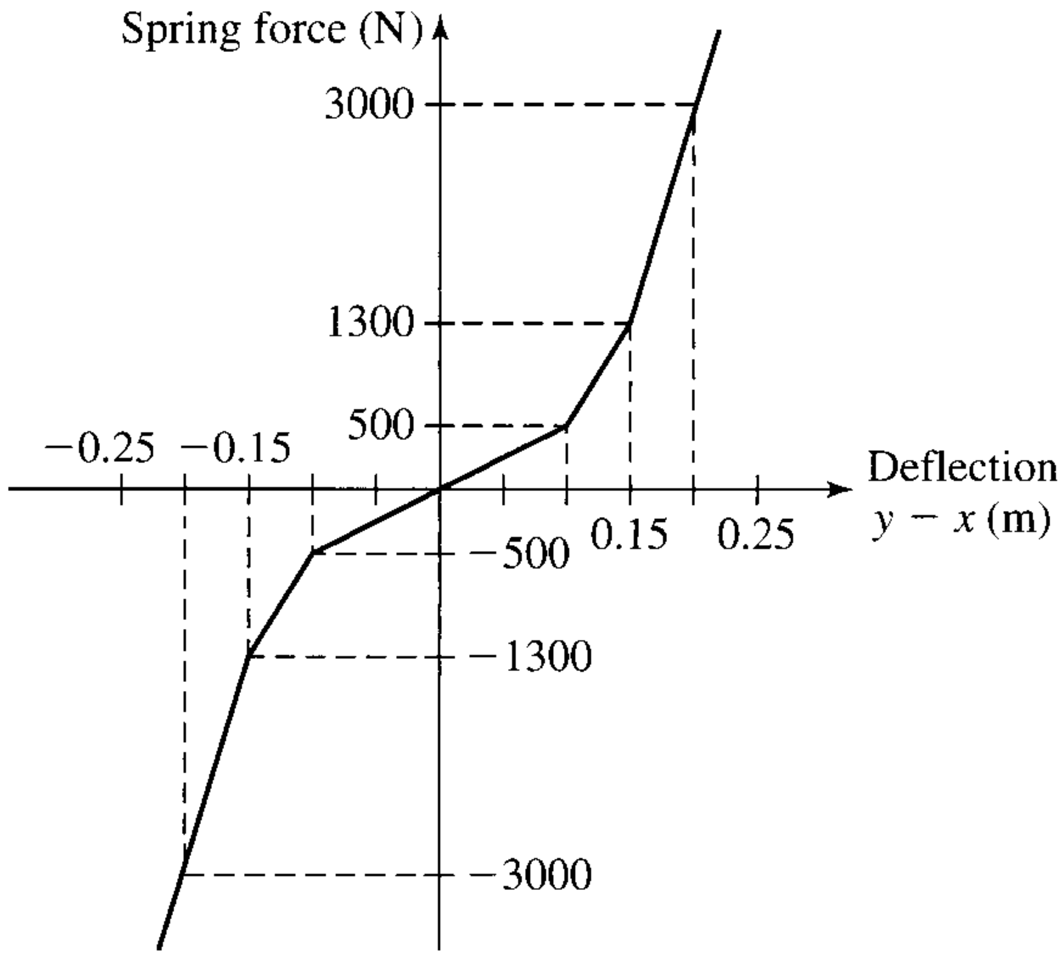
a. Develop a subsystem block for one mass.

b. Use the subsystem block to construct a Simulink model of the entire system of three masses. Plot the displacements of the masses over $0 \leq t \leq 2$ s for if the initial displacement of m_1 is 0.1 m.

33. Refer to Figure P30. Suppose there is a dead time of 10 sec between the outflow of the top tank and the lower tank. Use the subsystem block developed in Section 10.7 to create a Simulink model of this system. Using the parameters given in Problem 30, plot the heights h_1 and h_2 versus time.

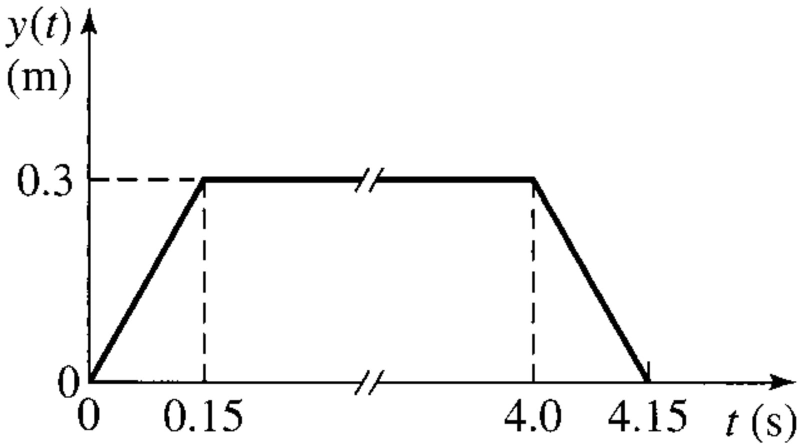
34. Redo the Simulink suspension model developed in Section 10.9, using the spring relation and input function shown in Figure P34 and the following damper relation.

$$f_d(v) = \begin{cases} -500|v|^{1.2} & v \le 0 \\ 50v^{1.2} & v > 0 \end{cases}$$



(a)

Figure P34



(b)

Use the simulation to plot the response. Evaluate the overshoot and undershoot.

35. Consider the system shown in Figure P35. The equations of motion are

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 - c_2\dot{x}_2 - k_2x_2 = 0$$
$$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - c_2\dot{x}_1 - k_2x_1 = f(t)$$

Suppose that $m_1 = m_2 = 1$, $c_1 = 3$, $c_2 = 1$, $k_1 = 1$, and $k_2 = 4$.

a. Develop a Simulink model of this system. In doing this, consider whether to use a state-variable representation or a transfer-function representation of the model.

b. Use the Simulink model to plot the response $x_1(t)$ for the following input. The initial conditions are zero.

$$f(t) = \begin{cases} t & 0 \le t \le 1 \\ 2 - t & 1 < t < 2 \\ 0 & t \ge 2 \end{cases}$$

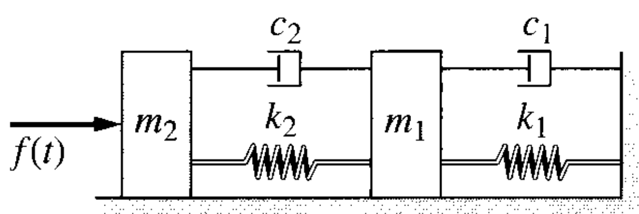


Figure P35