ME 1020 Engineering Programming with MATLAB

Chapter 9b Homework Solutions: 9.22, 9.23, 9.24, 9.25, 9.28

Problem 9.22:

22. Plot the solution of the equation

$$6\dot{y} + y = f(t)$$

if $f(t) = 0$ for $t < 0$ and $f(t) = 15$ for $t \geq 0$. The initial condition is $y(0) = 7$.

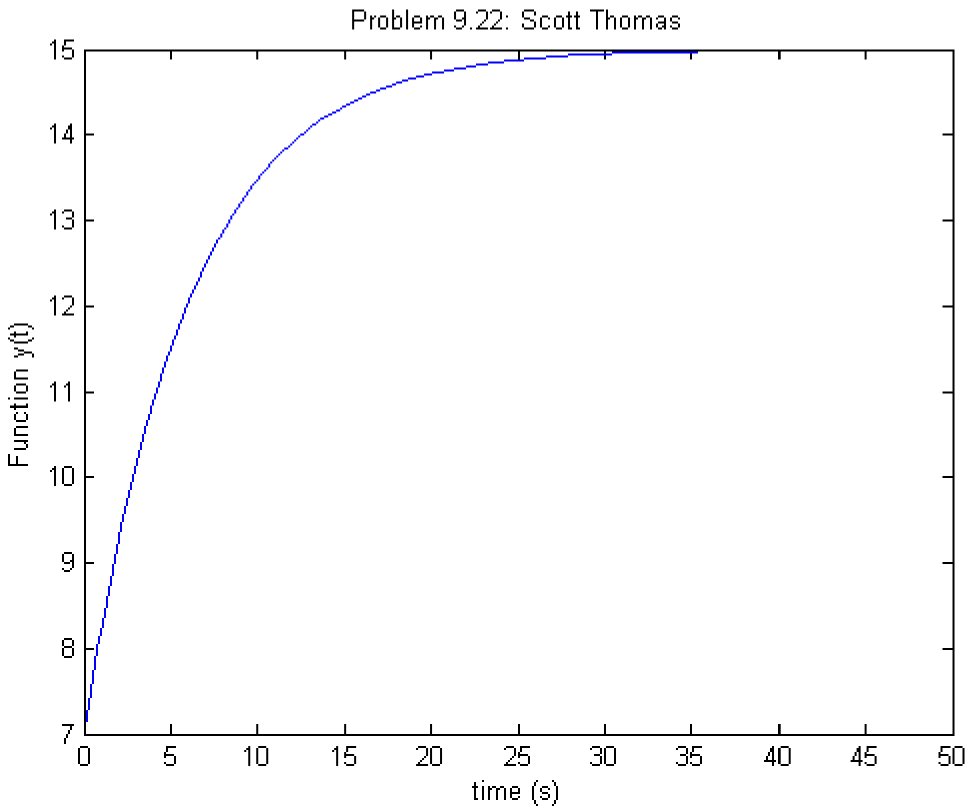
<u>Use the `ode45` Solver for this problem.</u>

$$\dot{y} = -\frac{y}{6} + \frac{f(t)}{6} = -\frac{y}{6} + \frac{15}{6}$$

```
function ydot = f922( ~,y )
ydot = -y/6 + 15/6;
end
```

```
% Problem 9.22
clear
clc
disp('Problem 9.22: Scott Thomas')

[t,y] = ode45(@f922, [0, 50], 7 );
y;
plot(t,y), xlabel('time (s)')
ylabel('Function y(t)')
title('Problem 9.22: Scott Thomas')
```

```
Problem 9.22: Scott Thomas
```


Problem 9.22: Scott Thomas
Function y(t)
time (s)
15
14
13
12
11
10
9
8
7
0
5
10
15
20
25
30
35
40
45
50

Problem 9.23:

23. The equation for the voltage y across the capacitor of an RC circuit is

$$RC\frac{dy}{dt} + y = v(t)$$

where $v(t)$ is the applied voltage. Suppose that $RC = 0.2$ s and that the capacitor voltage is initially 2 V. Suppose also that the applied voltage goes from 0 to 10 V at $t = 0$. Plot the voltage $y(t)$ for $0 \le t \le 1$ s.

Use Euler's Method to solve this problem.

Problem setup:

Solve the differential equation for $\dot{y}$:

$$\dot{y} = \frac{dy}{dt} = \left[-\frac{1}{RC}y + \frac{1}{RC}v(t)\right] = g(t,y)$$

Approximate the derivative as follows:

$$\frac{dy}{dt} \approx \frac{\big(y(t+\Delta t) - y(t)\big)}{\Delta t} = g(t,y)$$

where Δt is called the time step size. Solve for $y(t+\Delta t)$:

$$y(t+\Delta t) = y(t) + g(t,y)\Delta t$$

Put this equation into a form appropriate for computer solution. This is called the Difference Equation:

$$y(t_{k+1}) = y(t_k) + g[t_k, y(t_k)]\Delta t; \quad \text{where} \quad t_{k+1} = t_k + \Delta t$$

The expression for the forcing function is $v(t) = 10$. This results in the following expression for g:

$$g(t,y) = -\frac{1}{RC}y + \frac{1}{RC}v(t) = -\frac{1}{RC}y + \frac{10}{RC} = \frac{1}{RC}(10 - y)$$

Substitute this expression into the difference equation:

$$y(t_{k+1}) = y(t_k) + \left[\frac{1}{RC}\big(10 - y(t_k)\big)\right]\Delta t$$

In the problem statement, the initial condition is $y(0) = 2$. The resistance-capacitance product is $RC = 0.2$. Let $\Delta t = 0.01$.

For $k = 0$:

$$y(t_1) = y(t_0) + \left[\frac{1}{RC}\left(10 - y(t_0)\right)\right]\Delta t = 2.0 + \left[\frac{1}{0.2}(10 - 2.0)\right](0.01) = 2.4$$

For $k = 1$:

$$y(t_2) = y(t_1) + \left[\frac{1}{RC}\left(10 - y(t_1)\right)\right]\Delta t = 2.4 + \left[\frac{1}{0.2}(10 - 2.4)\right](0.01) = 2.78$$

For $k = 2$:

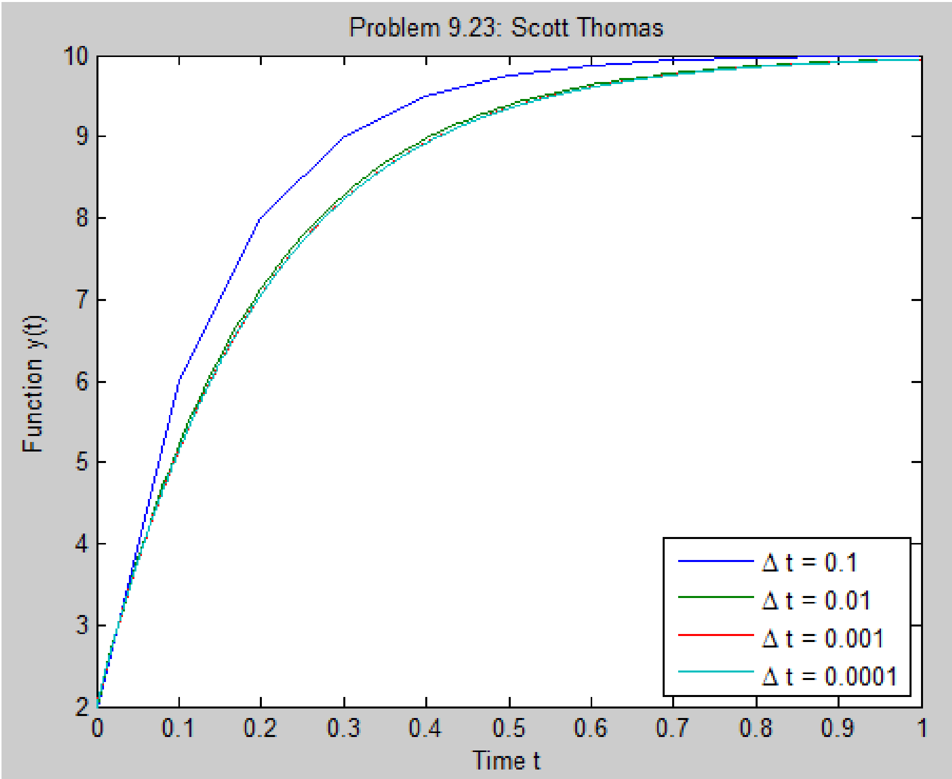
$$y(t_3) = y(t_2) + \left[\frac{1}{RC}\left(10 - y(t_2)\right)\right]\Delta t = 2.78 + \left[\frac{1}{0.2}(10 - 2.78)\right](0.01) = 3.141$$

```
% Problem 9.23
clear
clc
disp('Problem 9.23: Scott Thomas')

N = 10;
delta_t = 0.01;
y = zeros(1,N);
t = zeros(1,N);

y(1) = 2.0;
for k = 1:N
    y(k+1) = y(k) + 1/0.2*(10 - y(k))*delta_t;
    t(k+1) = t(k) + delta_t;
end
t
y
```

```
Problem 9.23: Scott Thomas

t =

        0    0.0100    0.0200    0.0300    0.0400    0.0500    0.0600    0.0700    0.0800    0.0900    0.1000


y =

   2.0000    2.4000    2.7800    3.1410    3.4840    3.8098    4.1193    4.4133    4.6926    4.9580    5.2101

fx >>
```

This shows that the code is working correctly. The next process is to do a time-step independence study, where the time step Δt is varied until the solution becomes independent of Δt.

```
% Problem 9.23
clear
clc
disp('Problem 9.23: Scott Thomas')

N = 100;
delta_t = 0.01;
y = zeros(1,N);
t = zeros(1,N);
y(1) = 2.0;
for k = 1:N
    y(k+1) = y(k) + 1/0.2*(10 - y(k))*delta_t;
    t(k+1) = t(k) + delta_t;
end

N2 = 10;
delta_t2 = 0.1;
y2 = zeros(1,N2);
t2 = zeros(1,N2);
y2(1) = 2.0;
for k = 1:N2
    y2(k+1) = y2(k) + 1/0.2*(10 - y2(k))*delta_t2;
    t2(k+1) = t2(k) + delta_t2;
end

N3 = 1000;
delta_t3 = 0.001;
y3 = zeros(1,N3);
t3 = zeros(1,N3);
y3(1) = 2.0;
for k = 1:N3
    y3(k+1) = y3(k) + 1/0.2*(10 - y3(k))*delta_t3;
    t3(k+1) = t3(k) + delta_t3;
end

N4 = 10000;
delta_t4 = 0.0001;
y4 = zeros(1,N4);
t4 = zeros(1,N4);
y4(1) = 2.0;
for k = 1:N4
    y4(k+1) = y4(k) + 1/0.2*(10 - y4(k))*delta_t4;
    t4(k+1) = t4(k) + delta_t4;
end

plot(t2, y2, t, y, t3,y3, t4, y4)%, t2, y2, t4, y4),
xlabel('Time t'), ylabel('Function y(t)')
title('Problem 9.23: Scott Thomas')
legend('\Delta t = 0.1', '\Delta t = 0.01', '\Delta t = 0.001','\Delta t = 0.0001','Location', 'SouthEast')
%'\Delta t = 0.01', 'Exact Solution', 'Location', 'SouthEast')
axis([0 1 2 10])

```



Problem 9.23: Scott Thomas
Function y(t)
Time t
Δ t = 0.1
Δ t = 0.01
Δ t = 0.001
Δ t = 0.0001

Problem 9.24:

24. The following equation describes the temperature $T(t)$ of a certain object immersed in a liquid bath of constant temperature T_b.

$$10\frac{dT}{dt} + T = T_b$$

Suppose the object's temperature is initially $T(0) = 70°\text{F}$ and the bath temperature is $T_b = 170°\text{F}$.

a. How long will it take for the object's temperature T to reach the bath temperature?

b. How long will it take for the object's temperature T to reach 168°F?

c. Plot the object's temperature $T(t)$ as a function of time.

Use Euler's Method to solve this problem.

Problem setup:

Solve the differential equation for dT/dt:

$$\frac{dT}{dt} = -\frac{1}{10}T + \frac{1}{10}T_b = \frac{1}{10}(T_b - T) = g(t,T)$$

Approximate the derivative as follows:

$$\frac{dT}{dt} \approx \frac{\big(T(t+\Delta t) - T(t)\big)}{\Delta t} = g(t,T)$$

where Δt is called the time step size. Solve for $T(t+\Delta t)$:

$$T(t+\Delta t) = T(t) + g(t,T)\Delta t$$

Put this equation into a form appropriate for computer solution. This is called the Difference Equation:

$$T(t_{k+1}) = T(t_k) + g[t_k, T(t_k)]\Delta t; \quad \text{where} \quad t_{k+1} = t_k + \Delta t$$

Substitute the expression for $g(t,T)$ into the difference equation:

$$T(t_{k+1}) = T(t_k) + \frac{1}{10}\big(T_b - T(t_k)\big)\Delta t$$

In the problem statement, the initial condition is $T(0) = 70$. Let $\Delta t = 0.1$.

For $k = 0$:

$$T(t_1) = T(t_0) + \left[\frac{1}{10}\big(T_b - T(t_0)\big)\right]\Delta t = 70.0 + \left[\frac{1}{10}(170 - 70.0)\right](0.1) = 71.0$$

For $k = 1$:

$$T(t_2) = T(t_1) + \left[\frac{1}{10}\left(T_b - T(t_1)\right)\right]\Delta t = 71.0 + \left[\frac{1}{10}(170 - 71.0)\right](0.1) = 71.99$$

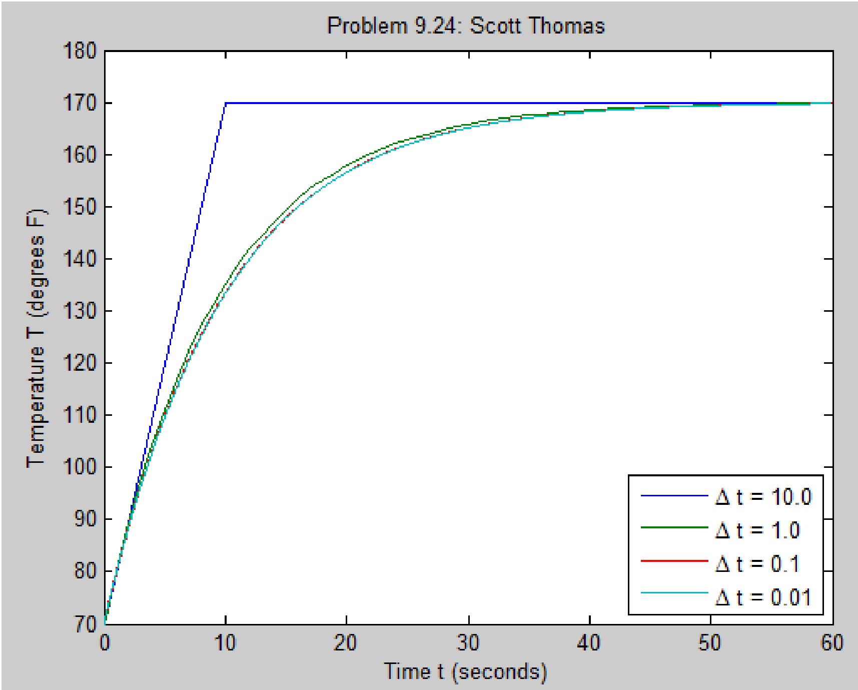
For $k = 2$:

$$T(t_3) = T(t_2) + \left[\frac{1}{10}\left(T_b - T(t_2)\right)\right]\Delta t = 71.99 + \left[\frac{1}{10}(170 - 71.99)\right](0.1) = 72.9701$$

```
% Problem 9.24
clear
clc
disp('Problem 9.24: Scott Thomas')

N = 10;
delta_t = 0.1;
T = zeros(1,N);
t = zeros(1,N);
T(1) = 70.0;
for k = 1:N
    T(k+1) = T(k) + 1/10*(170 - T(k))*delta_t;
    t(k+1) = t(k) + delta_t;
end
t
T

```

```
Problem 9.24: Scott Thomas

t =

         0    0.1000    0.2000    0.3000    0.4000    0.5000    0.6000    0.7000    0.8000    0.9000    1.0000


T =

   70.0000   71.0000   71.9900   72.9701   73.9404   74.9010   75.8520   76.7935   77.7255   78.6483   79.5618

>>
```

```
% Problem 9.24
clear
clc
disp('Problem 9.24: Scott Thomas')

N = 600;
delta_t = 0.1;
T = zeros(1,N);
t = zeros(1,N);
T(1) = 70.0;
for k = 1:N
    T(k+1) = T(k) + 1/10*(170 - T(k))*delta_t;
    t(k+1) = t(k) + delta_t;
end

N1 = 6000;
delta_t1 = 0.01;
T1 = zeros(1,N1);
t1 = zeros(1,N1);
T1(1) = 70.0;
for k = 1:N1
    T1(k+1) = T1(k) + 1/10*(170 - T1(k))*delta_t1;
    t1(k+1) = t1(k) + delta_t1;
end


N2 = 60;
delta_t2 = 1.0;
T2 = zeros(1,N2);
t2 = zeros(1,N2);
T2(1) = 70.0;
for k = 1:N2
    T2(k+1) = T2(k) + 1/10*(170 - T2(k))*delta_t2;
    t2(k+1) = t2(k) + delta_t2;
end

N3 = 6;
delta_t3 = 10.0;
T3 = zeros(1,N3);
t3 = zeros(1,N3);
T3(1) = 70.0;
for k = 1:N3
    T3(k+1) = T3(k) + 1/10*(170 - T3(k))*delta_t3;
    t3(k+1) = t3(k) + delta_t3;
end

plot(t3,T3, t2, T2, t, T, t1, T1)
xlabel('Time t (seconds)'), ylabel('Temperature T (degrees F)')
title('Problem 9.24: Scott Thomas')
%legend('\Delta t = 0.1','Location', 'SouthEast')
legend('\Delta t = 10.0', '\Delta t = 1.0','\Delta t = 0.1', '\Delta t = 0.01', 'Location', 'SouthEast')
axis([0 60 70 180])
```


Problem 9.24: Scott Thomas
Temperature T (degrees F)
Time t (seconds)
Δ t = 10.0
Δ t = 1.0
Δ t = 0.1
Δ t = 0.01
180
170
160
150
140
130
120
110
100
90
80
70
0
10
20
30
40
50
60

Problem 9.25:

25.* The equation of motion of a rocket-propelled sled is, from Newton's law,

$$m\dot{v} = f - cv$$

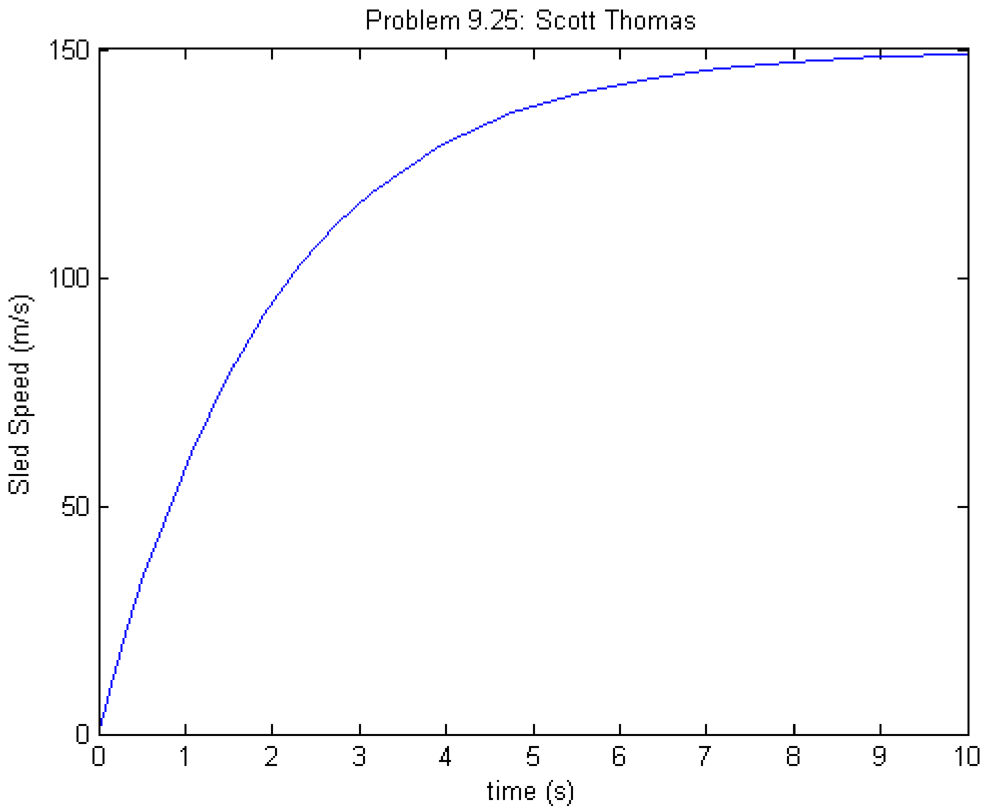
where m is the sled mass, f is the rocket thrust, and c is an air resistance coefficient. Suppose that $m = 1000$ kg and $c = 500$ N · s/m. Suppose also that $v(0) = 0$ and $f = 75{,}000$ N for $t \geq 0$. Determine the speed of the sled at $t = 10$ s.

Use the ode15s Solver for this problem.

```
function vdot = f925( ~,v )

f = 75000;% N
m = 1000;% kg
c = 500;% (N-s)/m

vdot = f/m - c/m*v;
end
```

```
% Problem 9.25
clear
clc
disp('Problem 9.25: Scott Thomas')

[t,v] = ode15s(@f925, [0, 10], 0 );
v;
plot(t,v), xlabel('time (s)')
ylabel('Sled Speed (m/s)')
title('Problem 9.25: Scott Thomas')

fprintf('Sled Velocity at t = 10 seconds: v = %f\n', v(length(v)));
```

```
Problem 9.25: Scott Thomas
Sled Velocity at t = 10 seconds: v = 148.997458
```



Problem 9.25: Scott Thomas
Sled Speed (m/s)
time (s)
0
50
100
150
0
1
2
3
4
5
6
7
8
9
10

Problem 9.28:

28. The equation describing the water height h in a spherical tank with a drain at the bottom is

$$\pi(2rh - h^2)\frac{dh}{dt} = -C_d A\sqrt{2gh}$$

Suppose the tank's radius is $r = 3$ m and the circular drain hole has a radius of 2 cm. Assume that $C_d = 0.5$ and that the initial water height is $h(0) = 5$ m. Use $g = 9.81$ m/s^2.

a. Use an approximation to estimate how long it takes for the tank to empty.

b. Plot the water height as a function of time until $h(t) = 0$.

Use the Euler Method to solve this problem.

Problem setup:

Solve the differential equation for dh/dt:

$$\frac{dh}{dt} = -\frac{C_d A\sqrt{2gh}}{\pi(2rh - h^2)}$$

Approximate the derivative as follows:

$$\frac{dh}{dt} \approx \frac{\left(h(t+\Delta t) - h(t)\right)}{\Delta t} = -\frac{C_d A\sqrt{2gh}}{\pi(2rh - h^2)}$$

where Δt is the time step size. Solve for $h(t+\Delta t)$:

$$h(t+\Delta t) = h(t) - \frac{C_d A\sqrt{2gh}}{\pi(2rh - h^2)}\Delta t$$

Put this equation into a form appropriate for computer solution. This is called the **Difference Equation**:

$$h(t_{k+1}) = h(t_k) - \frac{C_d A\sqrt{2gh}}{\pi(2rh - h^2)}\Delta t; \quad \text{where} \quad t_{k+1} = t_k + \Delta t$$

In the problem statement, the initial condition is $h(0) = 5.0\text{ m},\ r_{\text{Tank}} = 3.0\text{ m},\ r_{\text{Drain}} = 2.0\text{ cm} = 0.02\text{ m}.$
Let $\Delta t = 100.0$.

For $k = 1$:

$$h(1) = 5.0$$

$$t(1) = 0.0$$

$$A = \pi r_{\text{Drain}}^2 = \pi(0.02)^2 = 1.2566 \times 10^{-3}\ \text{m}^2$$

$$h(2) = h(1) - \frac{C_d A\sqrt{2gh(1)}}{\pi[2rh(1) - h(1)^2]}\Delta t$$

$$h(2) = (5.0) - \frac{(0.5)(1.2566 \times 10^{-3})\sqrt{2(9.81)(5.0)}}{\pi[2(3.0)(5.0) - (5.0)^2]}(100.0) = 4.9604$$

$$t_2 = t_1 + \Delta t = 0.0 + 100.0 = 100.0$$

For $k = 2$:

$$h(3) = h(2) - \frac{C_d A\sqrt{2gh(2)}}{\pi[2rh(2) - h(2)^2]}\Delta t$$

$$h(3) = (4.9604) - \frac{(0.5)(1.2566 \times 10^{-3})\sqrt{2(9.81)(4.9604)}}{\pi[2(3.0)(4.9604) - (4.9604)^2]}(100.0) = 4.9221$$

$$t_2 = t_1 + \Delta t = 100.0 + 100.0 = 200.0$$

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problem9_28_Euler.m

```
% Problem 9.28
clear
clc
disp('Problem 9.28: Scott Thomas')

r = 3.0;% m
rdrain = 0.02; % m
g = 9.81; %m/s^2
Cd = 0.5;
A = pi*rdrain^2;
N1 = 2;
delta_t1 = 100.0;
t1(1) = 0;
h1(1) = 5.0;
for k = 1:N1
    h1(k+1) = h1(k) - Cd*A*sqrt(2*g*h1(k))/(pi*(2*r*h1(k) - h1(k)^2))*delta_t1;
    t1(k+1) = t1(k) + delta_t1;
    if h1(k+1) < 0
        h1(k+1) = 0;
    end
end
t1
h1
plot(t1,h1)
xlabel('Time t (seconds)'), ylabel('Water Height (m)')
title('Problem 9.28: Scott Thomas')
```

Command Window

```
Problem 9.28: Scott Thomas

t1 =

     0   100   200


h1 =

    5.0000    4.9604    4.9221
```

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problem9_28_Euler.m*

```
% Problem 9.28
clear
clc
disp('Problem 9.28: Scott Thomas')

r = 3.0;% m
rdrain = 0.02; % m
g = 9.81; %m/s^2
Cd = 0.5;
A = pi*rdrain^2;

N1 = 253;
delta_t1 = 100.0;
t1(1) = 0;
h1(1) = 5.0;
for k = 1:N1
    h1(k+1) = h1(k) - Cd*A*sqrt(2*g*h1(k))/(pi*(2*r*h1(k) - h1(k)^2))*delta_t1;
    t1(k+1) = t1(k) + delta_t1;
    if h1(k+1) < 0
        h1(k+1) = 0;
    end
end

N2 = 2530;
delta_t2 = 10.0;
t2(1) = 0;
h2(1) = 5.0;
for k = 1:N2
    h2(k+1) = h2(k) - Cd*A*sqrt(2*g*h2(k))/(pi*(2*r*h2(k) - h2(k)^2))*delta_t2;
    t2(k+1) = t2(k) + delta_t2;
    if h2(k+1) < 0
        h2(k+1) = 0;
    end
end
```

```matlab

N3 = 25;
delta_t3 = 1000.0;
t3(1) = 0;
h3(1) = 5.0;
for k = 1:N3
    h3(k+1) = h3(k) - Cd*A*sqrt(2*g*h3(k))/(pi*(2*r*h3(k) - h3(k)^2))*delta_t3;
    t3(k+1) = t3(k) + delta_t3;
    if h3(k+1) < 0
        h3(k+1) = 0;
    end
end

plot(t3,h3, t1,h1, t2,h2)
xlabel('Time t (seconds)'), ylabel('Water Height (m)')
title('Problem 9.28: Scott Thomas')
legend('\Delta t = 1000 sec','\Delta t = 100 sec','\Delta t = 10 sec',...
    'Location','Best')
```