ME 1020 Engineering Programming with MATLAB

Chapter 9a Homework Solutions: 9.2, 9.4, 9.6, 9.7, 9.9, 9.18

Problem 9.2: The total distance traveled by an object moving at velocity $v(t)$ from the time $t = a$ to the time $t = b$ is

$$x(b) = \int_a^b v(t)\,dt + x(a)$$

Suppose an object starts at time $t = 0$ and moves with a velocity of $v(t) = \cos(\pi t)$ m. Find the object's location at $t = 1$ second if $x(0) = 2$ m.

The total distance traveled by an object moving at velocity $v(t)$ from time $t = a$ to $t = b$ is:

$$x(b) = \int_a^b v(t)dt + x(a)$$

For this problem, $a = 0,\ b = 1$ second. The position at $a = 0$ seconds is $x(t = 0) = 2.0$ meters:

$$x(1) = \int_0^1 v(t)dt + x(0)$$

$$x(1) = \int_0^1 \cos(\pi t)\,dt + 2$$

This function can be integrated directly:

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C$$

$$x(1) = \left[\left(\frac{1}{\pi}\right)\sin(\pi t)\right]_0^1 + 2$$

$$x(1) = \left(\frac{1}{\pi}\right)\{\sin(\pi \cdot 1) - \sin(\pi \cdot 0)\} + 2$$

$$x(1) = \left(\frac{1}{\pi}\right)(0 - 0) + 2 = 2.0$$

The distance traveled in the positive x direction is cancelled by the distance traveled in the negative x direction, so the object ends up where it started from.

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problem9_2.m*

```matlab
% Problem 9.2
clear
clc
disp('Problem 9.2: Scott Thomas')

N = 100
t = linspace(0,1.0,N);
v = cos(pi*t);
plot(t,v), xlabel('t (sec)'), ylabel('v (m/s)')
title('Problem 9.2: Scott Thomas')

sum = 2.0;
for k = 1:N-1
    sum = sum + 1/2*(t(k+1) - t(k))*(v(k+1) + v(k));
end
format long
sum
```

Command Window

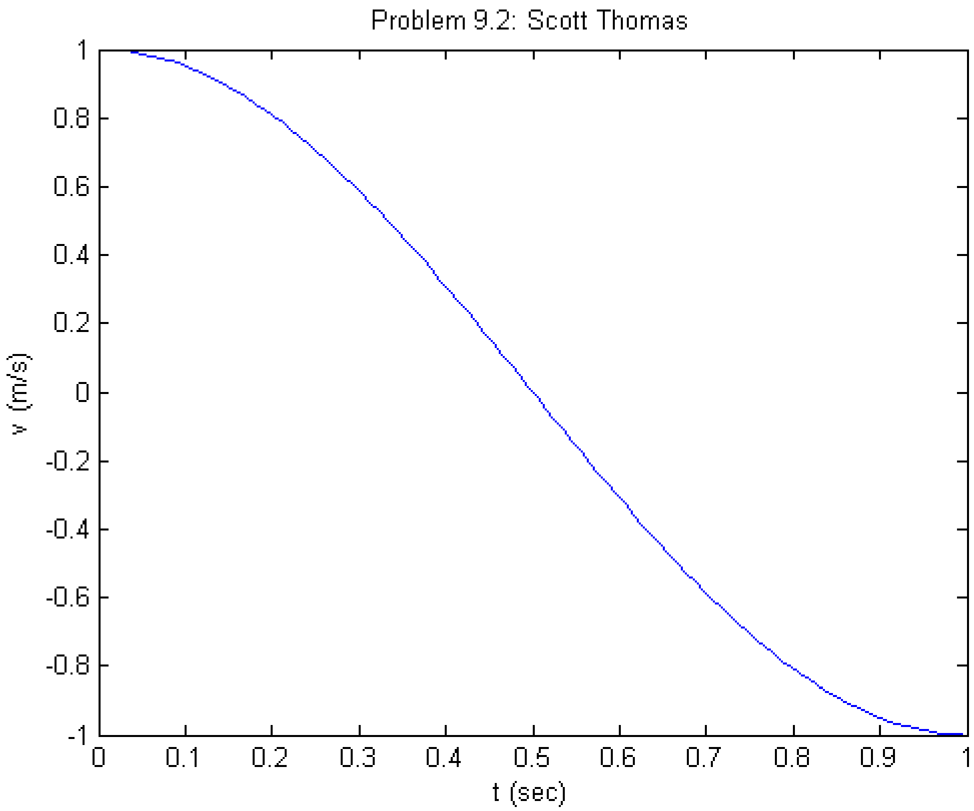
```
Problem 9.2: Scott Thomas

N =

   100


sum =

   1.999999999999999
```


Problem 9.2: Scott Thomas
v (m/s)
t (sec)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Problem 9.4:

4. The equation for the voltage $v(t)$ across a capacitor as a function of time is

$$v(t) = \frac{1}{C}\int_0^t i(t)dt + Q_0$$

where $i(t)$ is the applied current and Q_0 is the initial charge. A certain capacitor has a capacitance of $C = 10^{-2}$ F. If a current of $i(t) = 0.2[1 + \sin(200t)]$ A is applied to the capacitor, compute and plot the current and the voltage as functions of time over the period $0 \le t \le 3\pi/400$ seconds if the initial charge on the capacitor is zero ($Q_0 = 0$). Also, determine the voltage at $t = 3\pi/400$ seconds.

Problem setup:

From Wikipedia,

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax + C$$

$$v(t) = \frac{1}{C}\int_0^t i(t)dt + Q_0 = \frac{1}{0.01}\int_0^t 0.2[1 + \sin(200t)]dt + 0$$

$$v(t) = 20\left[t - \frac{1}{200}\cos(200t)\right]_0^t$$

$$v(t) = 20\left\{\left[t - \frac{1}{200}\cos(200t)\right] - \left[(0) - \frac{1}{200}\cos(200\cdot 0)\right]\right\}$$

$$v(t) = 20\left\{t - \frac{1}{200}\cos(200t) + \frac{1}{200}\right\}$$

$$v(t) = 20\left\{t + \frac{1}{200}[1 - \cos(200t)]\right\}$$

$$v\left(t = \frac{3\pi}{400}\right) = 20\left\{\left(\frac{3\pi}{400}\right) + \frac{1}{200}\left[1 - \cos\left(200\cdot\frac{3\pi}{400}\right)\right]\right\} = 0.571239$$

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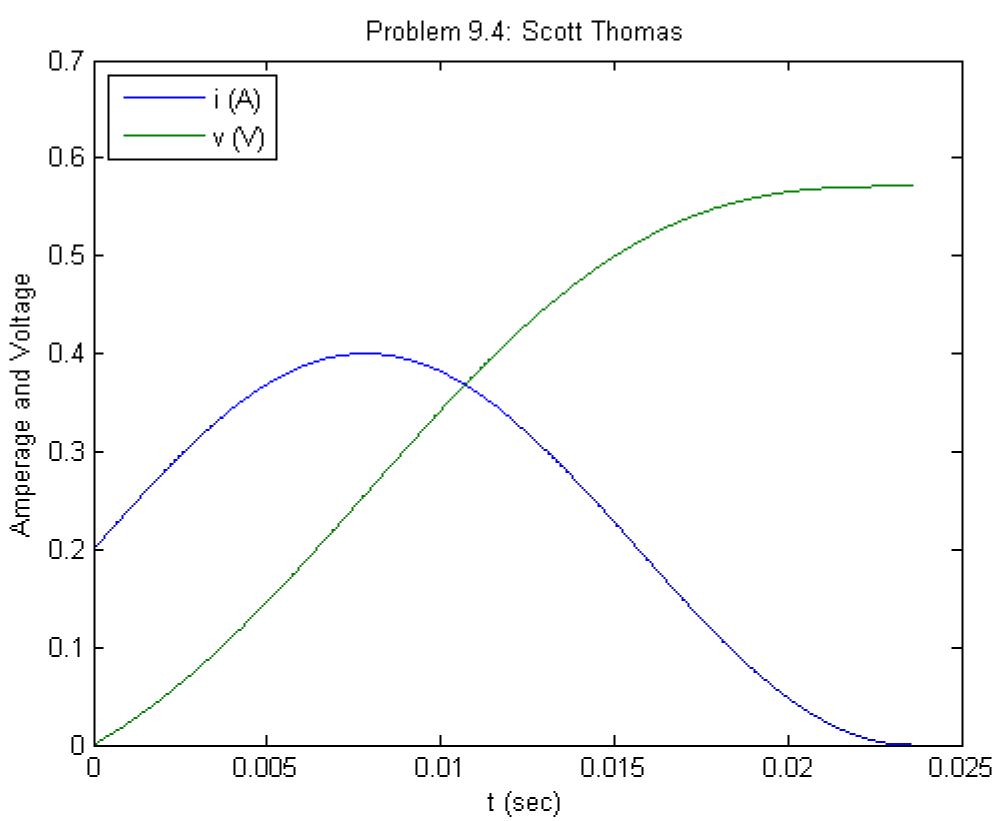
problem9_4.m

```
% Problem 9.4
clear
clc
disp('Problem 9.4: Scott Thomas')

C = 1e-2;
N = 1001;
t = linspace(0,3*pi/400,N);
i = 0.2*(1 + sin(200*t));

v(1) = 0.0;
for k = 1:N-1
v(k+1) = v(k) + 0.5*(t(k+1) - t(k))*(i(k) + i(k+1));
end
v = v/C;
v(N)

plot(t,i, t,v), xlabel('t (sec)')
ylabel('Amperage and Voltage')
title('Problem 9.4: Scott Thomas')
legend('i (A)', 'v (V)', 'Location', 'NorthWest')
```

Command Window

```
Problem 9.4: Scott Thomas

ans =

   0.571238712983318
```



Problem 9.6:

A certain object moves with the velocity $v(t)$ given in the table below. Calculate the object's position $x(t)$ using numerical integration. The position of the object at the initial time is $x(t = 0) = 3.0$. Plot the object's velocity and position versus time t.

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/s)	0	2	5	7	9	12	15	18	22	20	17

Problem setup:

The total distance traveled by an object moving at velocity $v(t)$ from the time $t = a$ to the time $t = b$ is

$$x(b) = \int_a^b v(t)\,dt + x(a)$$

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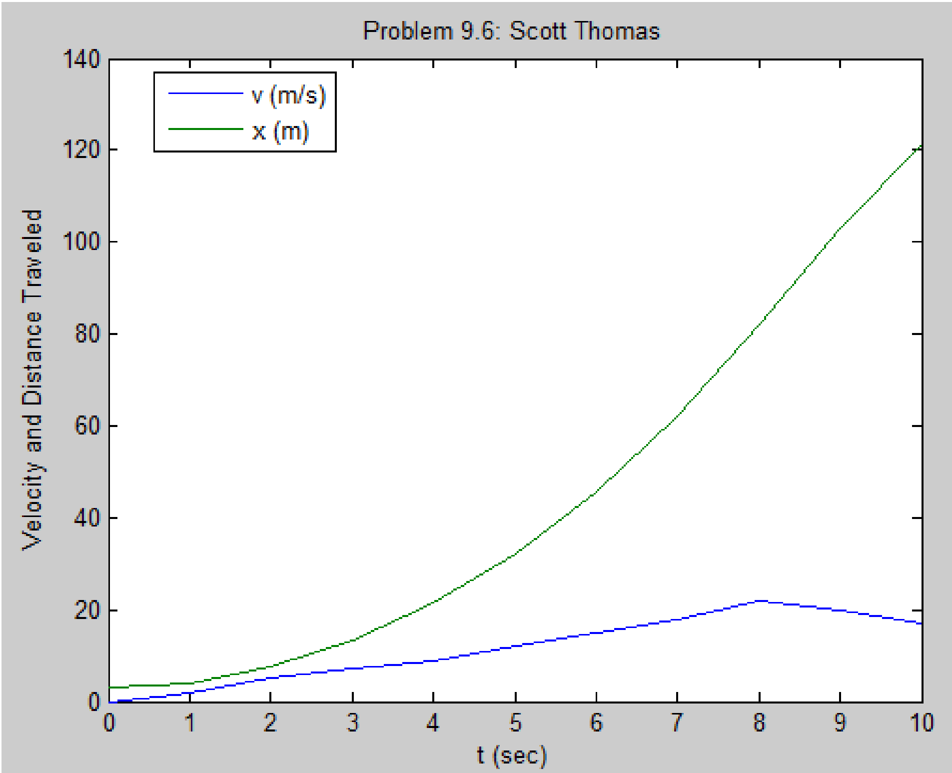
problem9_6_2.m

```
% Problem 9.6
clear
clc
disp('Problem 9.6: Scott Thomas')
v = [0 2 5 7 9 12 15 18 22 20 17];
t = linspace(0,10,11);
x(1) = 3.0;
for k = 1:10
x(k+1) = x(k) + 0.5*(t(k+1) - t(k))*(v(k) + v(k+1));
end
plot(t,v, t,x), xlabel('t (sec)')
ylabel('Velocity and Distance Traveled')
title('Problem 9.6: Scott Thomas')
legend('v (m/s)', 'x (m)','Location', 'Best')
```



Problem 9.6: Scott Thomas
v (m/s)
x (m)
Velocity and Distance Traveled
t (sec)
0
20
40
60
80
100
120
140
0
1
2
3
4
5
6
7
8
9
10

Problem 9.7:

A tank having vertical sides and a bottom area of 100 ft^2 is used to store water. The tank is initially empty. To fill the tank, water is pumped into the top at the rate given in the following table. Calculate the water height *h*(*t*) by numerical integration. Plot the volumetric flow rate and water height versus time.

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Flow Rate (ft^3/min)	0	80	130	150	150	160	165	170	160	140	120

Problem setup:

Volume within the tank at any time is

$$V(t) = \int_0^t \dot{V}\, dt + V(0)$$

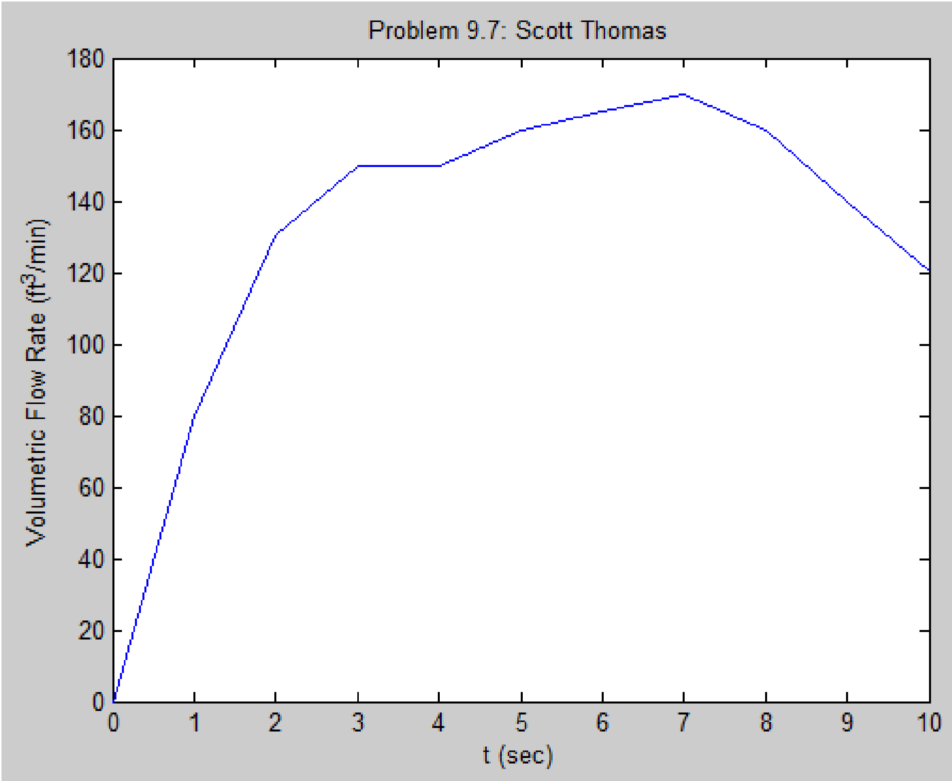
where $\dot{V}$ is the volumetric flow rate of water going into the tank, and $V(0)$ is the initial volume of water within the tank. The height of water is related to the volume by the area of the base of the tank.

$$V = hA$$

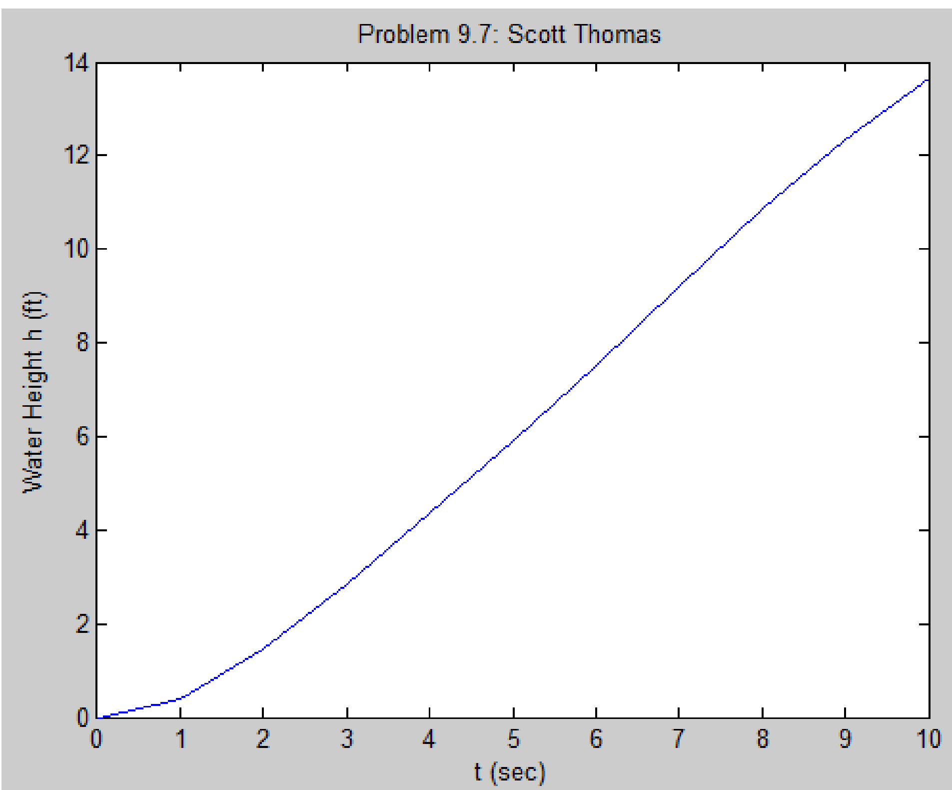
$$h = \frac{V}{A}$$

problem9_7_2.m

```
% Problem 9.7
clear
clc
disp('Problem 9.7: Scott Thomas')
A = 100; %ft^2
Vdot = [0 80 130 150 150 160 165 170 160 140 120];
t = linspace(0,10,11);
V(1) = 0.0;
for k = 1:10
V(k+1) = V(k) + 0.5*(t(k+1) - t(k))*(Vdot(k) + Vdot(k+1));
end
h=V/A;
figure
plot(t,Vdot), xlabel('t (sec)')
ylabel('Volumetric Flow Rate (ft^3/min)')
title('Problem 9.7: Scott Thomas')
figure
plot(t,h), xlabel('t (sec)')
ylabel('Water Height h (ft)')
title('Problem 9.7: Scott Thomas')
```



Problem 9.7: Scott Thomas
Volumetric Flow Rate (ft^3/min)
t (sec)
0
20
40
60
80
100
120
140
160
180
1
2
3
4
5
6
7
8
9
10



Problem 9.7: Scott Thomas
Water Height h (ft)
t (sec)
0
2
4
6
8
10
12
14
1
2
3
4
5
6
7
8
9
10

Problem 9.9:

A certain object has a mass of 100 kg and is acted on by a force (in Newtons):

$$f(t) = 500(2 - 5e^{-t})$$

The mass is a rest at $t = 0$. Calculate the object's velocity using numerical integration and plot the velocity as a function of time.

Problem setup:

$$F = ma$$

$$a = \frac{F}{m} = \frac{500[2 - e^{-t}]}{100} = 5[2 - e^{-t}]$$

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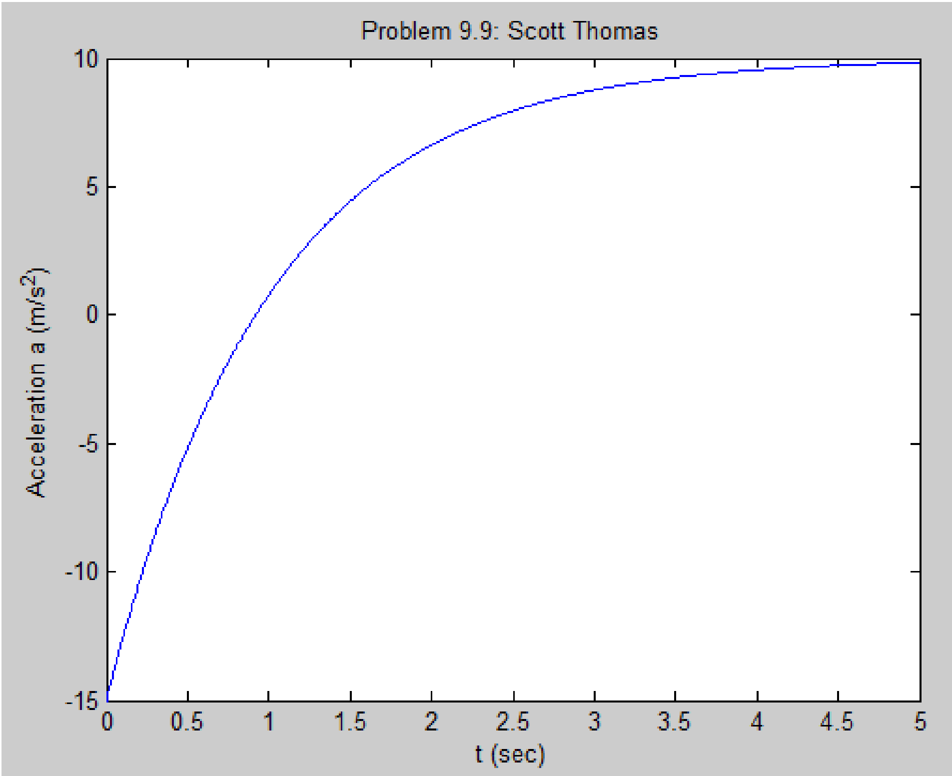
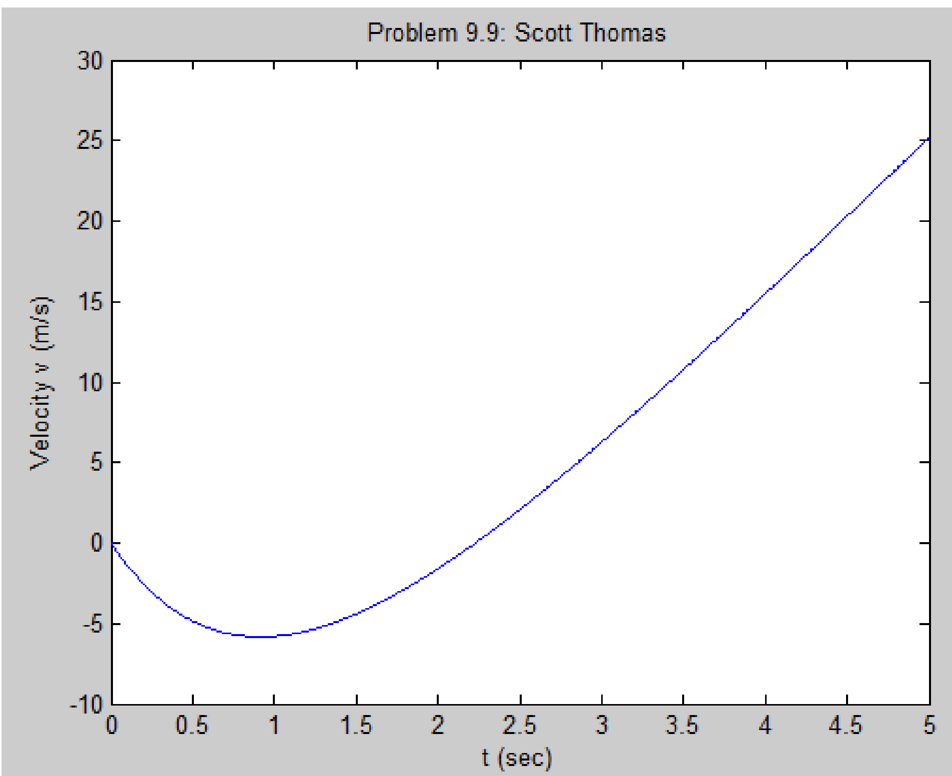
problem9_9.m*

```
% Problem 9.9
clear
clc
disp('Problem 9.9: Scott Thomas')
N = 1000;
t = linspace(0,5,N);
a = 5*(2 - 5*exp(-t));
v(1) = 0.0;
for k = 1:N-1
v(k+1) = v(k) + 0.5*(t(k+1) - t(k))*(a(k) + a(k+1));
end
figure
plot(t,a), xlabel('t (sec)')
ylabel('Acceleration a (m/s^2)')
title('Problem 9.9: Scott Thomas')
figure
plot(t,v), xlabel('t (sec)')
ylabel('Velocity v (m/s)')
title('Problem 9.9: Scott Thomas')
```



Problem 9.9: Scott Thomas
Acceleration a (m/s2)
10
5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t (sec)



Problem 9.9: Scott Thomas
Velocity v (m/s)
30
25
20
15
10
5
0
-5
-10
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t (sec)

Problem 9.18:

18. At a relative maximum of a curve $y(x)$, the slope dy/dx is zero. Use the following data to estimate the values of x and y that correspond to a maximum point.

x	0	1	2	3	4	5	6	7	8	9	10
y	0	2	5	7	9	10	8	7	6	8	10

Problem setup: Use the Backward, Forward and Central Differences to detect when $\frac{dy}{dx} = 0$. From the plot, $x = 5.0$ and $x = 7.5$.

Problem 9.18

```
clear
clc
disp('Problem 9.18: Scott Thomas')

N = 11;
k = 1:N;
x = 0:10;
y = [0 2 5 7 9 10 8 7 6 8 10];

% Backward difference
dydx_b = zeros(1,N);
for k=2:N
    dydx_b(k) = (y(k) - y(k-1))/(x(k) - x(k-1));
end
%disp('dydx_b');
x(2:N);
dydx_b(2:N);
```

```
% Forward difference
for k=1:N-1
    dydx_f(k) = (y(k+1) - y(k))/(x(k+1) - x(k));
end
%disp('dydx_f');
x(1:N-1);
dydx_f(1:N-1);

% Central difference
for k=2:N-1
    dydx_c(k) = (y(k+1) - y(k-1))/(x(k+1) - x(k-1));
end
%disp('dydx_c');
x(2:N-1);
dydx_c(2:N-1);

x2 = [0 10];
y2 = [0 0];

subplot(2,1,1)
plot(x,y,'o-'), xlabel('x'), ylabel('y'),title('Problem 9.18: Scott Thomas')
subplot(2,1,2)
plot(x(2:N),dydx_b(2:N), x(1:N-1),dydx_f(1:N-1), x(2:N-1),dydx_c(2:N-1), x2,y2,'m'),
xlabel('x'), ylabel('dy/dx')
legend('Backward','Forward','Central','Location','SouthWest')
```

```
Problem 9.18: Scott Thomas
```