Problem 10.17:

17. Consider the liquid-level system shown in Figure 10.7-1. The governing equation based on conservation of mass is Equation (10.7-2). Suppose that the height *h* is controlled by using a relay to switch the input flow rate between the values 0 and 50 kg/s. The flow rate is switched on when the height is less than 4.5 m and is switched off when the height reaches 5.5 m. Create a Simulink model for this application using the values $A = 2\ m^2$, $R =$ 400 <u>and</u> 4000 $m^{-1}\ s^{-1}$, $\rho = 1000\ kg/m^3$, and $h(0) = 1$ m. Obtain a plot of $h(t)$ for each *R* case.

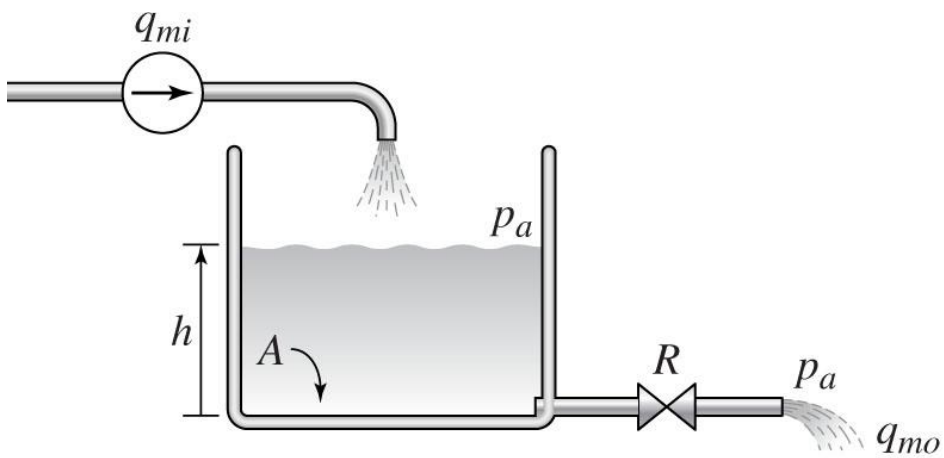


Figure 10.7–1 A hydraulic system with a flow source.

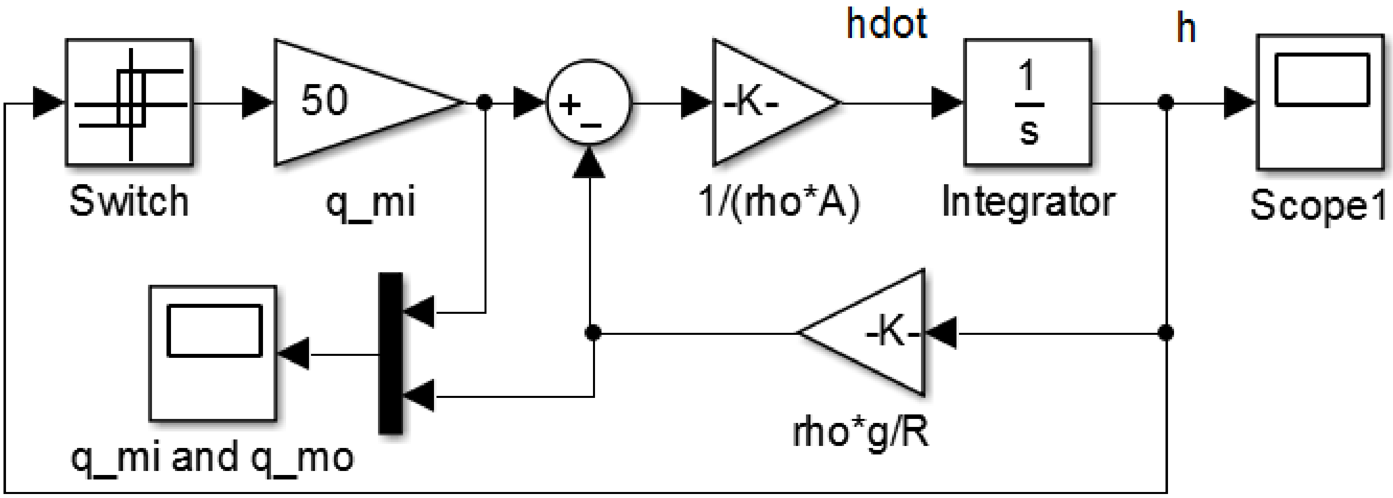
Equation 10.7-1:

$$\dot{h} = \left(\frac{1}{\rho A}\right)(q_{mi} - q_{mo})$$

$$q_{mo} = \left(\frac{\rho g}{R}\right) h$$

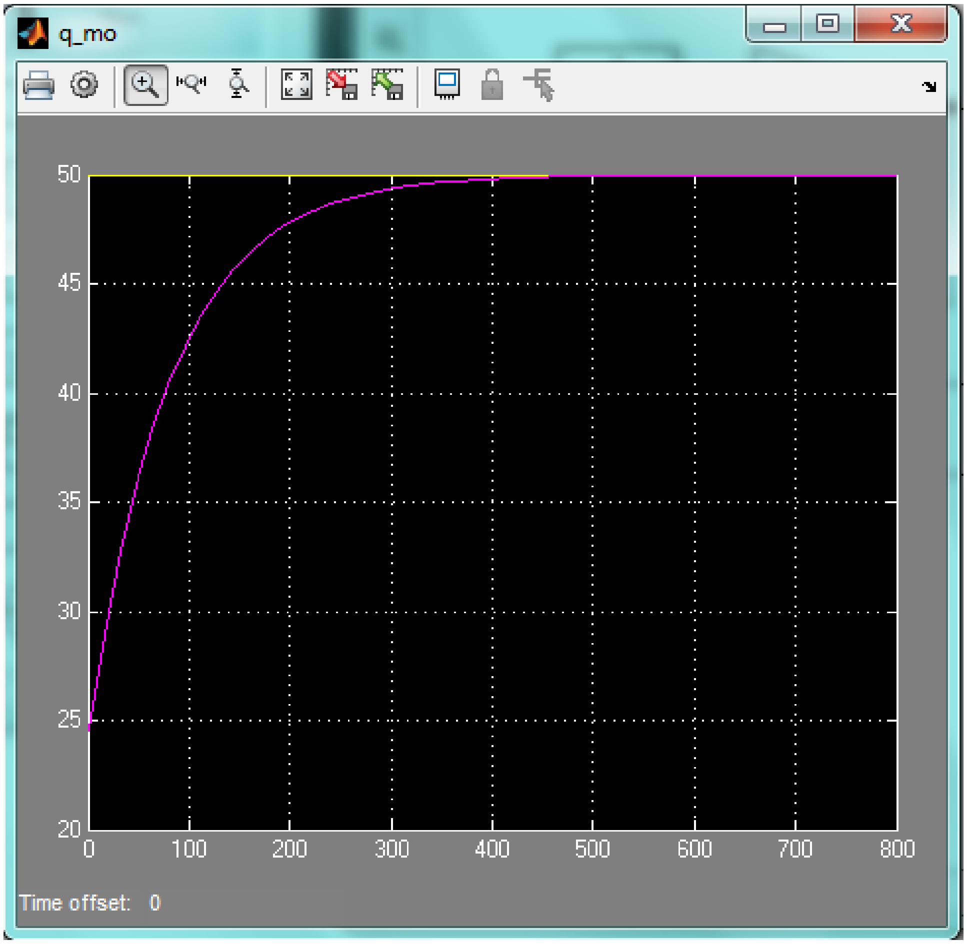
$$\dot{h} = \left(\frac{1}{\rho A}\right)\left[q_{mi} - \left(\frac{\rho g}{R}\right) h\right]$$

$$q_{mi} = \begin{cases} 50\ \text{kg/s}, & h \le 4.5\ \text{m} \\ 0\ \text{kg/s}, & h \ge 5.5\ \text{m} \end{cases}$$

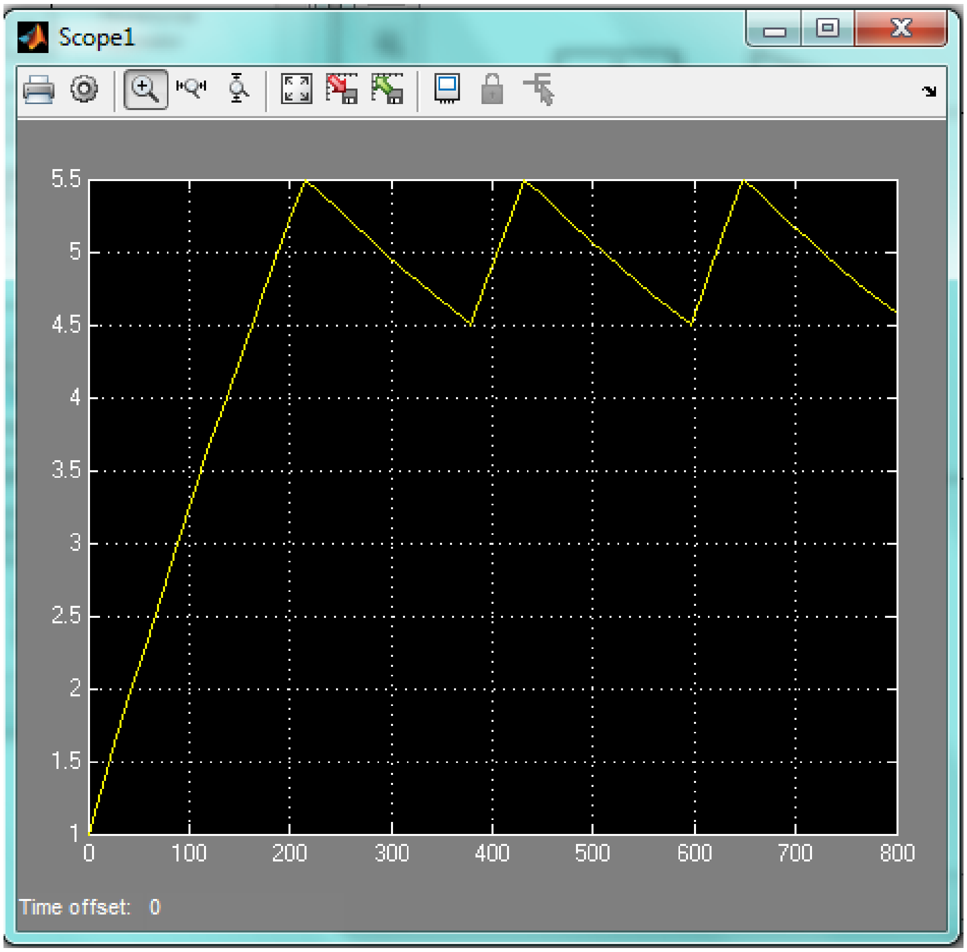


$h(t)$ plot for $R = 400$: The restriction at the outlet is so small that the liquid does not reach the point where the relay stops the input. In other words, the switch stays on and the tank is being filled continuously. When the *h* plot levels out, the flow out of the tank equals the flow into it [$q_{mo}(t) = q_{mi}(t) =$ 50 kg/s].

$q(t)$ plot for $R = 400$: Yellow line is the inlet flow rate $q_{mi}(t)$; purple line is the outlet flow rate $q_{mo}(t)$. The inlet flow rate $q_{mi}(t)$ is constant at 50 kg/s.



$h(t)$ plot for $R = 4000$: The restriction at the outlet is much higher, so the liquid level is maintained between 4.5 and 5.5 m. The slope of the filling line is much steeper than the draining portion. This means the tank drains slowly compared to how quickly it is refilled.



$q(t)$ plot for $R = 4000$: Yellow line is the inlet flow rate $q_{mi}(t)$; the purple line is the outlet flow rate $q_{mo}(t)$. The inlet flow rate pulses on and off, whereas the deadband for the outlet flow rate ranges from $q_{mo}(t) = 11$ to 13 kg/s, which is much smaller than the case with $R = 400$. This means that the outlet is more restrictive in this case.