ME 1020 Engineering Programming with MATLAB

Chapter 8 Homework Solutions: 8.1, 8.3, 8.8, 8.10, 8.12, 8.14, 8.16

Problem 8.1:

1. Solve the following problems using matrix inversion. Check your solutions by computing $\mathbf{A}^{-1}\mathbf{A}$.

a.
$$\begin{aligned} 2x + y &= 5 \\ 3x - 9y &= 7 \end{aligned}$$

b.
$$\begin{aligned} -8x - 5y &= 4 \\ -2x + 7y &= 10 \end{aligned}$$

c.
$$\begin{aligned} 12x - 5y &= 11 \\ -3x + 4y + 7x_3 &= -3 \\ 6x + 2y + 3x_3 &= 22 \end{aligned}$$

d.
$$\begin{aligned} 6x - 3y + 4x_3 &= 41 \\ 12x + 5y - 7x_3 &= -26 \\ -5x + 2y + 6x_3 &= 16 \end{aligned}$$

problem8_1.m

```
% Problem 8.1
clear
clc
disp('Problem 8.1: Scott Thomas')

disp('Part a:')
A = [2 1; 3 -9]
b = [5; 7]
x = inv(A)*b
checka = A*inv(A)

disp('Part b:')
A = [-8 -5; -2 7]
b = [4; 10]
x = inv(A)*b
checkb = A*inv(A)

disp('Part c:')
A = [12 -5 0; -3 4 7; 6 2 3]
b = [11; -3; 22]
x = inv(A)*b
checkc = A*inv(A)

disp('Part d:')
A = [6 -3 4; 12 5 -7; -5 2 6]
b = [41; -26; 16]
x = inv(A)*b
checkd = A*inv(A)
```

```
Command Window
Problem 8.1: Scott Thomas
Part a:

A =

     2     1
     3    -9


b =

     5
     7


x =

    2.4762
    0.0476


checka =

     1     0
     0     1
```

```
Command Window

Part b:

A =

    -8    -5
    -2     7


b =

     4
    10


x =

   -1.1818
    1.0909


checkb =

     1     0
     0     1
```

```
Command Window
Part c:

A =

    12    -5     0
    -3     4     7
     6     2     3


b =

    11
    -3
    22


x =

    3.0000
    5.0000
   -2.0000


checkc =

    1.0000    0.0000   -0.0000
   -0.0000    1.0000    0.0000
    0.0000         0    1.0000
```

Command Window

```
Part d:

A =

     6    -3     4
    12     5    -7
    -5     2     6


b =

    41
   -26
    16


x =

    2.0035
   -2.6848
    5.2312


checkd =

    1.0000         0   -0.0000
   -0.0000    1.0000   -0.0000
    0.0000   -0.0000    1.0000
```

fx

Problem 8.3:

3. Use MATLAB to solve the following problems.

a.
$$-2x + y = -5$$
$$-2x + y = 3$$

b.
$$-2x + y = 3$$
$$-8x + 4y = 12$$

c.
$$-2x + y = -5$$
$$-2x + y = -5.00001$$

d.
$$x_1 + 5x_2 - x_3 + 6x_4 = 19$$
$$2x_1 - x_2 + x_3 - 2x_4 = 7$$
$$-x_1 + 4x_2 - x_3 + 3x_4 = 30$$
$$3x_1 - 7x_2 - 2x_3 + x_4 = -75$$

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problem8_3.m

```
% Problem 8.3
clear
clc
disp('Problem 8.3: Scott Thomas')

disp('Part a: Parallel lines do not intersect, so no solution')

A = [-2 1; -2 1]
b = [-5; 3]
x = inv(A)*b

disp('Part b: Two equations are the same (Second equation = 4*first equation)')

A = [-2 1; -8 4]
b = [3; 12]
x = inv(A)*b

disp('Part c: Parallel lines do not intersect, so no solution')

A = [-2 1; -2 1]
b = [-5; -5.00001]
x = inv(A)*b

disp('Part d:')

A = [1 5 -1 6; 2 -1 1 -2; -1 4 -1 3; 3 -7 -2 1]
b = [19; 7; 30; -75;]
x = inv(A)*b

A_times_x = A*x
```

Command Window

```
Problem 8.3: Scott Thomas
Part a: Parallel lines do not intersect, so no solution

A =

    -2     1
    -2     1


b =

    -5
     3

Warning: Matrix is singular to working precision.
> In problem8_3 at 10

x =

   NaN
   NaN
```

```
Part b: Two equations are the same (Second equation = 4*first equation)

A =

    -2     1
    -8     4


b =

     3
    12

Warning: Matrix is singular to working precision.
> In problem8_3 at 16

x =

   Inf
   Inf
```

```
Part c: Parallel lines do not intersect, so no solution

A =

    -2     1
    -2     1


b =

   -5.0000
   -5.0000

Warning: Matrix is singular to working precision.
> In problem8_3 at 22

x =

  -Inf
  -Inf
```

```
Part d:

A =

     1     5    -1     6
     2    -1     1    -2
    -1     4    -1     3
     3    -7    -2     1


b =

    19
     7
    30
   -75


x =

    5.0000
   14.6250
  -12.1250
  -11.8750


A_times_x =

   19.0000
    7.0000
   30.0000
  -75.0000
```

Problem 8.8:

8.* Engineers must be able to predict the rate of heat loss through a building wall to determine the heating system requirements. They do this by using the concept of *thermal resistance* R, which relates the heat flow rate q through a material to the temperature difference ΔT across the material: $q = \Delta T/R$. This relation is like the voltage-current relation for an electric resistor: $i = v/R$. So the heat flow rate plays the role of electric current, and the temperature difference plays the role of the voltage difference. The SI unit for q is the *watt* (W), which is 1 joule/second (J/s).

The wall shown in Figure P8 consists of four layers: an inner layer of plaster/lathe 10 mm thick, a layer of fiberglass insulation 125 mm thick, a layer of wood 60 mm thick, and an outer layer of brick 50 mm thick. If we assume that the inner and outer temperatures T_i and T_o have remained constant for some time, then the heat energy stored in the layers is constant, and thus the heat flow rate through each layer is the same. Applying conservation of energy gives the following equations.

$$q = \frac{1}{R_1}(T_i - T_1) = \frac{1}{R_2}(T_1 - T_2) = \frac{1}{R_3}(T_2 - T_3) = \frac{1}{R_4}(T_3 - T_o)$$

The thermal resistance of a solid material is given by $R = D/k$, where D is the material thickness and k is the material's *thermal conductivity.* For the given materials, the resistances for a wall area of 1 m^2 are $R_1 = 0.036$, $R_2 = 4.01$, $R_3 = 0.408$, and $R_4 = 0.038$ K/W.

Suppose that $T_i = 20°C$ and $T_o = -10°C$. Find the other three temperatures and the heat loss rate q, in watts. Compute the heat loss rate if the wall's area is 10 m^2.

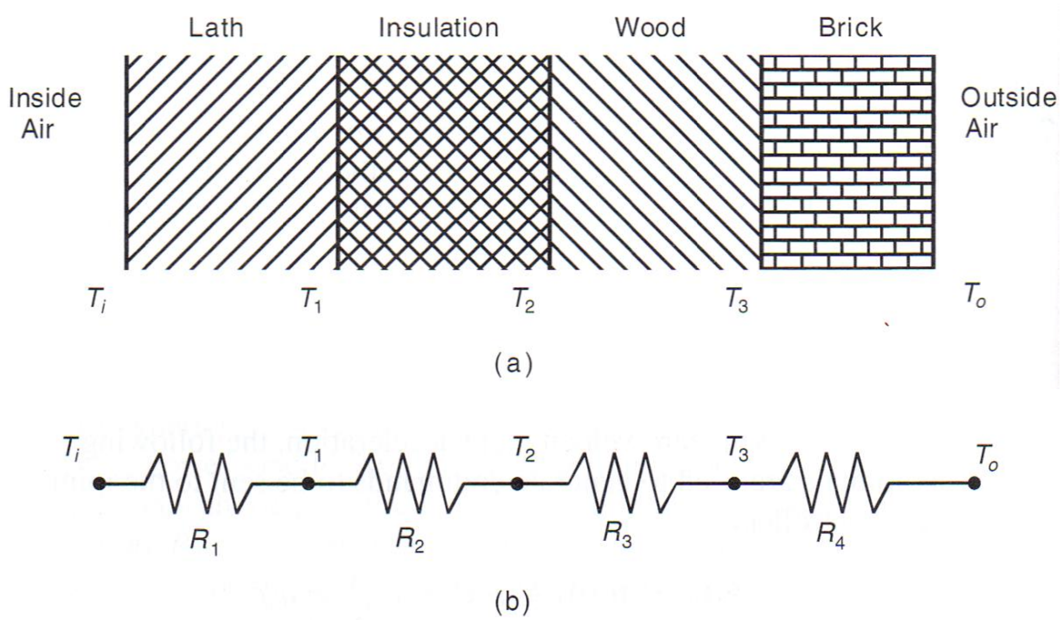

Figure P8

Problem setup:

The unknowns are $T_1, T_2, T_{3,}$ and q. Cast the equations above in terms of these unknowns.

$$(R_1)q + (1)T_1 + (0)T_2 + (0)T_3 = T_i$$

$$(R_2)q + (-1)T_1 + (1)T_2 + (0)T_3 = 0$$

$$(R_3)q + (0)T_1 + (-1)T_2 + (1)T_3 = 0$$

$$(R_4)q + (0)T_1 + (0)T_2 + (-1)T_3 = -T_o$$

$$x^T = [q \quad T_1 \quad T_2 \quad T_3]$$

$$B^T = [T_i \quad 0 \quad 0 \; -T_o]$$

$$R_1 = 0.036; \quad R_2 = 4.01; \quad R_3 = 0.408; \quad R_4 = 0.038$$

$$T_i = 20; \quad T_o = -10$$

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problem8_8.m ×

```
% Problem 8.8
clear
clc
disp('Problem 8.8: Scott Thomas')

R1 = 0.036;
R2 = 4.01;
R3 = 0.408;
R4 = 0.038;
Ti = 20;
To = -10;

A = [ R1     1     0     0; ...
      R2    -1     1     0; ...
      R3     0    -1     1;...
      R4     0     0    -1]
b = [Ti; 0; 0; -To]
A_inv = inv(A)
x = inv(A)*b
check = A_inv*A
```

Command Window

```
Problem 8.8: Scott Thomas

A =

    0.0360    1.0000         0         0
    4.0100   -1.0000    1.0000         0
    0.4080         0   -1.0000    1.0000
    0.0380         0         0   -1.0000


b =

    20
     0
     0
    10


A_inv =

    0.2226    0.2226    0.2226    0.2226
    0.9920   -0.0080   -0.0080   -0.0080
    0.0993    0.0993   -0.9007   -0.9007
    0.0085    0.0085    0.0085   -0.9915


x =

    6.6785
   19.7596
   -7.0214
   -9.7462


check =

    1.0000         0         0         0
    0.0000    1.0000         0         0
    0.0000         0    1.0000         0
         0    0.0000         0    1.0000
```

Problem 8.10:

10. Use the averaging principle developed in Problem 9 to find the temperature distribution of the plate shown in Figure P10, using the 3 × 3 grid and the given values $T_a = 150°C$ and $T_b = 20°C$.

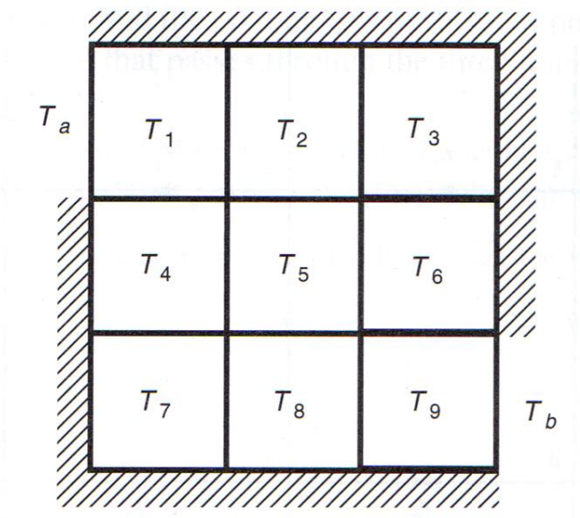
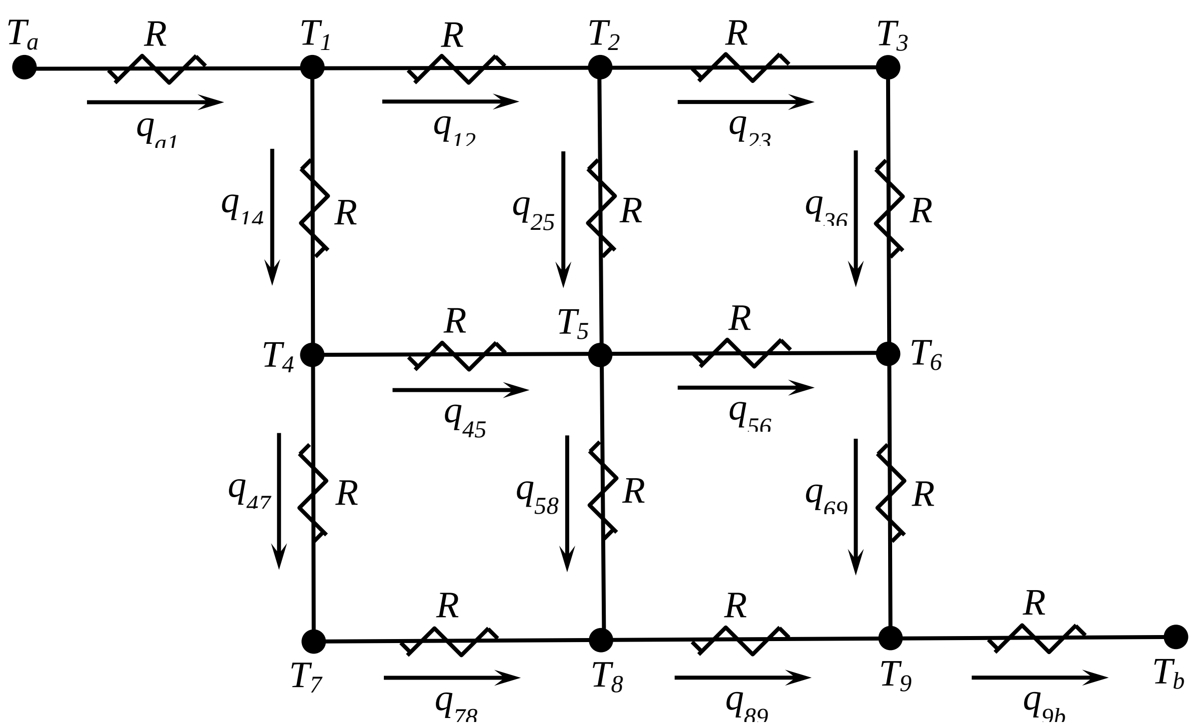


Figure P10

Problem setup: Assign the directions of the heat fluxes arbitrarily, but be consistent when deriving the equations.



Heat in = Heat out: Heat balance equations.

$$q_{a1} = q_{12} + q_{14}$$

$$q_{12} = q_{23} + q_{25}$$

$$q_{23} = q_{36}$$

$$q_{14} = q_{47} + q_{45}$$

$$q_{45} + q_{25} = q_{56} + q_{58}$$

$$q_{36} + q_{56} = q_{69}$$

$$q_{47} = q_{78}$$

$$q_{78} + q_{58} = q_{89}$$

$$q_{69} + q_{89} = q_{9b}$$

Fourier's Law of Heat Conduction: Heat transfer is proportional to the temperature difference.

$$q_{a1} = \frac{1}{R}(T_a - T_1)$$

$$q_{12} = \frac{1}{R}(T_1 - T_2)$$

$$q_{23} = \frac{1}{R}(T_2 - T_3)$$

$$q_{14} = \frac{1}{R}(T_1 - T_4)$$

$$q_{45} = \frac{1}{R}(T_4 - T_5)$$

$$q_{25} = \frac{1}{R}(T_2 - T_5)$$

$$q_{56} = \frac{1}{R}(T_5 - T_6)$$

$$q_{36} = \frac{1}{R}(T_3 - T_6)$$

$$q_{47} = \frac{1}{R}(T_4 - T_7)$$

$$q_{78} = \frac{1}{R}(T_7 - T_8)$$

$$q_{58} = \frac{1}{R}(T_5 - T_8)$$

$$q_{89} = \frac{1}{R}(T_8 - T_9)$$

$$q_{69} = \frac{1}{R}(T_6 - T_9)$$

$$q_{9b} = \frac{1}{R}(T_9 - T_b)$$

Substitute the q expressions into the heat balance equations. The interior temperatures are the unknowns.

$$(3)T_1 + (-1)T_2 + (0)T_3 + (-1)T_4 + (0)T_5 + (0)T_6 + (0)T_7 + (0)T_8 + (0)T_9 = T_a$$

$$(-1)T_1 + (3)T_2 + (-1)T_3 + (0)T_4 + (-1)T_5 + (0)T_6 + (0)T_7 + (0)T_8 + (0)T_9 = 0$$

$$(0)T_1 + (-1)T_2 + (2)T_3 + (0)T_4 + (0)T_5 + (-1)T_6 + (0)T_7 + (0)T_8 + (0)T_9 = 0$$

$$(-1)T_1 + (0)T_2 + (0)T_3 + (3)T_4 + (-1)T_5 + (0)T_6 + (-1)T_7 + (0)T_8 + (0)T_9 = 0$$

$$(0)T_1 + (-1)T_2 + (0)T_3 + (-1)T_4 + (4)T_5 + (-1)T_6 + (0)T_7 + (-1)T_8 + (0)T_9 = 0$$

$$(0)T_1 + (0)T_2 + (-1)T_3 + (0)T_4 + (-1)T_5 + (3)T_6 + (0)T_7 + (0)T_8 + (-1)T_9 = 0$$

$$(0)T_1 + (0)T_2 + (0)T_3 + (-1)T_4 + (0)T_5 + (0)T_6 + (2)T_7 + (-1)T_8 + (0)T_9 = 0$$

$$(0)T_1 + (0)T_2 + (0)T_3 + (0)T_4 + (-1)T_5 + (0)T_6 + (-1)T_7 + (3)T_8 + (-1)T_9 = 0$$

$$(0)T_1 + (0)T_2 + (0)T_3 + (0)T_4 + (0)T_5 + (-1)T_6 + (0)T_7 + (-1)T_8 + (3)T_9 = T_b$$

$$x^T = [T_1 \;\; T_2 \;\; T_3 \;\; T_4 \;\; T_5 \;\; T_6 \;\; T_7 \;\; T_8 \;\; T_9]$$

$$T_a = 150; \quad T_b = 20$$

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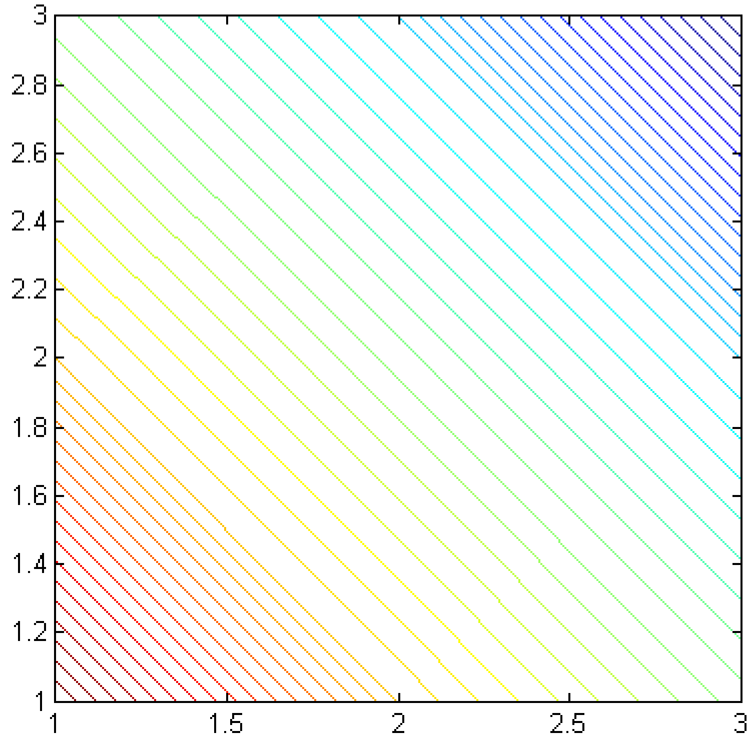
problem8_10.m*

```matlab
% Problem 8.10
clear
clc
disp('Problem 8.10: Scott Thomas')

Ta = 150;
Tb = 20;

A = [ 3.0   -1.0    0.0   -1.0    0.0    0.0    0.0    0.0    0.0; ...
     -1.0    3.0   -1.0    0.0   -1.0    0.0    0.0    0.0    0.0; ...
      0.0   -1.0    2.0    0.0    0.0   -1.0    0.0    0.0    0.0;...
     -1.0    0.0    0.0    3.0   -1.0    0.0   -1.0    0.0    0.0;...
      0.0   -1.0    0.0   -1.0    4.0   -1.0    0.0   -1.0    0.0;...
      0.0    0.0   -1.0    0.0   -1.0    3.0    0.0    0.0   -1.0;...
      0.0    0.0    0.0   -1.0    0.0    0.0    2.0   -1.0    0.0;...
      0.0    0.0    0.0    0.0   -1.0    0.0   -1.0    3.0   -1.0;...
      0.0    0.0    0.0    0.0    0.0   -1.0    0.0   -1.0    3.0]
b = [Ta; 0; 0; 0; 0; 0; 0; 0; Tb]

x = A\b

x1 = [x(1) x(2) x(3); x(4) x(5) x(6); x(7) x(8) x(9); ]
y = 1:3
z = 1:3

contour(y,z,x1,50), axis square
```

Command Window

```
Problem 8.10: Scott Thomas

A =

     3    -1     0    -1     0     0     0     0     0
    -1     3    -1     0    -1     0     0     0     0
     0    -1     2     0     0    -1     0     0     0
    -1     0     0     3    -1     0    -1     0     0
     0    -1     0    -1     4    -1     0    -1     0
     0     0    -1     0    -1     3     0     0    -1
     0     0     0    -1     0     0     2    -1     0
     0     0     0     0    -1     0    -1     3    -1
     0     0     0     0     0    -1     0    -1     3


b =

   150
     0
     0
     0
     0
     0
     0
     0
    20
```

```
x =

  112.8571
   94.2857
   85.0000
   94.2857
   85.0000
   75.7143
   85.0000
   75.7143
   57.1429


x1 =

  112.8571   94.2857   85.0000
   94.2857   85.0000   75.7143
   85.0000   75.7143   57.1429


y =

     1     2     3


z =

     1     2     3
```



3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1
1.5
2
2.5
3

Problem 8.12:

12. The following table shows how many hours in process reactors A and B are required to produce 1 ton each of chemical products 1, 2, and 3. The two reactors are available for 35 and 40 hrs per week, respectively.

Hours	Product 1	Product 2	Product 3
Reactor A	6	2	10
Reactor B	3	5	2

Let *x*, *y*, and *z* be the number of tons each of products 1, 2, and 3 that can be produced in one week.

a. Use the data in the table to write two equations in terms of *x*, *y*, and *z*. Determine whether a unique solution exists. If not, use MATLAB to find the relations between *x*, *y*, and *z*.

b. Note that negative values *x*, *y*, and *z* have no meaning here. Find the allowable ranges for *x*, *y*, and *z*.

c. Suppose the profits for each product are \$200, \$300, and \$100 for products 1, 2, and 3, respectively. Find the values of *x*, *y*, and *z* to maximize the profit.

d. Suppose the profits for each product are \$200, \$500, and \$100 for products 1, 2, and 3, respectively. Find the values of *x*, *y*, and *z* to maximize the profit.

Problem setup:

Reactor A:

$$\left(6\ \frac{\text{hr}}{\text{ton}}\right)\left(x\ \frac{\text{ton}}{\text{wk}}\right)+\left(2\ \frac{\text{hr}}{\text{ton}}\right)\left(y\ \frac{\text{ton}}{\text{wk}}\right)+\left(10\ \frac{\text{hr}}{\text{ton}}\right)\left(z\ \frac{\text{ton}}{\text{wk}}\right)=35\ \frac{\text{hr}}{\text{wk}}$$

Reactor B:

$$\left(3\ \frac{\text{hr}}{\text{ton}}\right)\left(x\ \frac{\text{ton}}{\text{wk}}\right)+\left(5\ \frac{\text{hr}}{\text{ton}}\right)\left(y\ \frac{\text{ton}}{\text{wk}}\right)+\left(2\ \frac{\text{hr}}{\text{ton}}\right)\left(z\ \frac{\text{ton}}{\text{wk}}\right)=40\ \frac{\text{hr}}{\text{wk}}$$

$$6x+2y+10z=35$$

$$3x+5y+2z=40$$

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problem8_10.m | problem8_12.m

```
% Problem 8.12
clear
clc
disp('Problem 8.12: Scott Thomas')

A = [6 2 10; 3 5 2]
b = [35; 40]

%x1 = inv(A)*b
% Using inv(A)*b, the following error is given:
%Error using inv
%Matrix must be square.

x2 = A\b
% Using A\b, the result is x = [0; 7.1739; 2.0652], which is a solution in
% which one of the unknowns is set to zero. This is not the general
% solution.

RankA = rank(A)
RankAb = rank([A b])

%since rank(A) = rank([A b])= 2, a solution exists, but the system is
% underdetermined. This means it has an infinite number of solutions.
% Use rref([A b]) to get the reduced equation set.
x3 = rref([A b])

x4 = pinv(A)*b
% Using pinv(A)*b gives one solution where the norm is minimized.
% x = [2.7255; 6.1074; 0.6432]
```

Command Window

```
Problem 8.12: Scott Thomas

A =

     6     2    10
     3     5     2


b =

    35
    40


x2 =

         0
    7.1739
    2.0652

RankA =

     2


RankAb =

     2


x3 =

    1.0000         0    1.9167    3.9583
         0    1.0000   -0.7500    5.6250


x4 =

    2.7255
    6.1074
    0.6432
```

Discussion of results:

The row-reduced echelon form is:

$$(1)x + (0)y + 1.9176z = 3.9583$$

$$(0)x + (1)y - 0.7500z = 5.6250$$

These equations can be reduced:

$$x = -1.9176z + 3.9583$$

$$y = 0.7500z + 5.6250$$

For this problem, we must have $x \geq 0,\ y \geq 0,$ and $z \geq 0$.

$$\text{for } z \geq 0,\ \ x \leq 3.9583$$

$$\text{for } z \geq 0,\ \ y \geq 5.6250$$

$$\text{for } x \geq 0,\ \ -1.9176z + 3.9583 \geq 0 \ \text{ or } \ z \leq 2.0648$$

$$\text{for } y \geq 0,\ \ 0.7500z + 5.6250 \geq 0 \ \text{ or } \ z \geq -7.5 \ \text{ (not a valid solution)}$$

Ranges of variables:

$$0 \leq x \leq 3.9583$$

$$y \geq 5.6250$$

$$0 \leq z \leq 2.0648$$

Part c): Profit analysis:

$$P = 200x + 300y + 100z$$

$$P = 200(-1.9176z + 3.9583) + 300(0.7500z + 5.6250) + 100z$$

$$P = 2479 - 58.52z$$

For maximum profit, set $z = 0$. This gives $P = 2479$.

$$x = 3.9583$$

$$y = 5.6250$$

Part d): Profit analysis:

$$P = 200x + 500y + 100z$$

$$P = 200(-1.9176z + 3.9583) + 500(0.7500z + 5.6250) + 100z$$

$$P = 3604 + 91.48z$$

For maximum profit, set z to its maximum value: $z = 2.0648$. This gives

$$P = 3604 + 91.48(2.0648) = 3793$$

$$x = -1.9176(2.0648) + 3.9583 = 0$$

$$y = 0.7500(2.0648) + 5.6250 = 7.1736$$

Problem 8.14:

14.* Use MATLAB to solve the following problem:

$$\begin{aligned} x - 3y &= 2 \\ x + 5y &= 18 \\ 4x - 6y &= 20 \end{aligned}$$

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problem8_14.m

```
% Problem 8.14
clear
clc
disp('Problem 8.14: Scott Thomas')

A = [ 1 -3; 1 5; 4 -6]
b = [2; 18; 20]

%x1 = inv(A)*b
% Using inv(A)*b, the following error is given:
%Error using inv
%Matrix must be square.

RankA = rank(A)
RankAb = rank([A b])

% The system is overdetermined since the # equations > # unknowns.
% Since rank(A) = rank([A b])= 2, a solution exists.
% Since rank(A) = # of unknowns, a unique solution exists.
% Use x2 = A\b to compute the solution.

x2 = A\b
```

Command Window

```
Problem 8.14: Scott Thomas

A =

     1    -3
     1     5
     4    -6


b =

     2
    18
    20


RankA =

     2


RankAb =

     2


x2 =

    8.0000
    2.0000
```

Problem 8.16:

16. *a.* Use MATLAB to find the coefficients of the quadratic polynomial $y = ax^2 + bx + c$ that passes through the three points $(x, y) = (1, 4)$, $(4, 73)$, $(5, 120)$.

b. Use MATLAB to find the coefficients of the cubic polynomial $y = ax^3 + bx^2 + cx + d$ that passes through the three points given in part *a*.

Problem setup:

Part a):

$$y = ax^2 + bx + c$$

$$\text{Point 1: } (x, y) = (1,4)\text{:} \quad 4 = a(1)^2 + b(1) + c; \quad (1)a + (1)b + (1)c = 4$$

$$\text{Point 2: } (x, y) = (4,73)\text{:} \quad 73 = a(4)^2 + b(4) + c; \quad (16)a + (4)b + (1)c = 73$$

$$\text{Point 3: } (x, y) = (5,120)\text{:} \quad 120 = a(5)^2 + b(5) + c; \quad (25)a + (5)b + (1)c = 120$$

$$x^T = [a \quad b \quad c]$$

Part b):

$$y = ax^3 + bx^2 + cx + d$$

$$\text{Point 1: } (x, y) = (1,4)\text{:} \quad 4 = a(1)^3 + b(1)^2 + c(1) + d; \quad (1)a + (1)b + (1)c + (1)d = 4$$

$$\text{Point 2: } (x, y) = (4,73)\text{:} \quad 73 = a(4)^3 + b(4)^2 + c(4) + d; \quad (64)a + (16)b + (4)c + (1)d = 73$$

$$\text{Point 3: } (x, y) = (5,120)\text{:} \quad 120 = a(5)^3 + b(5)^2 + c(5) + d; \quad (125)a + (25)b + (5)c + (1)d = 120$$

$$x^T = [a \quad b \quad c]$$

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problem8_16.m

```
% Problem 8.16
clear
clc
disp('Problem 8.16: Scott Thomas')

disp('Part a):' )

A = [ 1 1 1; 16 4 1; 25 5 1]
b = [4; 73; 120]

% The system is overdetermined since the # equations > # unknowns.

RankA = rank(A)
RankAb = rank([A b])

% Since rank(A) = 3 and rank([A b])= 3, a unique solution exists.
% The left division method can be used to compute the solution.

x1 = A\b

disp('Part b):' )

A = [ 1 1 1 1; 64 16 4 1; 125 25 5 1]
b = [4; 73; 120]

% The system is underdetermined since the # equations < # unknowns.

RankA = rank(A)
RankAb = rank([A b])

% Since rank(A) = 3 and rank([A b])= 3, a unique solution exists.
% The row reduced echelong form method can be used to compute the solution
% in terms of d.

x1 = rref([A,b])

% The left division method can be used to compute the least squares
% solution.

x2 = A\b
```

Command Window

```
Problem 8.16: Scott Thomas
Part a):

A =

     1     1     1
    16     4     1
    25     5     1


b =

     4
    73
   120


RankA =

     3


RankAb =

     3


x1 =

    6.0000
   -7.0000
    5.0000
```

```
Part b):

A =

     1     1     1     1
    64    16     4     1
   125    25     5     1


b =

     4
    73
   120


RankA =

     3


RankAb =

     3


x1 =

    1.0000         0         0    0.0500    0.2500
         0    1.0000         0   -0.5000    3.5000
         0         0    1.0000    1.4500    0.2500

x2 =

    0.2414
    3.5862
         0
    0.1724
```

The exact solution (in terms of *d*) is:

$$(1)a + (0)b + (0)c + (0.05)d = 0.25$$

$$(0)a + (1)b + (0)c + (0.05)d = 3.5$$

$$(0)a + (0)b + (1)c + (1.45)d = 0.25$$

$$a + 0.05d = 0.25 \text{ or } a = -0.05d + 0.25$$

$$b + 0.05d = 3.5 \text{ or } b = -0.05d + 3.5$$

$$c + 1.45d = 0.25 \text{ or } c = -1.45d + 0.25$$

$$y = (-0.05d + 0.25)x^3 + (-0.05d + 3.5)x^2 + (-1.45d + 0.25)x + d$$

The least squares solution is (inexact solution):

$$y = (0.2414)x^3 + (3.5862)x^2 + (0)x + 0.1724$$