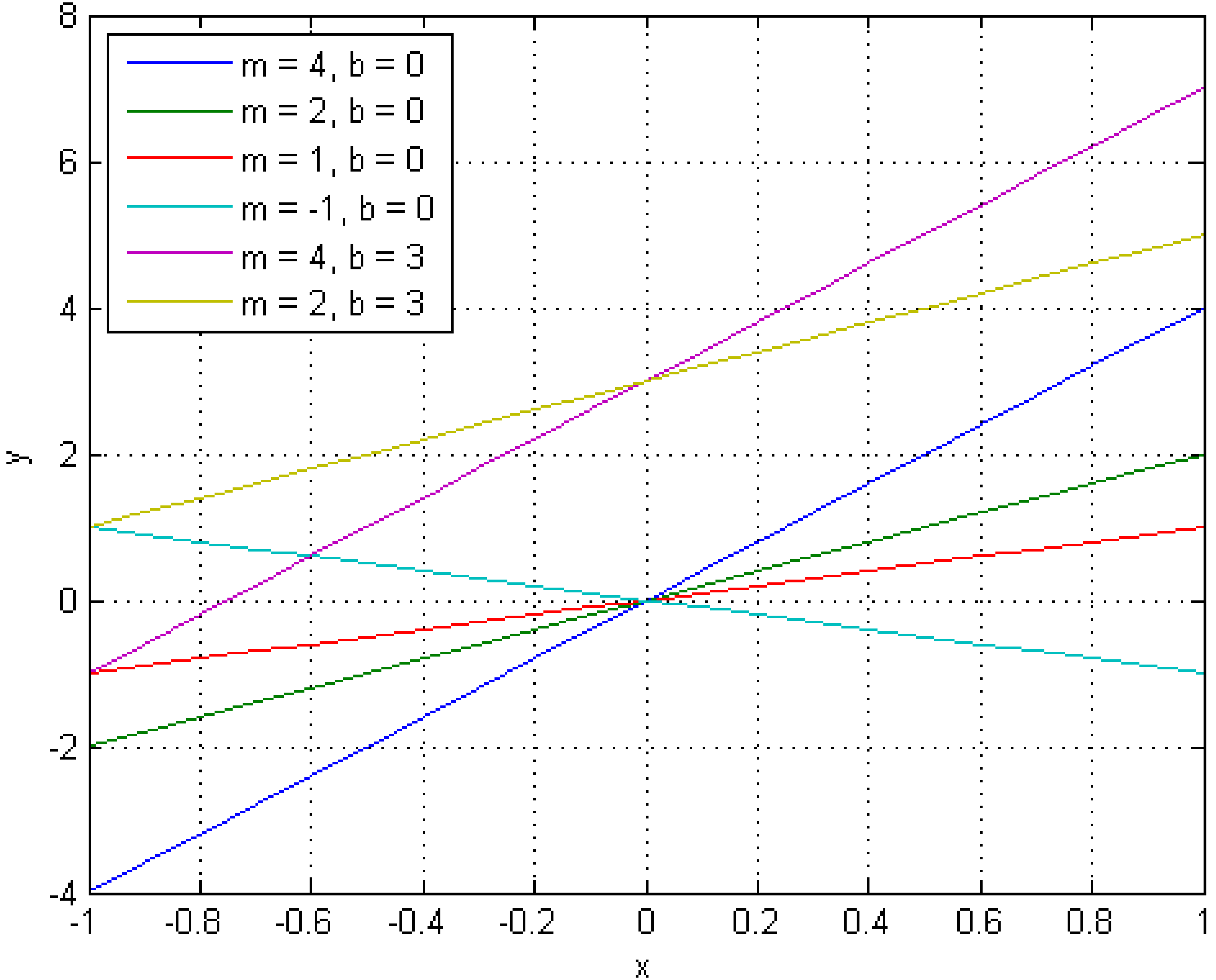
Chapter 6: Model Building and Regression

- Engineers take experimentally determined data and attempt to fit curves to it for analysis.

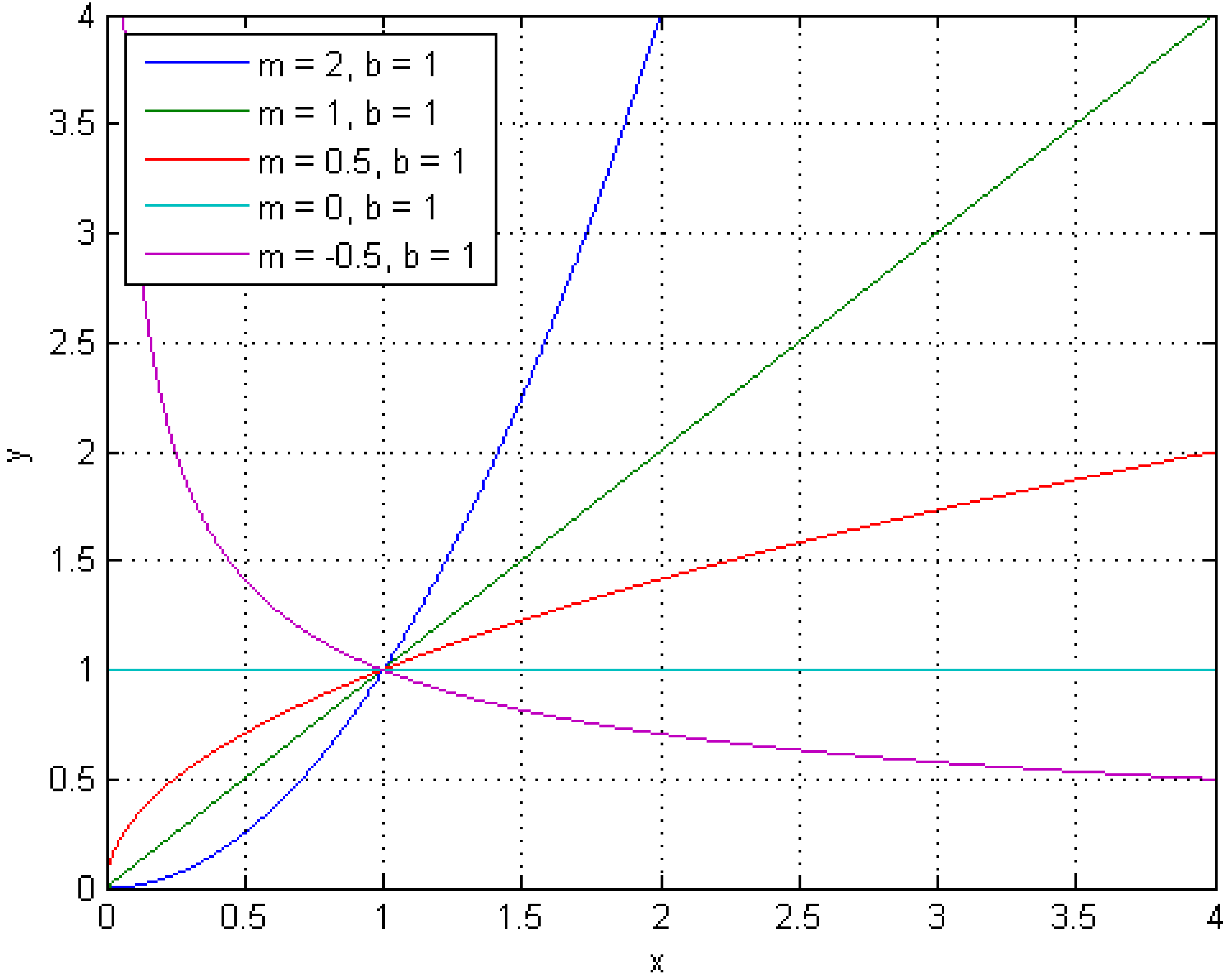
  - Linear: $y(x) = mx + b$ (m = slope, b = y-intercept)
  - Power: $y(x) = bx^m$
  - Exponential: $y(x) = b(10)^{mx}$ or $y(x) = be^{mx}$ where e is the base of the natural logarithm ($\ln e = 1$)

- Regression uses the Least-Squares Method to find an equation that best fits the given data.

Linear Function: $y(x) = mx + b$

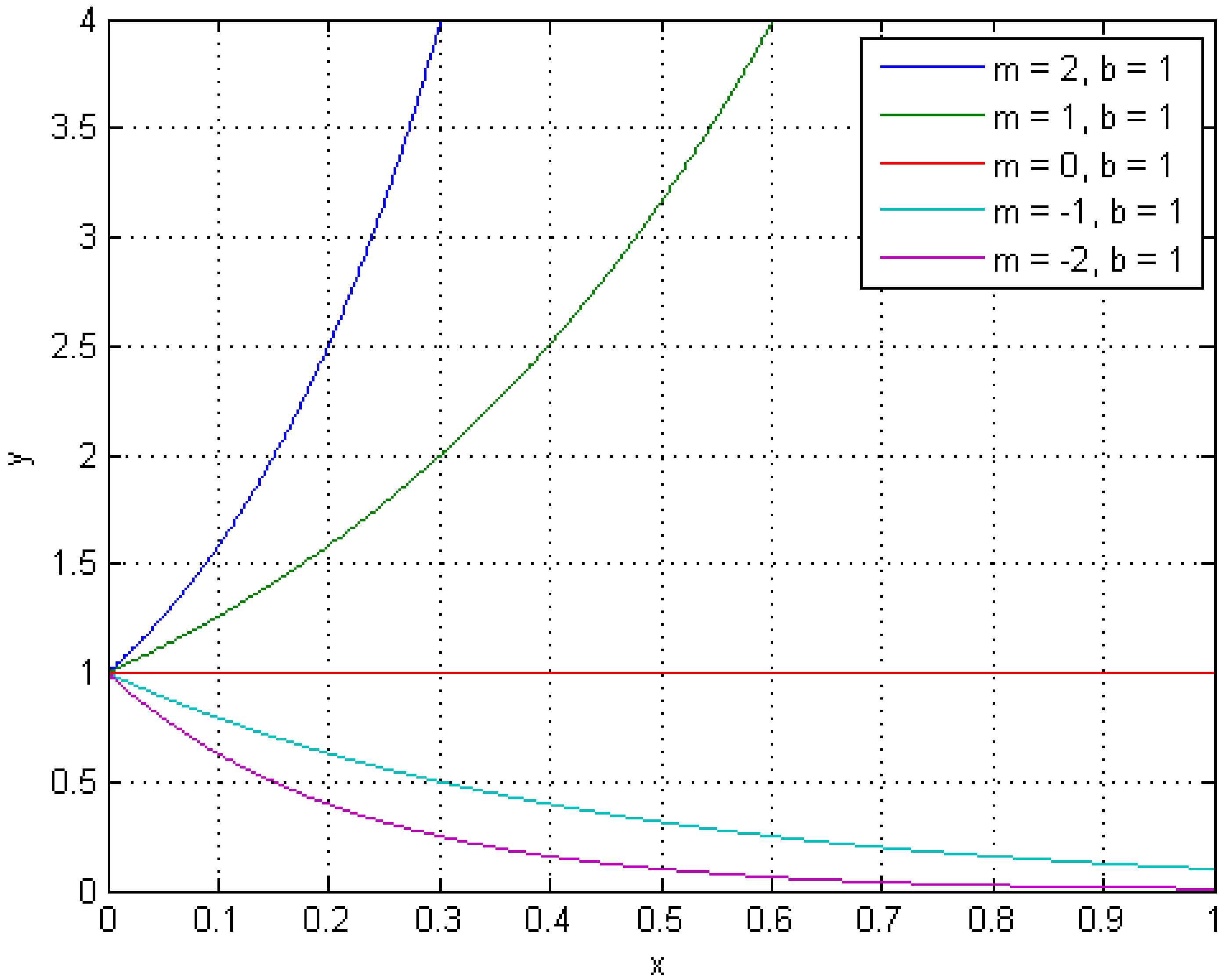


Power Function: $y(x) = bx^m$



Exponential Function:

$$y(x) = b(10)^{mx}$$



Function Discovery

Linear Functions: Linear on rectilinear plot (x,y)
Power-Law Functions: Linear on log-log plot (log10 x, log10 y)
Exponential Functions: Linear on semi-log y plot (x, log10 y)

Once the function type is determined, use the polyfit function to determine the curve fit equation.

For an original data set (*x*,*y*), the polyfit function returns coefficients for the linear curve fit model $w = p_1 z + p_2$

$$p = \text{polyfit}(x, y, 1)$$

where $p_1 = p(1)\ and\ p_2 = p(2)$

Function Discovery

Linear Functions:

$y(x) = mx + b$ (m = slope, b = y-intercept)

$p = \text{polyfit}(x, y, 1)$

$w(z) = p(1)z + p(2)$

Power-Law Functions: $y(x) = bx^m$

$p = \text{polyfit}(\log10(x), \log10(y), 1)$

$w(z) = 10^{p(2)} z^{p(1)}$

Exponential Functions: $y(x) = b(10)^{mx}$

$p = \text{polyfit}(x, \log10(y), 1)$

$w(z) = 10^{p(2)} (10)^{p(1)z}$

Function Discovery

Open a new MATLAB Script file. Type in the following data:

t	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
w	6.00	4.83	3.70	3.15	2.41	1.83	1.49	1.21	0.96	0.73	0.64

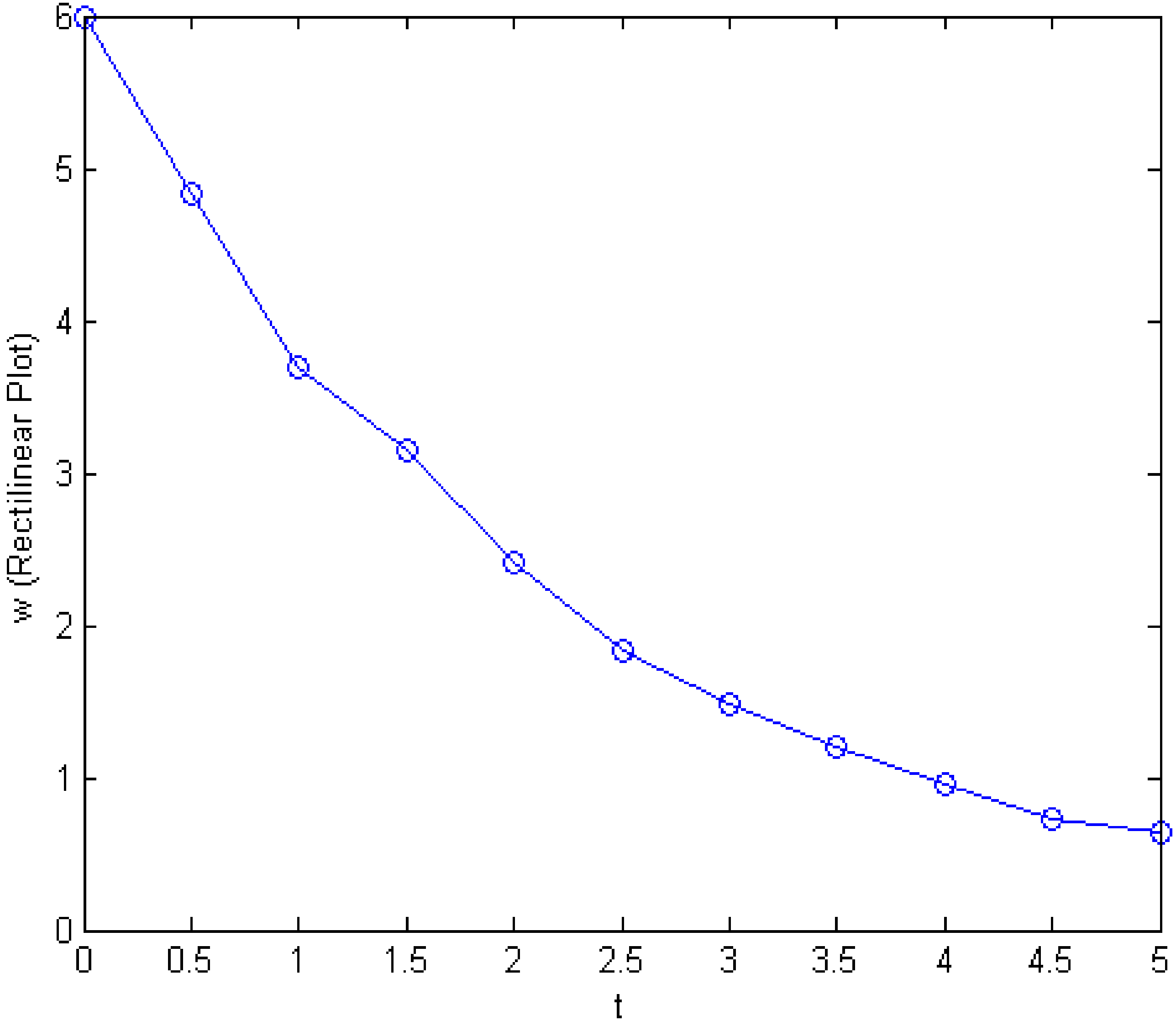
Plot the data using rectilinear coordinates, as shown below.

```
t = 0:0.5:5.0;
w = [6 4.83 3.7 3.15 2.41 1.83 1.49 1.21 0.96 0.73 0.64];

% Linear Fit
figure
plot( t, w,'-o'), xlabel('t'),ylabel('w (Rectilinear Plot)')
```

Function Discovery

This plot shows that the data is not a Linear Function.



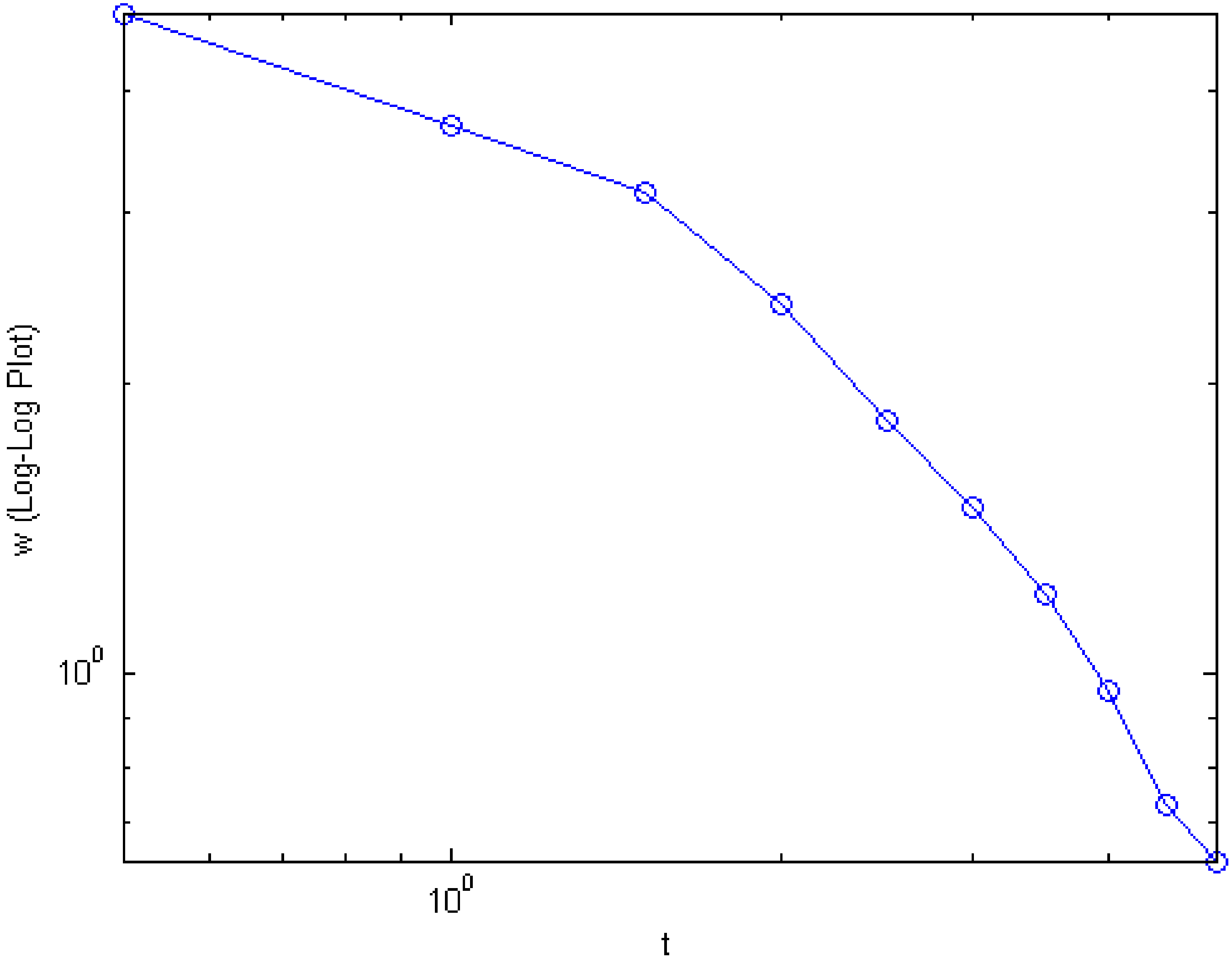
Function Discovery

Now plot the data using log-log coordinates, as shown below.

```
% Power-Law Fit
figure
loglog( t, w,'-o'), xlabel('t'),ylabel('w (Log-Log Plot)')
```

Function Discovery

This plot shows that the data is not a Power-Law Function because it is not linear on log-log coordinates.



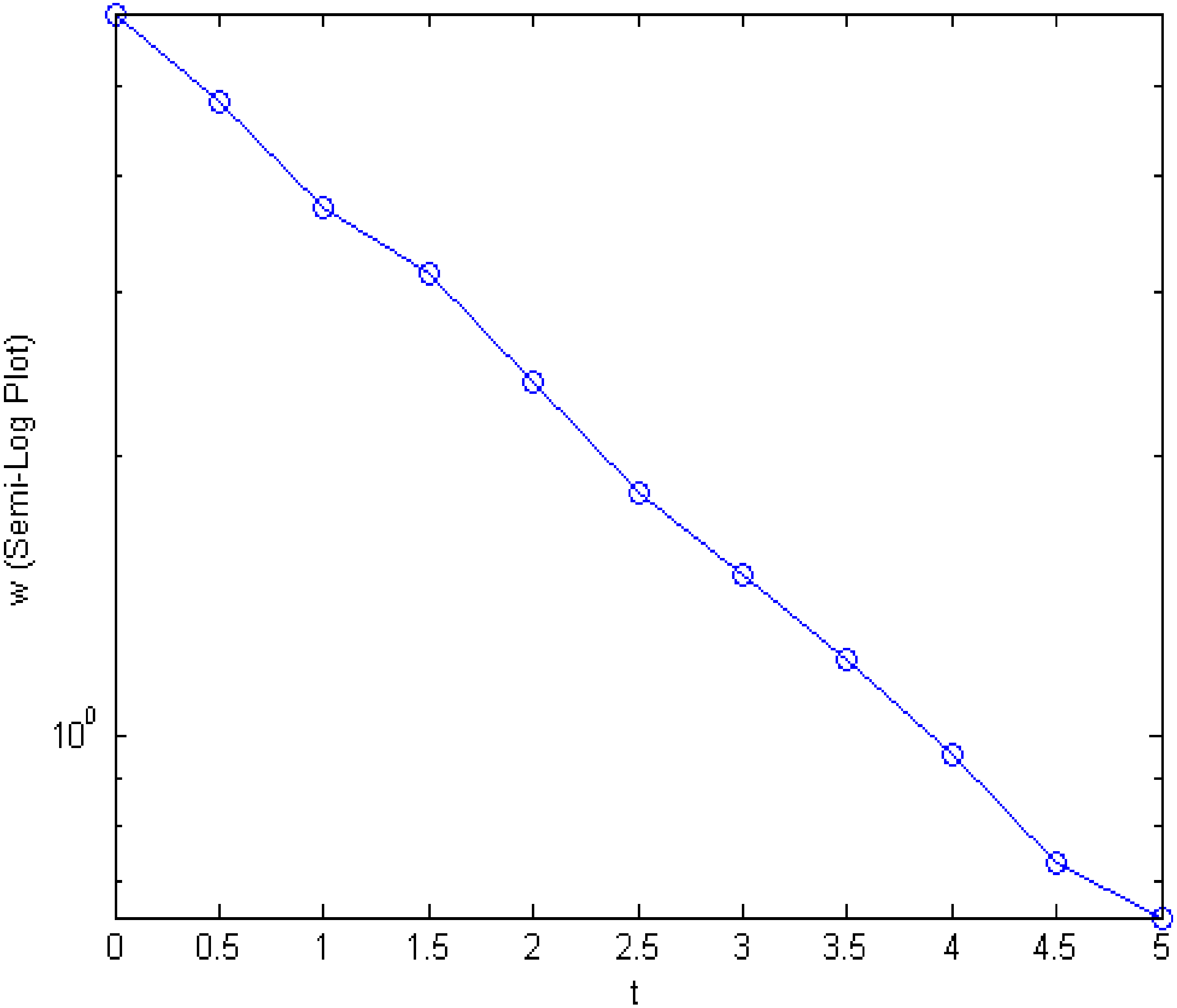
Function Discovery

Now plot the data using semi-log y coordinates, as shown below.

```
% Exponential Fit
figure
semilogy( t, w,'-o'), xlabel('t'),ylabel('w (Semi-Log Plot)')
```

Function Discovery

This plot shows that the data is an Exponential Function because it is linear on semi-log y coordinates.



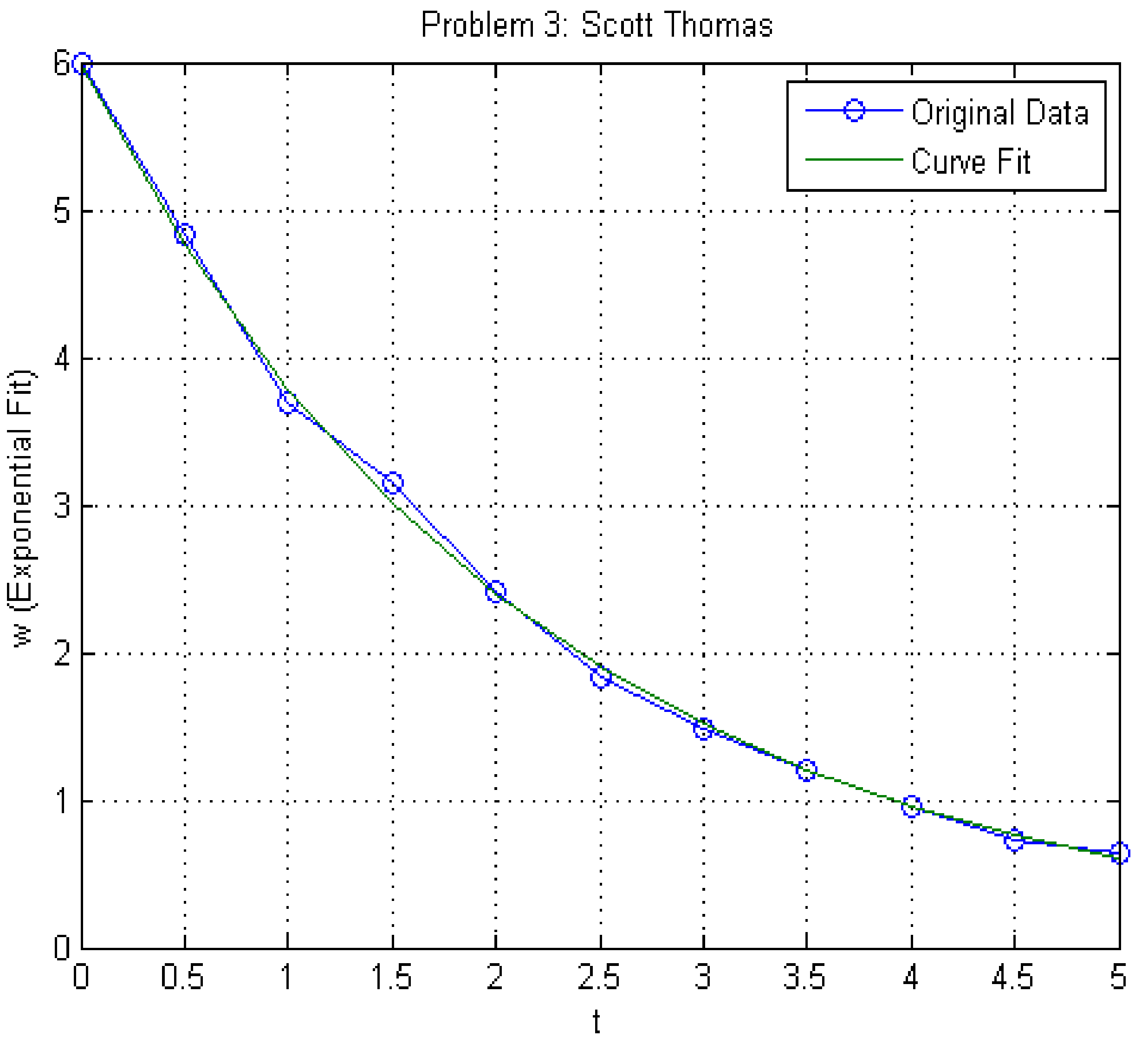
Function Discovery

Now use the polyfit command to construct an Exponential Function that can be used to approximate the original data. Plot the original data and the curve-fit model on the same graph. Use this model to estimate the value of *w* at *t* = 0.25:

```
% Exponential Fit
p = polyfit(t, log10(w),1); % generates coefficients for curve fit
t2 = linspace(0,5,100); % generates a new t vector for curve fit
w2 = 10^(p(2))*10.^(p(1)*t2); % generates new w vector using t2
% Estimate w at t = 0.25:
t_025 = 0.25;
w_025 = 10^(p(2))*10.^(p(1)*t_025)
figure
plot(t,w,'o',t2,w2,t_025,w_025), xlabel('t'),ylabel('w (Exponential Fit)')
legend('Original Data', 'Curve Fit', ‘w @ t = 2.5 s' )
```

Function Discovery

w_025 = 5.3410



Regression

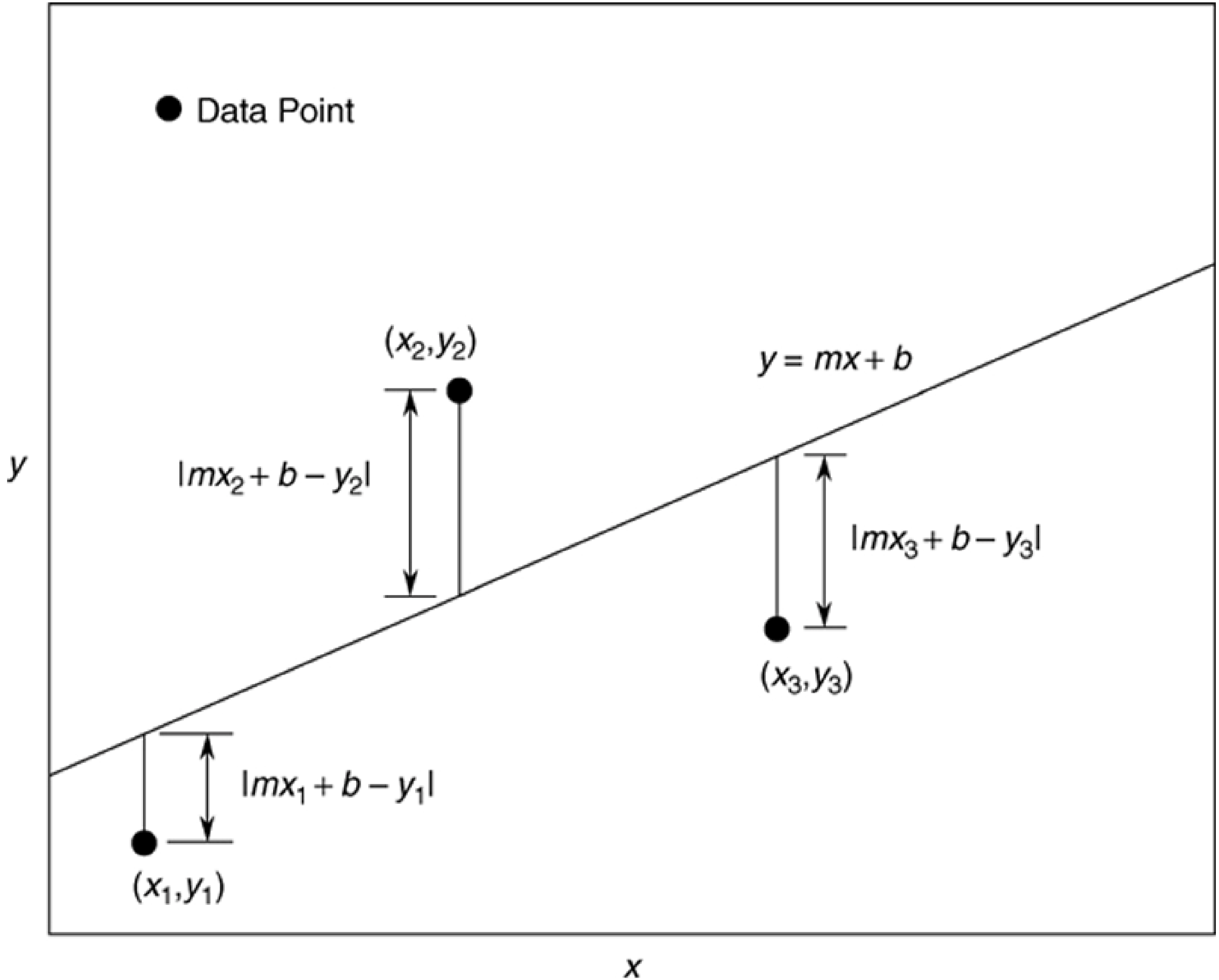
The Least-Squares Method minimizes the vertical differences (Residuals) between the data points and the predictive equation. This gives the line that best fits the data. For a linear curve (First Order) fit:

$$J = \sum_{i=1}^{n} (mx_i + b - y_i)^2$$

where the equation of a straight line is

$$y(x) = mx + b$$

Regression

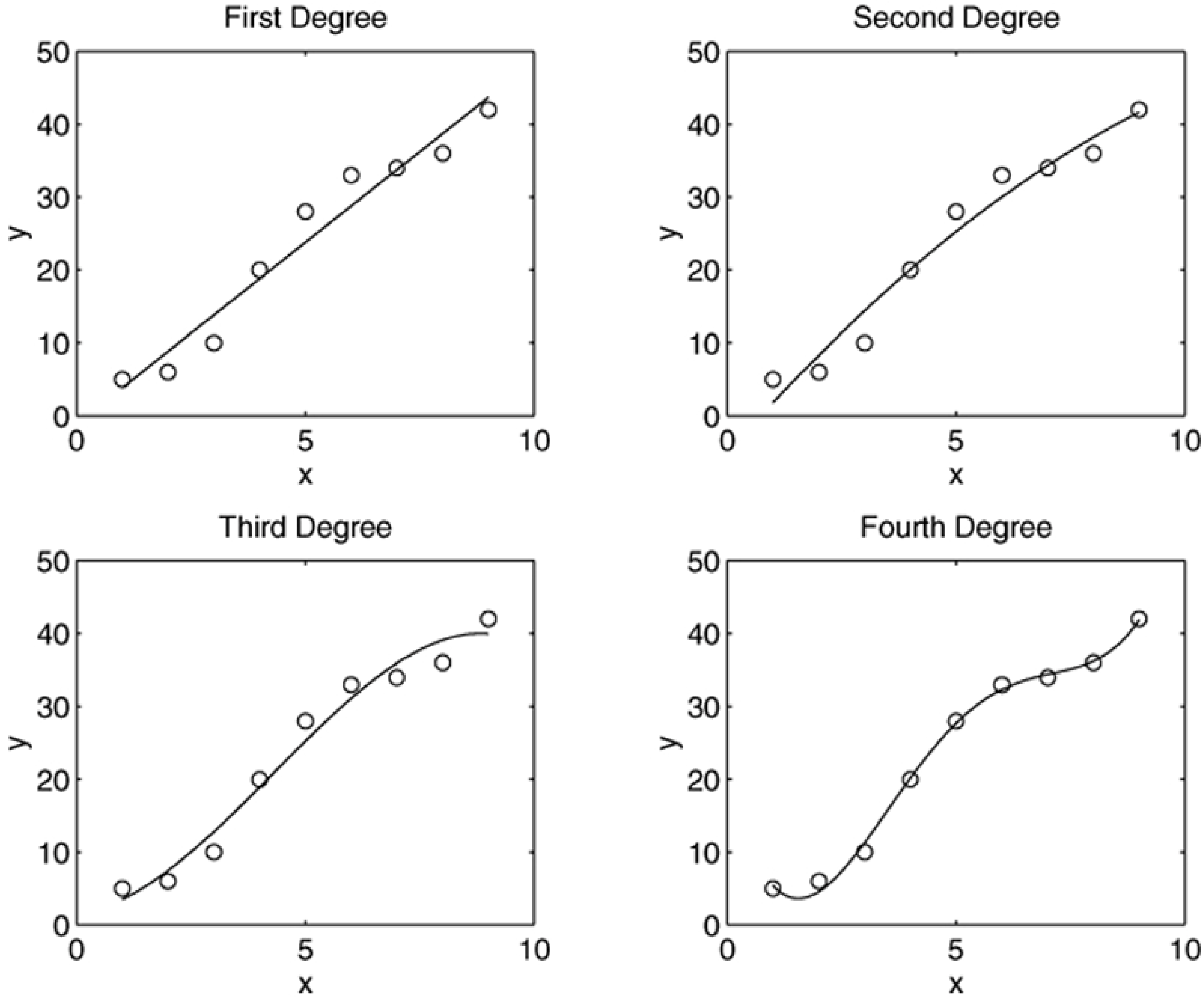


Regression

The curve fit can be improved by increasing the order of the polynomial. Increasing the degree of the polynomial increases the number of coefficients:

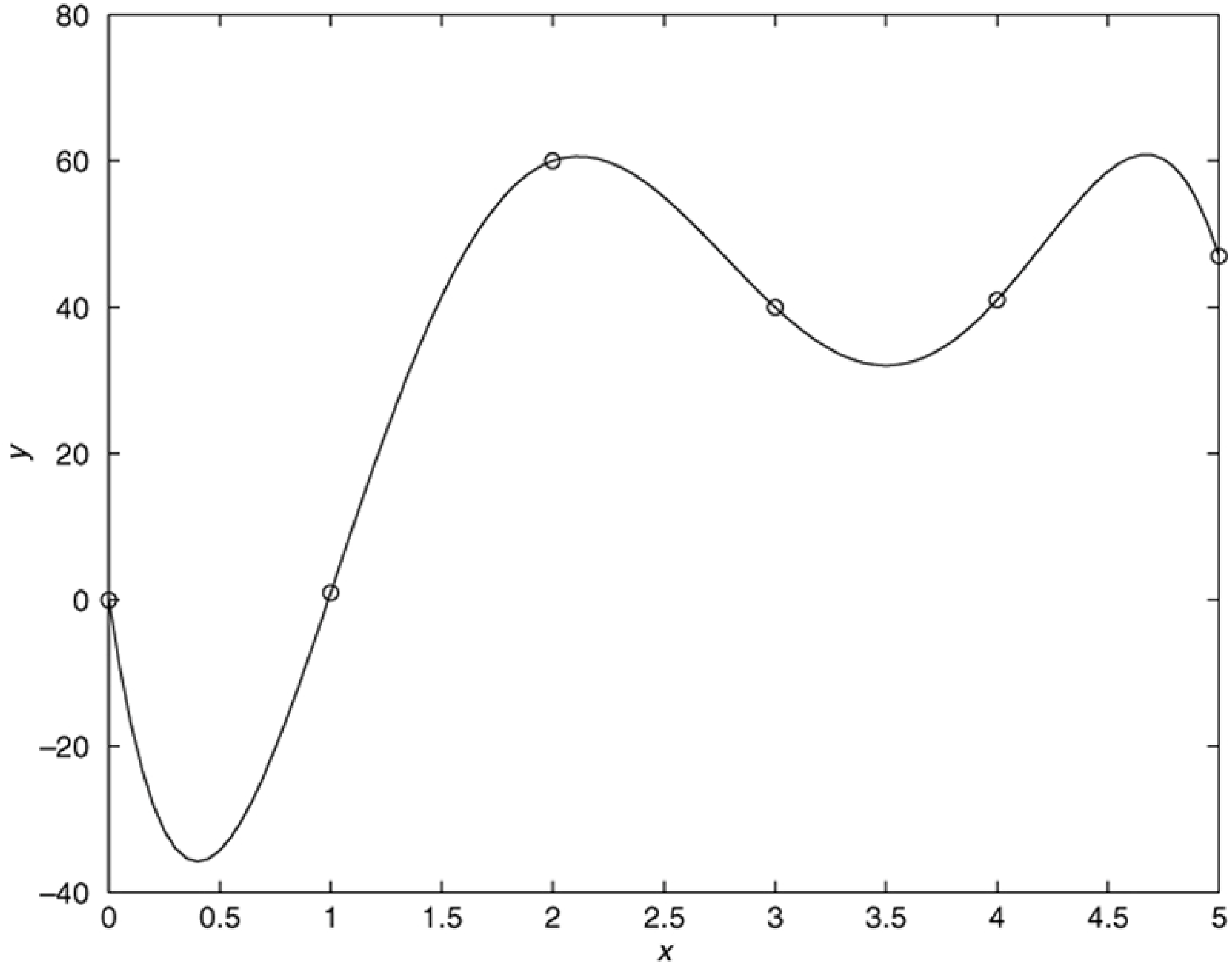
- First Degree: $y(x) = a_1x + a_0$
- Second Degree: $y(x) = a_2x^2 + a_1x + a_0$
- Third Degree: $y(x) = a_3x^3 + a_2x^2 + a_1x + a_0$
- Fourth Degree: $y(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

Regression


First Degree
Second Degree
Third Degree
Fourth Degree
y
x
0
5
10
10
20
30
40
50

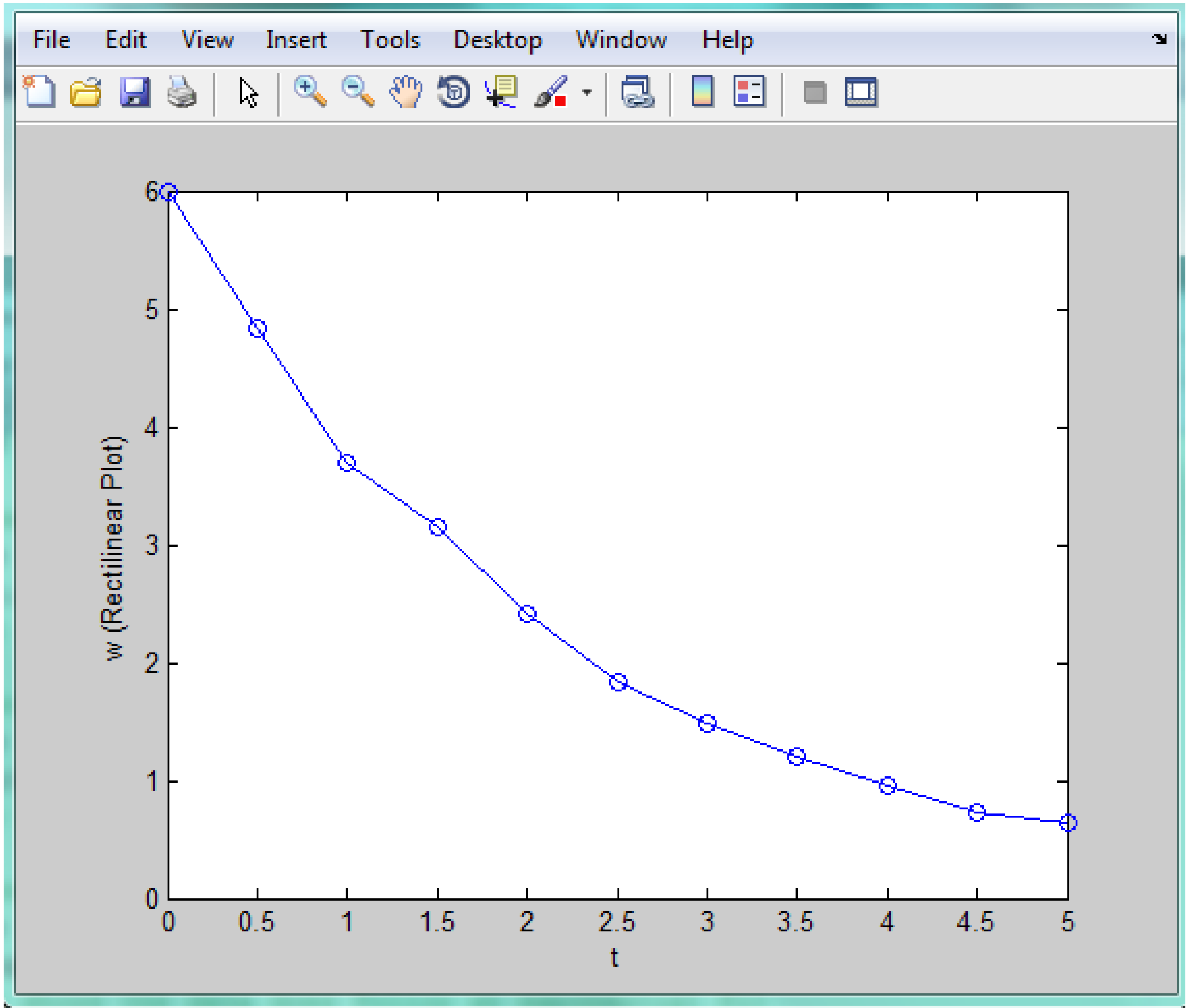
Regression

Having a very high-order polynomial doesn't necessarily mean a better fit. The objective is to be able to use the equation to predict values between the data points.



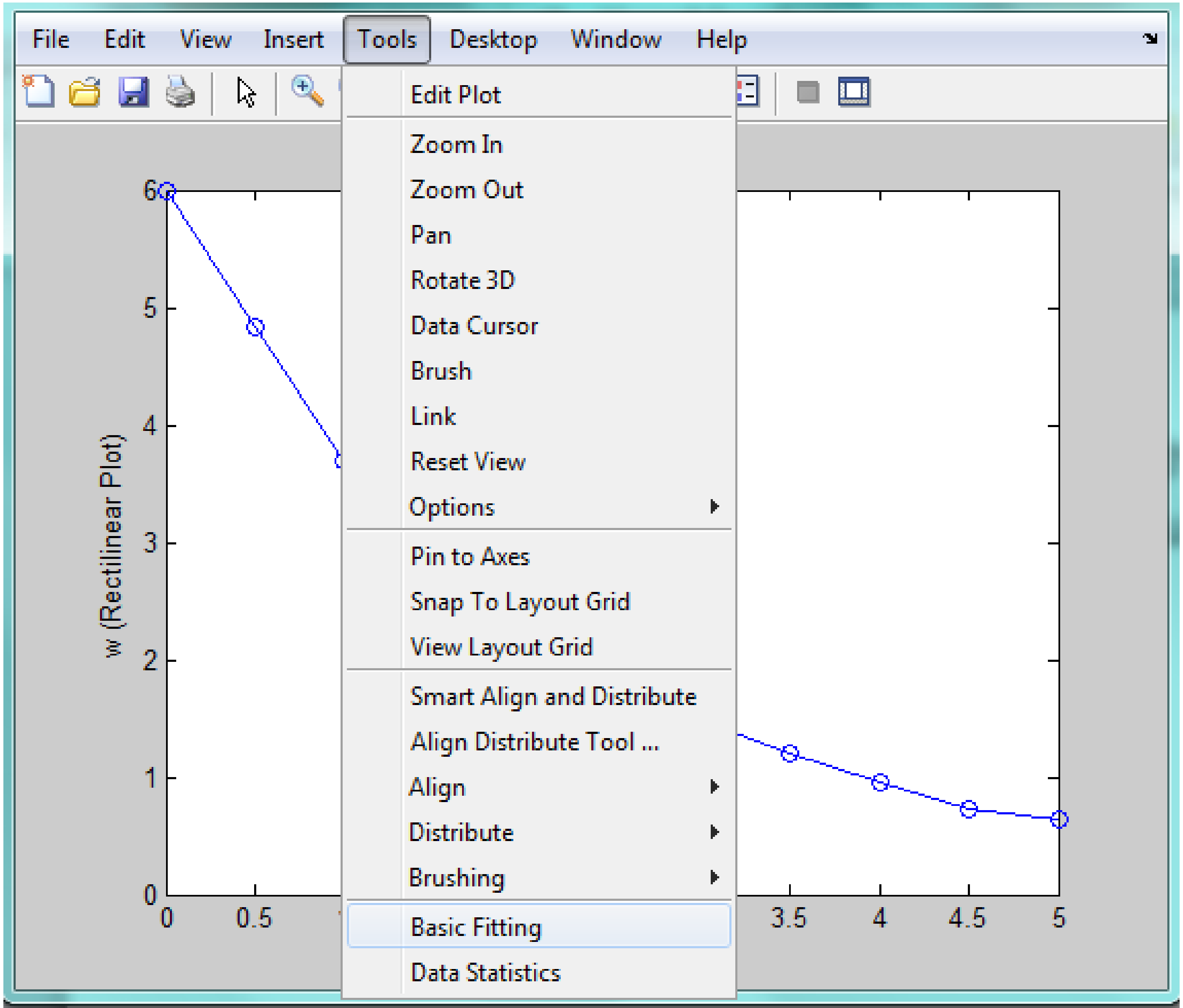
Basic Fitting Interface

Use the previously developed Script File to use the Basic Fitting Interface.



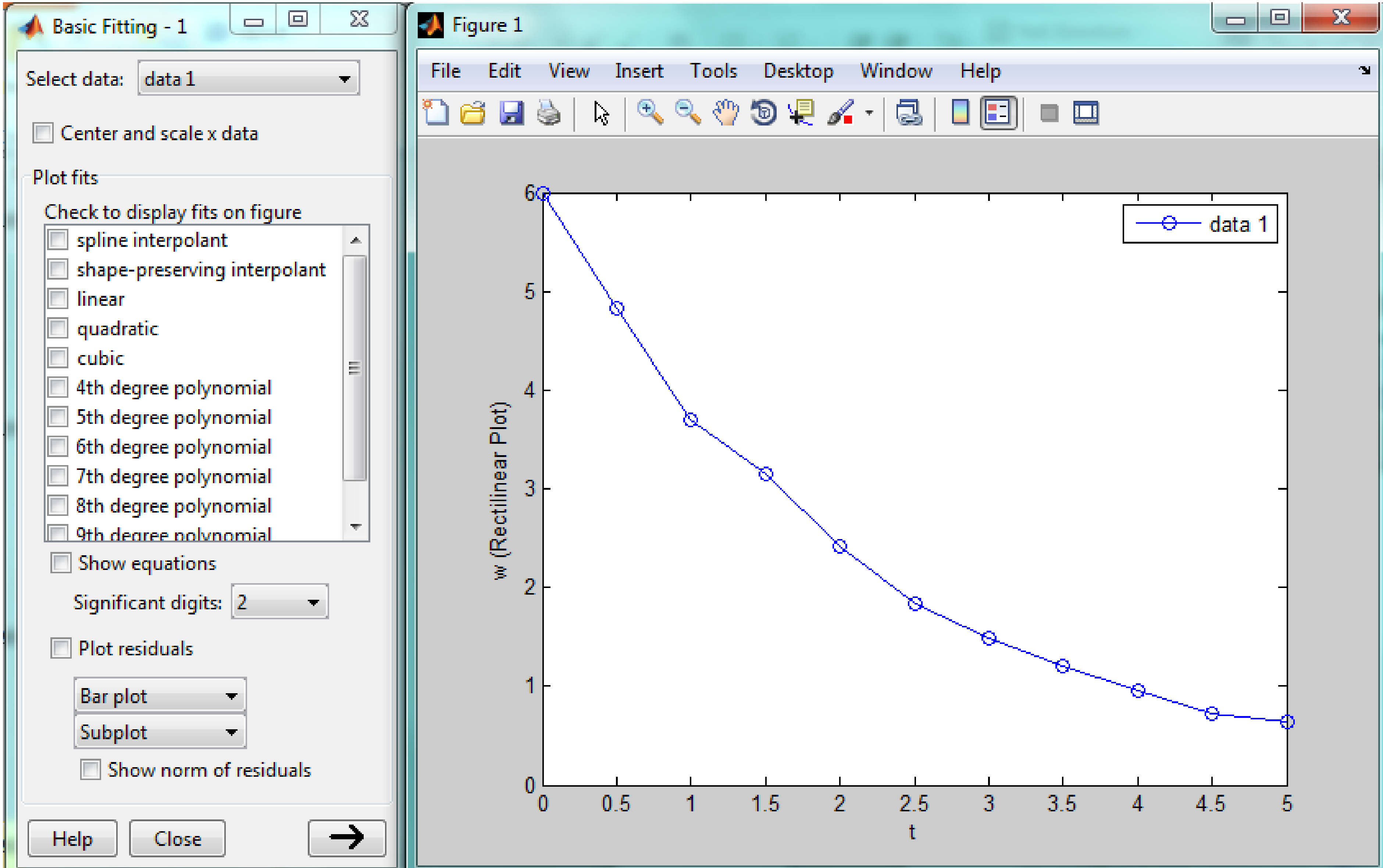
Basic Fitting Interface

Use the Tools Drop-Down Menu and go to Basic Fitting.



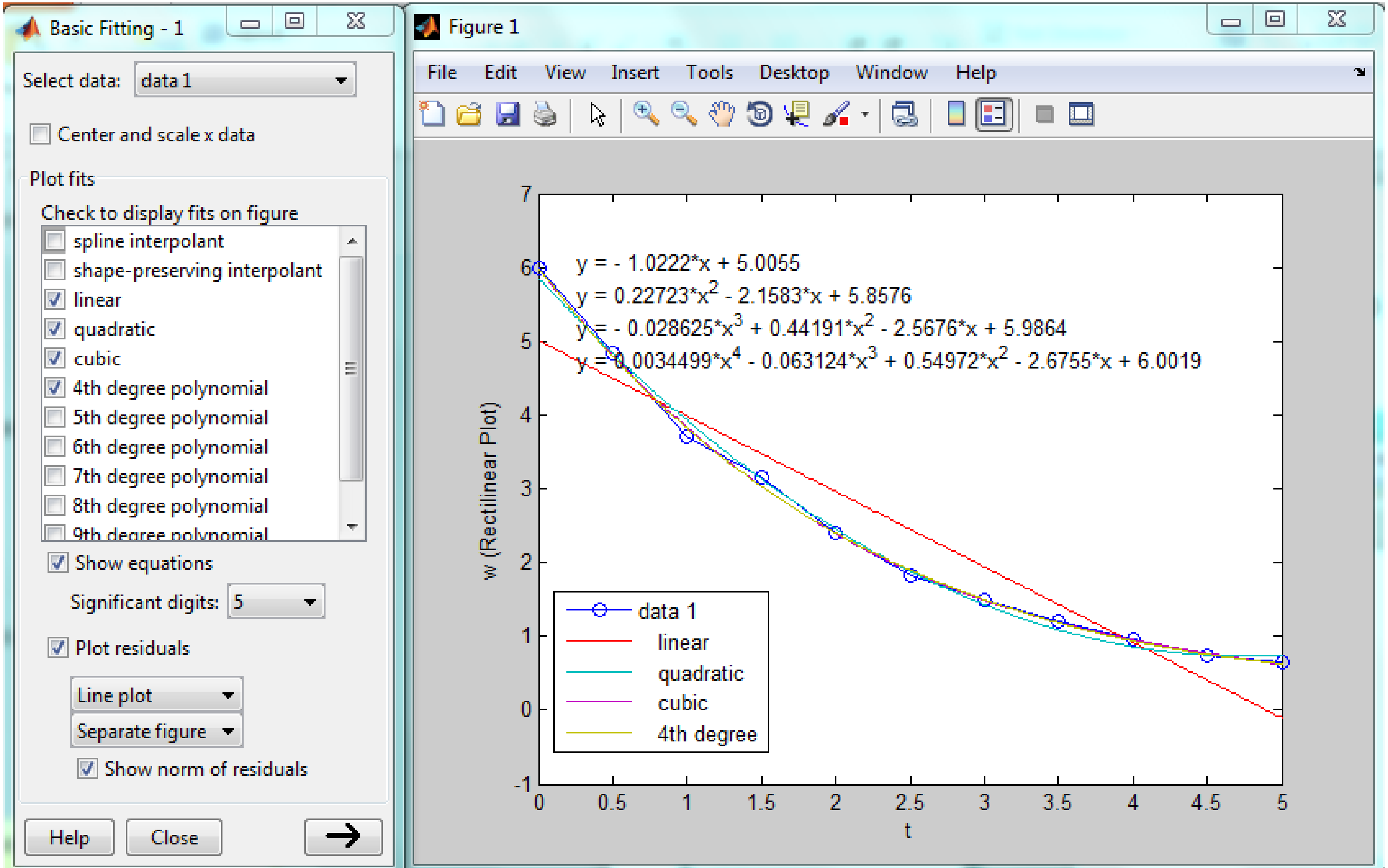
Basic Fitting Interface

Use the Tools Drop-Down Menu and go to Basic Fitting.



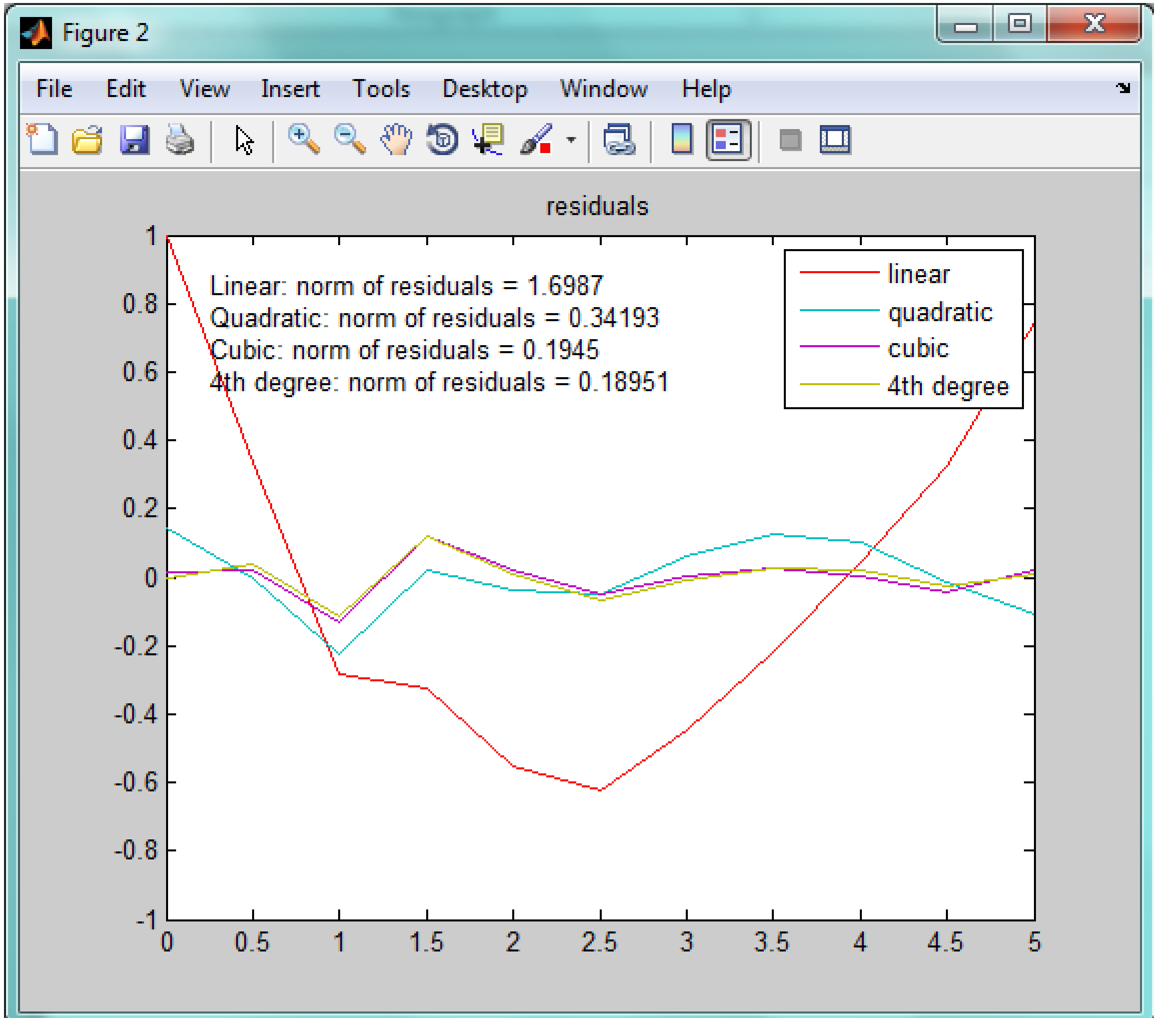
Basic Fitting Interface

Check the boxes indicated below. Change the number of Significant Digits to 5.



Basic Fitting Interface

The Residuals Plot is shown below. The norm of the residuals is a measure of the "Goodness of Fit." A smaller value is preferable.

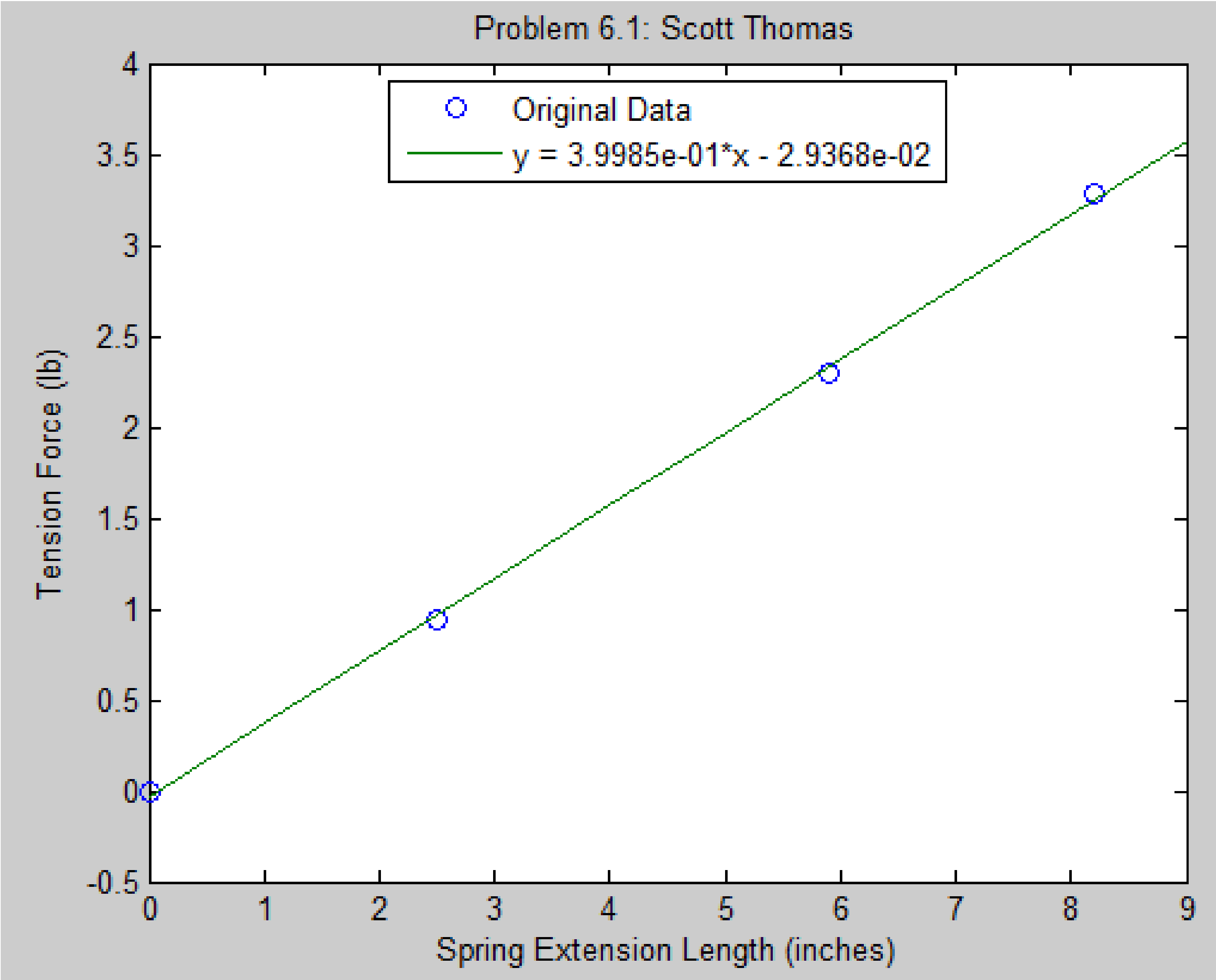


Problem 6.1:

The distance a spring stretches from its "free length" is a function of how much tension force is applied to it. The following table gives the spring length y that the given applied force f produced in a particular spring. The spring's free length is 4.7 in. Find a functional relation between f and x, the extension from the free length ($x = y - 4.7$).

Force f (lb)	Spring length y (in.)
0	4.7
0.94	7.2
2.30	10.6
3.28	12.9

Problem 6.1:



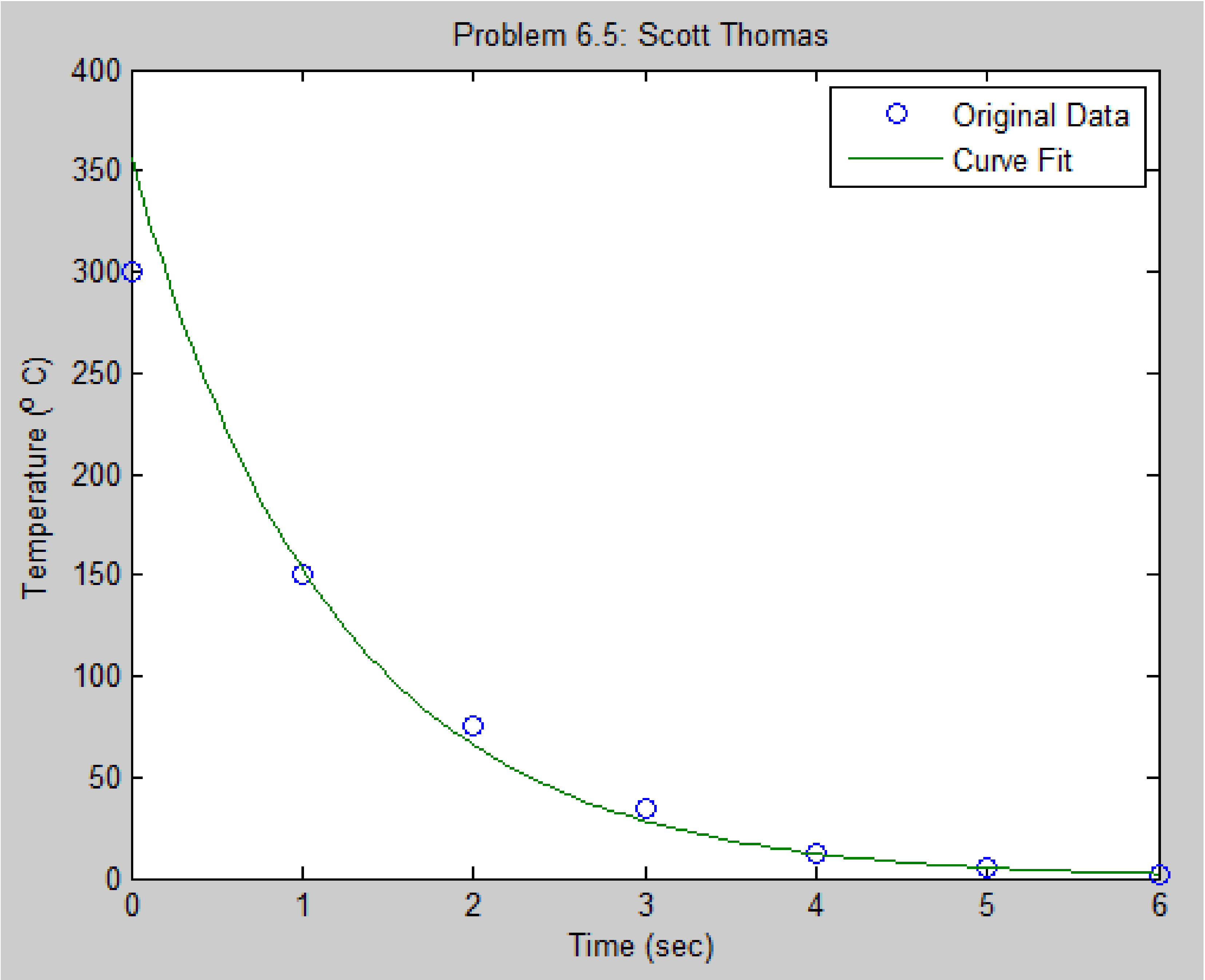
Problem 6.1: Scott Thomas
Original Data
y = 3.9985e-01*x - 2.9368e-02
Tension Force (lb)
Spring Extension Length (inches)
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
6
7
8
9

Problem 6.5:

Quenching is the process of immersing a hot metal object in a bath for a specified time to obtain certain properties such as hardness. A copper sphere 25 mm in diameter, initially at 300°C, is immersed in a bath at 0°C. The following table gives measurements of the sphere's temperature versus time. Find a functional description of these data. Plot the function and the data on the same plot.

Time (s)	0	1	2	3	4	5	6
Temperature (°C)	300	150	75	35	12	5	2

Problem 6.5:

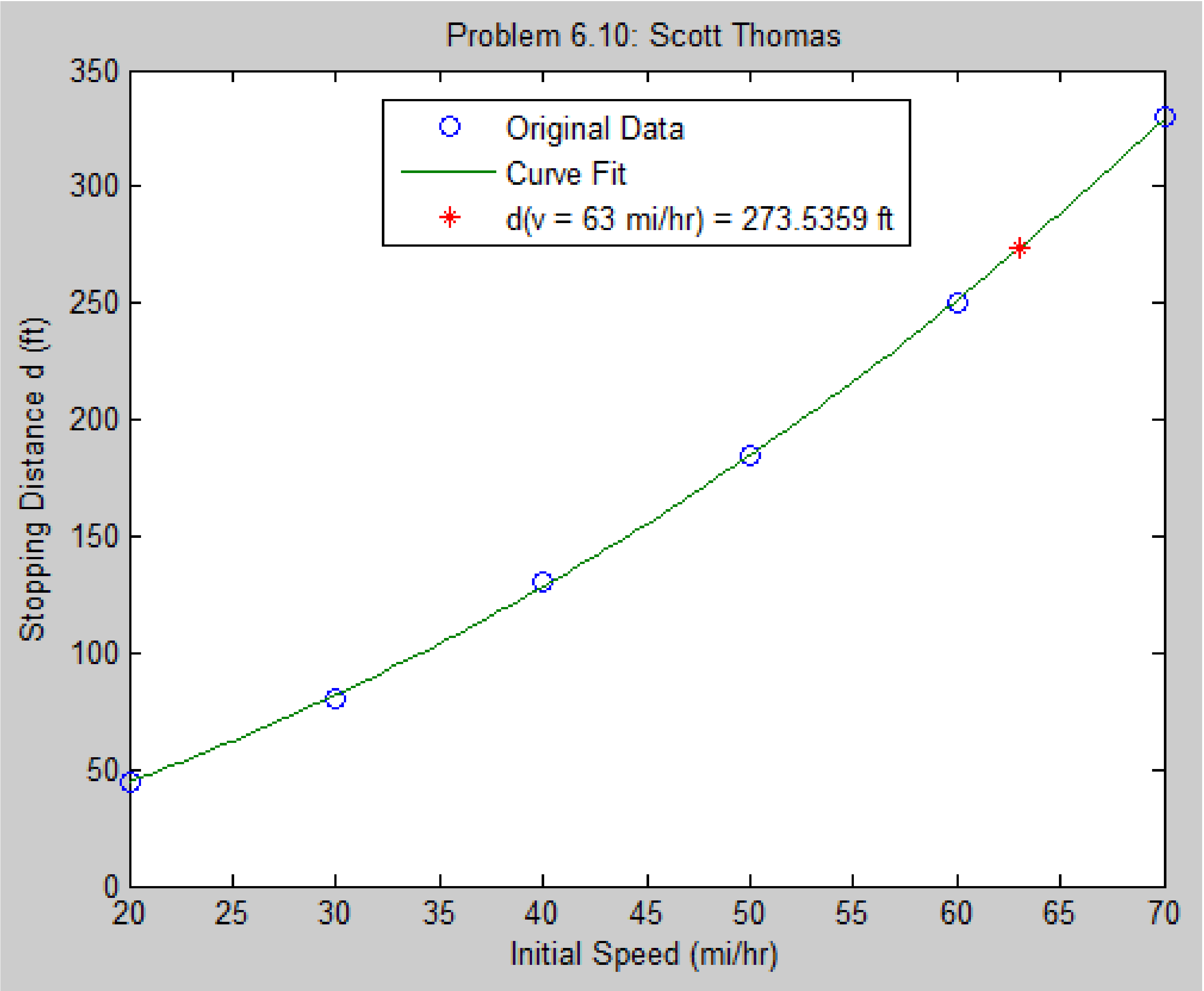

Problem 6.5: Scott Thomas
Original Data
Curve Fit
Temperature (° C)
Time (sec)
0
50
100
150
200
250
300
350
400
0
1
2
3
4
5
6

Problem 6.10:

The following data give the stopping distance *d* as a function of the initial speed *v*, for a certain car model. Using the **polyfit** command, find a third-order polynomial that fits the data. Show the original data and the curve fit on a plot. Using the curve fit, estimate the stopping distance for an initial speed of 63 mi/hr.

v (mi/hr)	20	30	40	50	60	70
d (ft)	45	80	130	185	250	330

Problem 6.10:

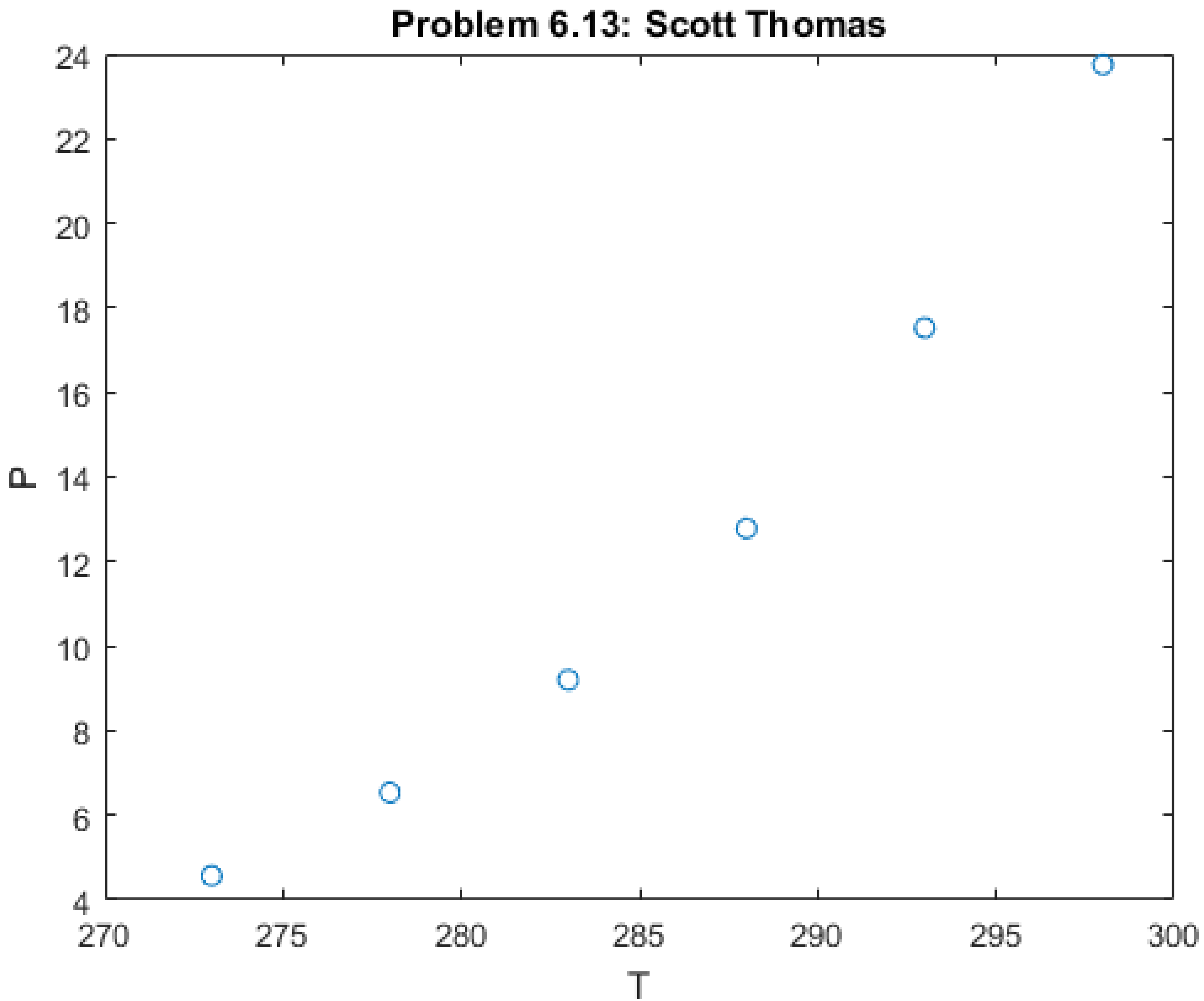

Problem 6.10: Scott Thomas
Original Data
Curve Fit
d(v = 63 mi/hr) = 273.5359 ft
Stopping Distance d (ft)
Initial Speed (mi/hr)
0
50
100
150
200
250
300
350
20
25
30
35
40
45
50
55
60
65
70

Problem 6.13:

Data on the vapor pressure P of water as a function of temperature T are given in the following table. From theory we know that $\ln P$ is proportional to $1/T$. Obtain a curve fit for $P(T)$ from these data using the polyfit command. Use the fit to estimate the vapor pressure at $T = 285$ K.

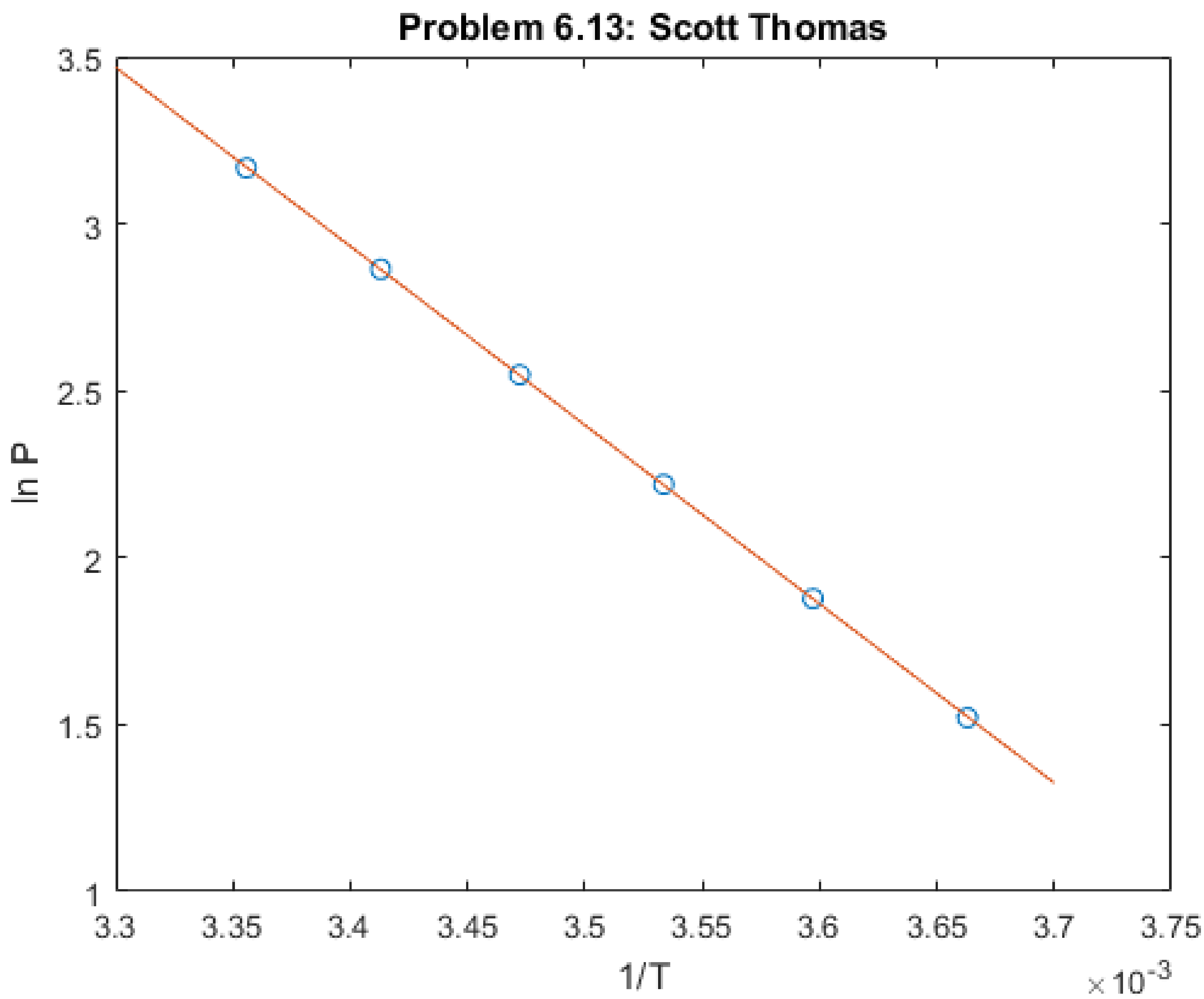
T (K)	P (torr)
273	4.579
278	6.543
283	9.209
288	12.788
293	17.535
298	23.756

Problem 6.13:

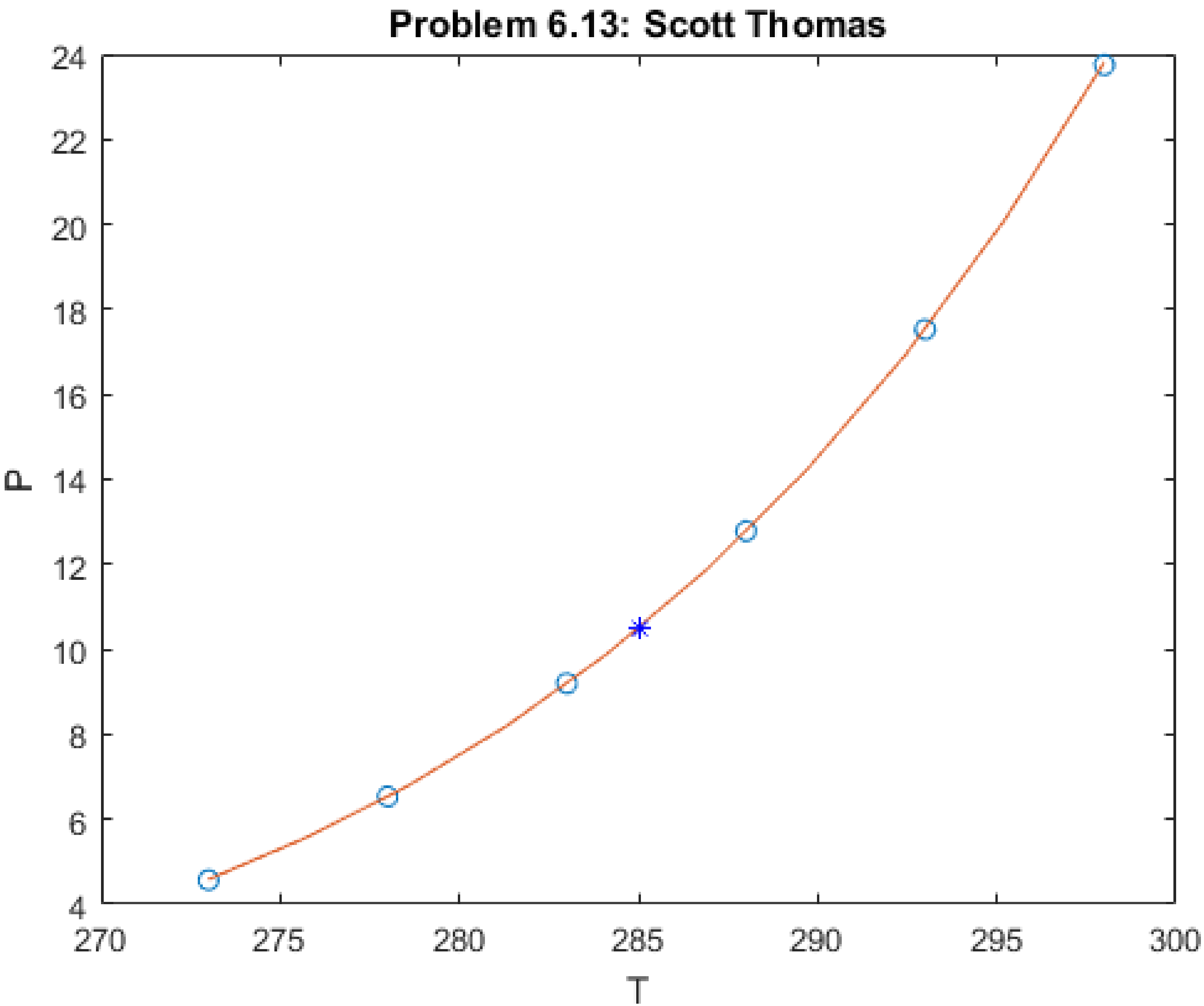


Problem 6.13: Scott Thomas
24
22
20
18
16
14
12
10
8
6
4
P
270
275
280
285
290
295
300
T

Problem 6.13:


Problem 6.13: Scott Thomas
3.5
3
2.5
2
1.5
1
ln P
3.3
3.35
3.4
3.45
3.5
3.55
3.6
3.65
3.7
3.75
1/T
× 10^-3

Problem 6.13:


Problem 6.13: Scott Thomas
P
T
24
22
20
18
16
14
12
10
8
6
4
270
275
280
285
290
295
300