ME 1020 Engineering Programming with MATLAB

Chapter 9c Homework Solutions: 9.26, 9.31, 9.32, 9.36, 9.40

Topics Covered:

- Higher-Order Differential Equations
  - Euler Method
  - MATLAB ODE Solver ode45
  - ode45/Matrix Method
- Matrix Methods for Linear Equations
  - Free Response
  - Impulse Response
  - Step Response
  - Arbitrary Input Response

Problem 9.26:

26. The following equation describes the motion of a mass connected to a spring, with viscous friction on the surface.

$$m\ddot{y} + c\dot{y} + ky = 0$$

Plot $y(t)$ for $y(0) = 10$, $\dot{y}(0) = 5$ if

a. $m = 3$, $c = 18$, and $k = 102$

b. $m = 3$, $c = 39$ and $k = 120$

<u>Use the Euler Method to solve this problem.</u>

Problem setup:

This is a second-order ordinary differential equation. Rewrite the equation by solving for the second derivative.

$$\ddot{y} = -\frac{c}{m}\dot{y} - \frac{k}{m}y$$

$$\text{Let } \; x_1 = y \; \text{ and } \; x_2 = \dot{y}$$

Taking the derivative of the first equation gives

$$\dot{x}_1 = \dot{y} = x_2 \;\text{ or }\; \dot{x}_1 = x_2$$

Taking the derivative of the second equation gives

$$\dot{x}_2 = \ddot{y} = -\left(\frac{c}{m}\right)x_2 - \left(\frac{k}{m}\right)x_1$$

The original second order ordinary differential equation is now converted into two first order ordinary differential equations that are coupled.

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -\left(\frac{c}{m}\right)x_2 - \left(\frac{k}{m}\right)x_1, \quad x_1(0) = 10, \;\; x_2(0) = 5$$

Use the Euler method for this problem. The system of equations can be discretized as follows:

$$x_{1,k+1} = x_{1,k} + \Delta t \cdot x_{2,k}$$

$$x_{2,k+1} = x_{2,k} + \Delta t\left[-\left(\frac{c}{m}\right)x_{2,k} - \left(\frac{k}{m}\right)x_{1,k}\right]$$

In the problem statement, the initial conditions are $y(0) = x_1(0) = 10$ and $\dot{y}(0) = x_2(0) = 5$. Let $\Delta t = 0.001$.

For $k = 1$:

$$x_{1,2} = x_{1,1} + \Delta t \cdot x_{2,1} = (10.0) + (0.001)(5.0) = 10.005$$

$$x_{2,2} = x_{2,1} + \Delta t\left[-\left(\frac{c}{m}\right)x_{2,1} - \left(\frac{k}{m}\right)x_{1,1}\right]$$

$$x_{2,2} = (5.0) + (0.001)\left[-\left(\frac{18}{3}\right)(5.0) - \left(\frac{102}{3}\right)(10.0)\right] = 4.63$$

For $k = 2$:

$$x_{1,3} = x_{1,2} + \Delta t \cdot x_{2,2} = (10.005) + (0.001)(4.63) = 10.00963$$

$$x_{2,3} = x_{2,2} + \Delta t\left[-\left(\frac{c}{m}\right)x_{2,2} - \left(\frac{k}{m}\right)x_{1,2}\right]$$

$$x_{2,3} = (4.63) + (0.001)[-(6)(4.63) - (34)(10.005)] = 4.26205$$

For $k = 3$:

$$x_{1,4} = x_{1,3} + \Delta t \cdot x_{2,3} = (10.00963) + (0.001)(4.26205) = 10.01389$$

$$x_{2,4} = x_{2,3} + \Delta t\left[-\left(\frac{c}{m}\right)x_{2,3} - \left(\frac{k}{m}\right)x_{1,3}\right]$$

$$x_{2,4} = (4.26205) + (0.001)[-(6)(4.26205) - (34)(10.00963)] = 3.89615$$

Editor - C:\Laptop Backup\matlab\Homework Solutions\Chapter 09 Homework\problem9_26.m

problem9_26.m

```
% Problem 9.26
clear
clc
disp('Problem 9.26: Scott Thomas')
ca = 18;
ma = 3;
Ka = 102;
Na = 3;
delta_ta = 0.001;
ta(1) = 0;
x1a(1) = 10.0;
x2a(1) = 5.0;
for k = 1:Na
    x1a(k+1) = x1a(k) + x2a(k)*delta_ta;
    x2a(k+1) = x2a(k) + (-(ca/ma)*x2a(k) - (Ka/ma)*x1a(k))*delta_ta;
    ta(k+1) = ta(k) + delta_ta;
end
x1a
x2a

```

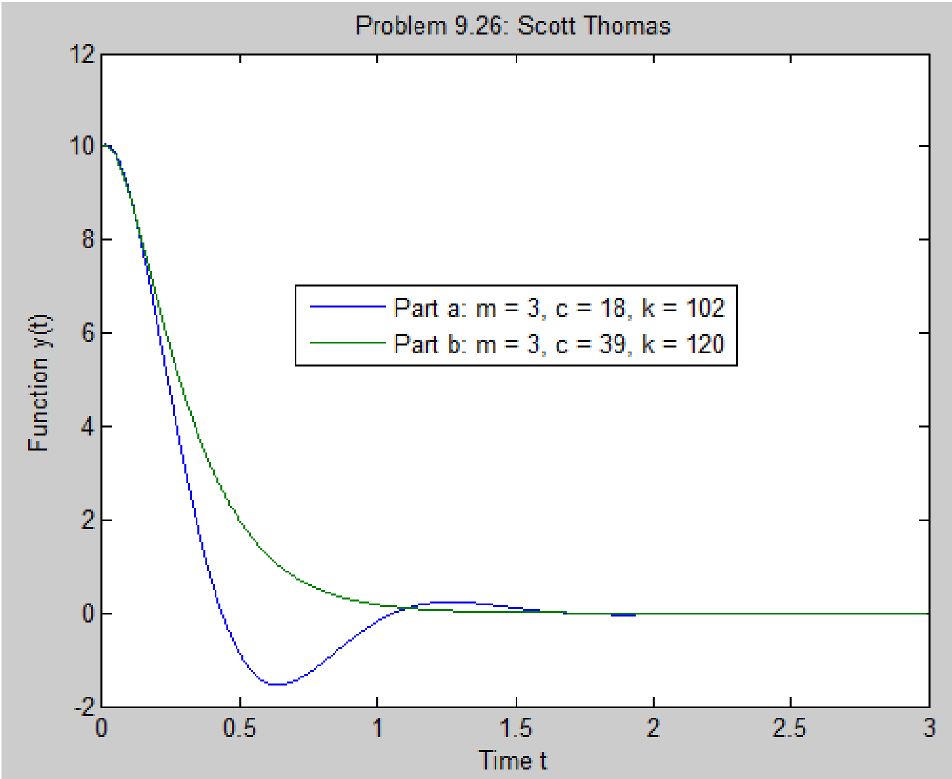
Command Window

```
Problem 9.26: Scott Thomas

x1a =

   10.0000   10.0050   10.0096   10.0139


x2a =

    5.0000    4.6300    4.2620    3.8962
```

Editor - C:\Laptop Backup\matlab\Homework Solutions\Chapter 09 Homework\problem9_26.m

problem9_26.m

```
% Problem 9.26
clear
clc
disp('Problem 9.26: Scott Thomas')

ca = 18;
ma = 3;
Ka = 102;
Na = 3000;
delta_ta = 0.001;
ta(1) = 0;
x1a(1) = 10.0;
x2a(1) = 5.0;
for k = 1:Na
    x1a(k+1) = x1a(k) + x2a(k)*delta_ta;
    x2a(k+1) = x2a(k) + (-(ca/ma)*x2a(k) - (Ka/ma)*x1a(k))*delta_ta;
    ta(k+1) = ta(k) + delta_ta;
end

cb = 39;
mb = 3;
Kb = 120;
Nb = 3000;
delta_tb = 0.001;
tb(1)=0;
x1b(1) = 10.0;
x2b(1) = 5.0;
for k = 1:Nb
    x1b(k+1) = x1b(k) + x2b(k)*delta_tb;
    x2b(k+1) = x2b(k) + (-(cb/mb)*x2b(k) - (Kb/mb)*x1b(k))*delta_tb;
    tb(k+1) = tb(k) + delta_tb;
end

plot(ta, x1a, tb, x1b)
xlabel('Time t'), ylabel('Function y(t)')
title('Problem 9.26: Scott Thomas')
legend('Part a: m = 3, c = 18, k = 102',...
    'Part b: m = 3, c = 39, k = 120','Location', 'Best')
```



Problem 9.26: Scott Thomas
Part a: m = 3, c = 18, k = 102
Part b: m = 3, c = 39, k = 120
Function y(t)
Time t
12
10
8
6
4
2
0
-2
0
0.5
1
1.5
2
2.5
3

Problem 9.31:

31. The following equation describes the motion of a certain mass connected to a spring, with viscous friction on the surface

$$3\ddot{y} + 39\dot{y} + 120y = f(t)$$

where $f(t)$ is an applied force. Suppose that $f(t) = 0$ for $t < 0$ and $f(t) = 10$ for $t \geq 0$.

a. Plot $y(t)$ for $y(0) = \dot{y}(0) = 0$.

b. Plot $y(t)$ for $y(0) = 0$ and $\dot{y}(0) = 10$. Discuss the effect of the nonzero initial velocity.

Use the `ode45` Solver for this problem.

This is a second-order ordinary differential equation. Rewrite the equation by solving for the second derivative.

$$\ddot{y} = -\frac{39}{3}\dot{y} - \frac{120}{3}y + 10/3 = -13\dot{y} - 40y + 10/3$$

$$\text{Let } x_1 = y \text{ and } x_2 = \dot{y}$$

Taking the derivative of the first equation gives

$$\dot{x}_1 = \dot{y} = x_2 \text{ or } \dot{x}_1 = x_2$$

Taking the derivative of the second equation gives

$$\dot{x}_2 = \ddot{y} = -13x_2 - 40x_1 + 10/3$$

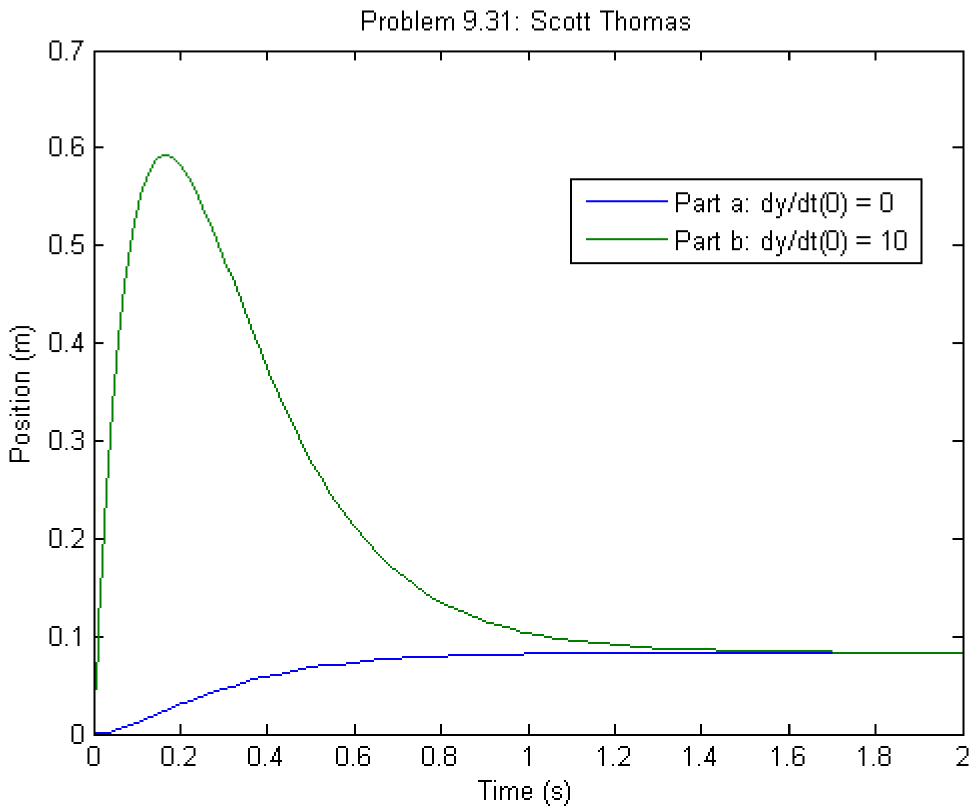
The original second order ordinary differential equation is now converted into two first order ordinary differential equations that are coupled.

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -13x_2 - 40x_1 + 10/3, \quad x_1(0) = 0, \; x_2(0) = 0$$

```
function xdot = f931( t,x )

xdot(1) = x(2);
xdot(2) = -13*x(2) - 40*x(1) + 10/3;
xdot = [xdot(1); xdot(2)];
end
```

```
% Problem 9.31: Solve using ode45/matrix method
clear
clc
disp('Problem 9.31: Scott Thomas')

[ta,xa] = ode45(@f931, [0, 2], [0, 0] );
[tb,xb] = ode45(@f931, [0, 2], [0, 10] );
plot(ta,xa(:,1),tb, xb(:,1)), xlabel('Time (s)')
ylabel('Position (m)')
title('Problem 9.31: Scott Thomas')
legend('Part a: dy/dt(0) = 0', 'Part b: dy/dt(0) = 10','Location', 'Best')
```

Problem 9.31: Scott Thomas



Problem 9.32:

32. The following equation describes the motion of a certain mass connected to a spring, with no friction

$$3\ddot{y} + 75y = f(t)$$

where $f(t)$ is an applied force. Suppose the applied force is sinusoidal with a frequency of ω rad/s and an amplitude of 10 N: $f(t) = 10\sin(\omega t)$. Suppose that the initial conditions are $y(0) = \dot{y}(0) = 0$. Plot $y(t)$ for $0 \le t \le 20$ s. Do this for the following three cases. Compare the results of each case.

a. $\omega = 1$ rad/s
b. $\omega = 5$ rad/s
c. $\omega = 10$ rad/s

<u>Use the `ode45` Solver with Matrix Method for this problem.</u>

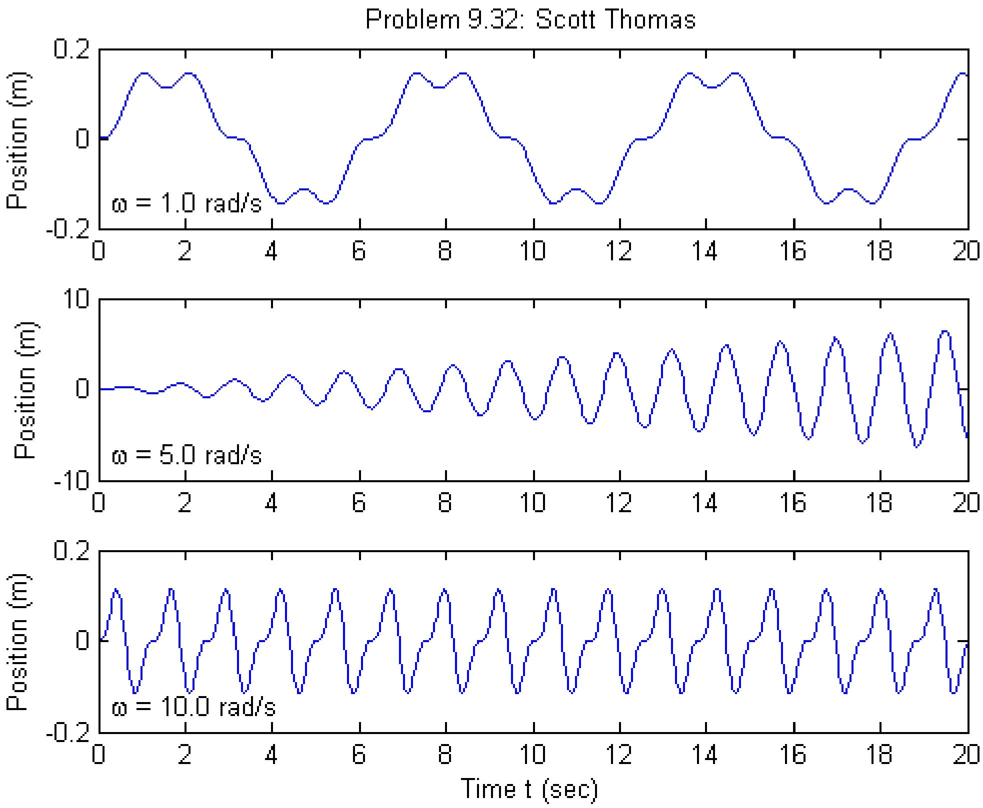
```
function xdot = f932a( t,x )
% Problem 9.32a: omega = 1 rad/s

omega = 1;

u = 10*sin(omega*t);
m = 3;
c = 0;
k = 75;

A = [0 1; -k/m -c/m];
B = [0; 1/m];
xdot = A*x + B*u;

end
```

```
function xdot = f932b( t,x )
% Problem 9.32a: omega = 5 rad/s

omega = 5;

u = 10*sin(omega*t);
m = 3;
c = 0;
k = 75;

A = [0 1; -k/m -c/m];
B = [0; 1/m];
xdot = A*x + B*u;

end
```

```
function xdot = f932c( t,x )
% Problem 9.32a: omega = 10 rad/s

omega = 10;

u = 10*sin(omega*t);
m = 3;
c = 0;
k = 75;

A = [0 1; -k/m -c/m];
B = [0; 1/m];
xdot = A*x + B*u;

end
```

```
% Problem 9.32
clear
clc
disp('Problem 9.32: Scott Thomas')

[ta,xa] = ode45(@f932a, [0 20], [0,0]);
[tb,xb] = ode45(@f932b, [0 20], [0,0]);
[tc,xc] = ode45(@f932c, [0 20], [0,0]);

subplot(3,1,1)
plot(ta, xa(:,1))
ylabel('Position (m)')
title('Problem 9.32: Scott Thomas')
text(0.3,-0.14,'\omega = 1.0 rad/s')

subplot(3,1,2)
plot(tb, xb(:,1))
ylabel('Position (m)')
text(0.3,-7,'\omega = 5.0 rad/s')

subplot(3,1,3)
plot(tc, xc(:,1))
xlabel('Time t (sec)'), ylabel('Position (m)')
text(0.3,-0.14,'\omega = 10.0 rad/s')
```

Problem 9.32: Scott Thomas



Problem 9.36:

36. The equations for an armature-controlled dc motor are the following. The motor's current is i and its rotational velocity is ω.

$$L\frac{di}{dt} = -Ri - K_e\omega + v(t) \tag{9.6–1}$$

$$I\frac{d\omega}{dt} = K_T i - c\omega \tag{9.6–2}$$

where L, R, and I are the motor's inductance, resistance, and inertia; K_T and K_e are the torque constant and back emf constant; c is a viscous damping constant; and $v(t)$ is the applied voltage.

Use the values $R = 0.8\ \Omega$, $L = 0.003$ H, $K_T = 0.05$ N · m/A, $K_e =$ 0.05 V · s/rad, $c = 0$, and $I = 8 \times 10^{-5}$ kg · m^2.

a. Suppose the applied voltage is 20 V. Plot the motor's speed and current versus time. Choose a final time large enough to show the motor's speed becoming constant.

b. Suppose the applied voltage is trapezoidal as given below.

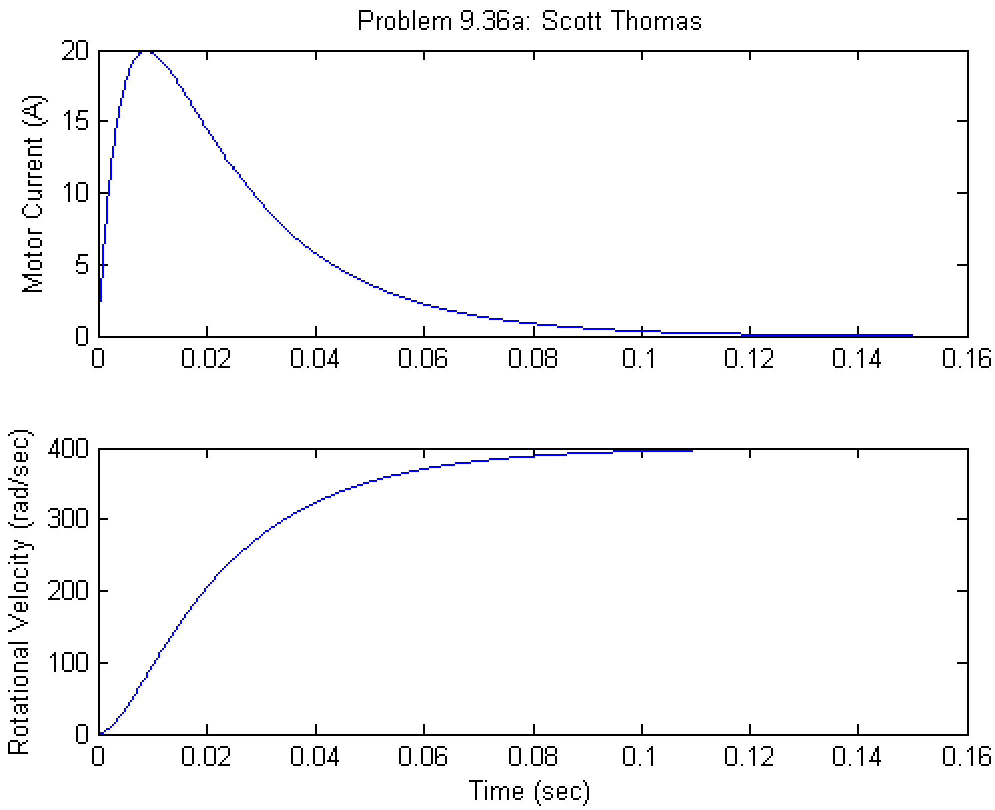
$$v(t) = \begin{cases} 400t & 0 \le t < 0.05 \\ 20 & 0.05 \le t \le 0.2 \\ -400(t - 0.2) + 20 & 0.2 < t \le 0.25 \\ 0 & t > 0.25 \end{cases}$$

Plot the motor's speed versus time for $0 \le t \le 0.3$ s. Also plot the applied voltage versus time. How well does the motor speed follow a trapezoidal profile?

<u>Use the Control System Toolbox for this problem.</u>

```
% Problem 9.36a

clc
clear
disp('Problem 9.36a: Scott Thomas')

R = 0.8;
L = 0.003;
KT = 0.05;
Ke = 0.05;
c = 0;
I = 8e-5;

A = [-R/L, -Ke/L; KT/I -c/I];
B = [1/L; 0];
C = [1 0];
D = 0;
sys = ss(A, B, C, D);
N = 1000;
t = linspace(0,0.15,N);
for k = 1:N
v(k) = 20;
end
[i,t] = lsim(sys,v,t);
subplot(2,1,1)
plot(t,i), ylabel('Motor Current (A)')
title('Problem 9.36a: Scott Thomas')

C = [0 1];
sys = ss(A, B, C, D);
[omega,t] = lsim(sys,v,t);
subplot(2,1,2)
plot(t,omega),xlabel('Time (sec)'),ylabel('Rotational Velocity (rad/sec)')
```

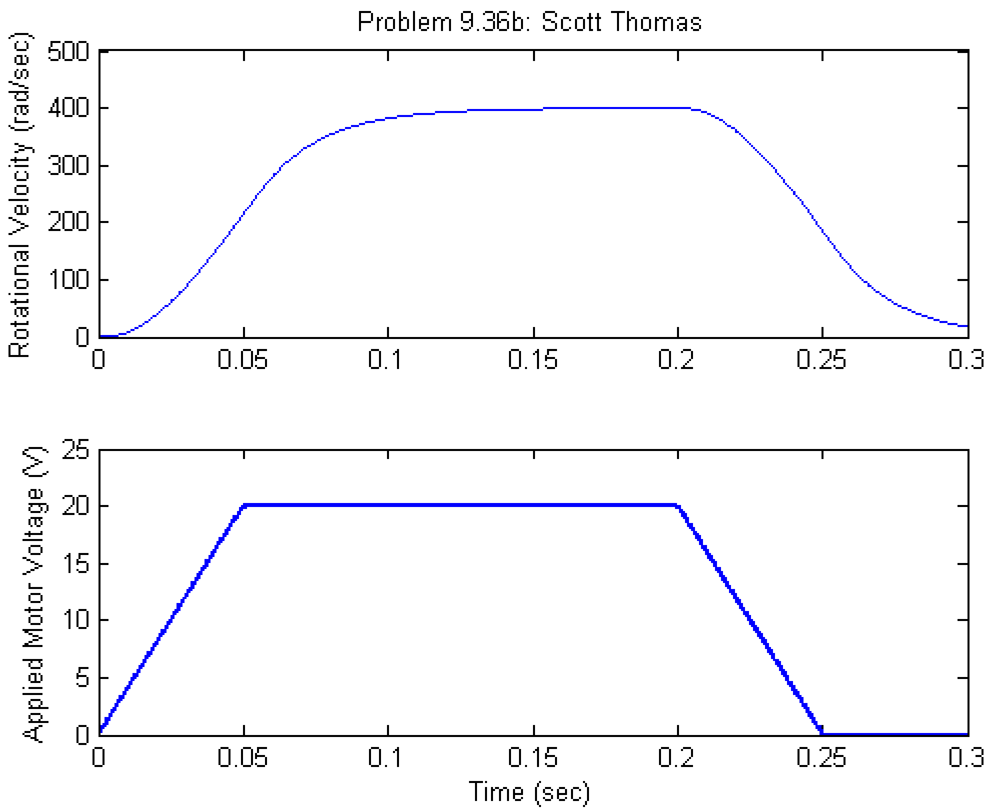
```
Problem 9.36a: Scott Thomas
```




```
% Problem 9.36b

clc
clear
disp('Problem 9.36b: Scott Thomas')

R = 0.8;
L = 0.003;
KT = 0.05;
Ke = 0.05;
c = 0;
I = 8e-5;

A = [-R/L, -Ke/L; KT/I -c/I];
B = [1/L; 0];
C = [0 1];
D = 0;
sys = ss(A, B, C, D);
N = 1000;
t = linspace(0,0.3,N);
```

```
for k = 1:N
    if t(k) < 0.05
        v(k) = 400*t(k);
    elseif t(k) > 0.05 & t(k) < 0.2
        v(k) = 20;
    elseif t(k) > 0.2 & t(k) < 0.25
        v(k) = -400*(t(k) - 0.2) + 20;
    else
        v(k) = 0;
    end
end
[omega,t] = lsim(sys,v,t);
subplot(2,1,1)
plot(t,omega),ylabel('Rotational Velocity (rad/sec)')
title('Problem 9.36b: Scott Thomas')
axis([0 0.3 0 500])

subplot(2,1,2)
plot(t,v, 'LineWidth',2), ylabel('Applied Motor Voltage (V)')
xlabel('Time (sec)')
axis([0 0.3 0 25])
```

Problem 9.36b: Scott Thomas



Problem 9.40:

40. The following state model describes the motion of a certain mass connected to a spring, with viscous friction on the surface, where $m = 1$, $c = 2$, and $k = 5$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t)$$

a. Use the `initial` function to plot the position x_1 of the mass, if the initial position is 5 and the initial velocity is 3.

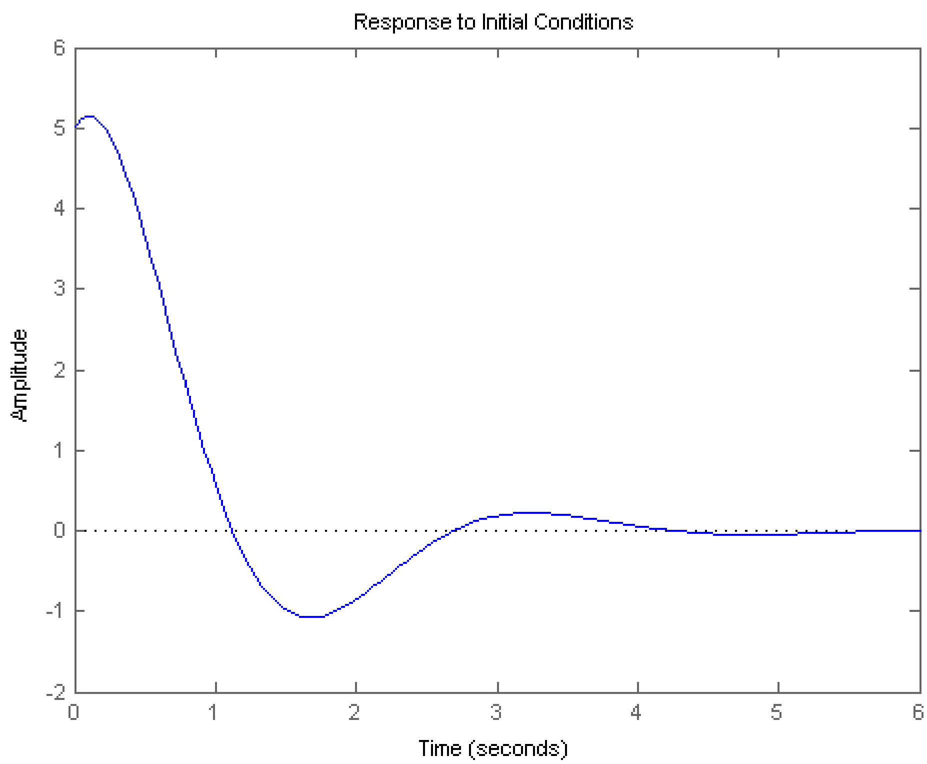
b. Use the `step` function to plot the step response of the position and velocity for zero initial conditions, where the magnitude of the step input is 10. Compare your plot with that shown in Figure 9.5–1.

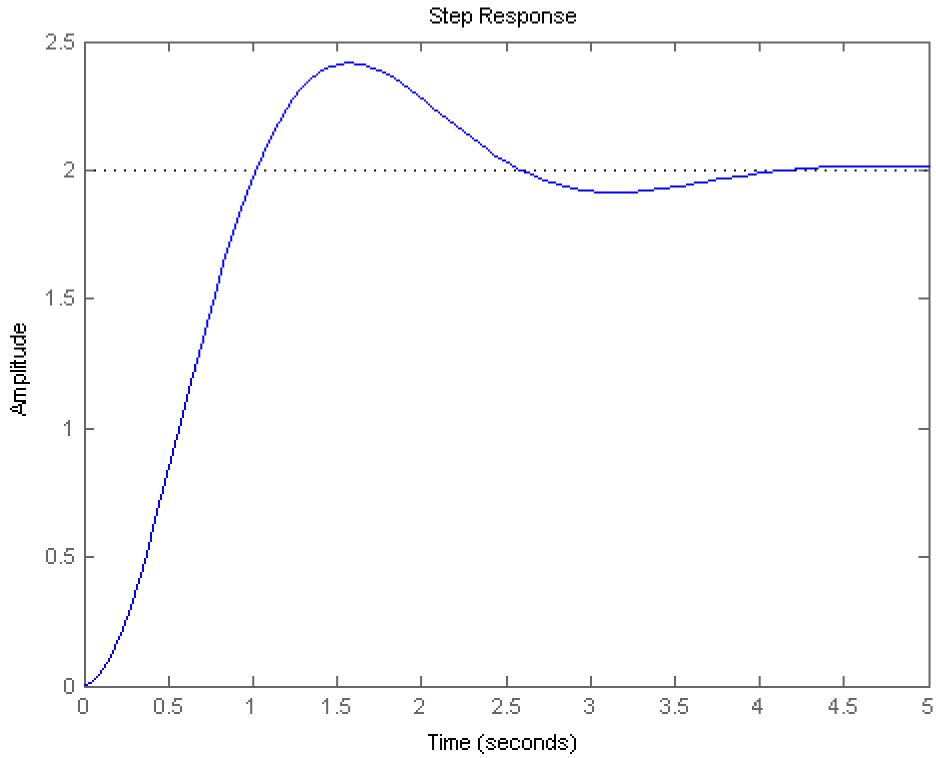
Use the Control System Toolbox for this problem.

```
% Problem 9.40

clc
clear
disp('Problem 9.40: Scott Thomas')

A = [0, 1; -5, -2]
B = [0; 1]
C = [1, 0]
D = 0
sys = ss(A,B,C,D)
x0 = [5, 3]
figure
initial(sys,x0)
figure
step(10*sys) See page 404
```

```
Problem 9.40: Scott Thomas

A =

     0     1
    -5    -2


B =

     0
     1


C =

     1     0
```

```
D =

     0


sys =

  a =
       x1  x2
   x1   0   1
   x2  -5  -2

  b =
       u1
   x1   0
   x2   1

  c =
       x1  x2
   y1   1   0

  d =
       u1
   y1   0

Continuous-time state-space model.


x0 =

     5     3
```




Step Response
Amplitude
Time (seconds)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5