Problem 10.28:

28. A cone-shaped paper drinking cup (like the kind used at water fountains) has a radius R and a height H. If the water height in the cup is h, the water volume is given by

$$V = \frac{1}{3}\pi\left(\frac{R}{H}\right)^2 h^3$$

Suppose that the cup's dimensions are $R=1.5$ in. and $H = 4$ in.

a. If the flow rate from the fountain into the cup is 2 in.^{3}/sec, use Simulink to determine how long will it take to fill the cup to the brim.

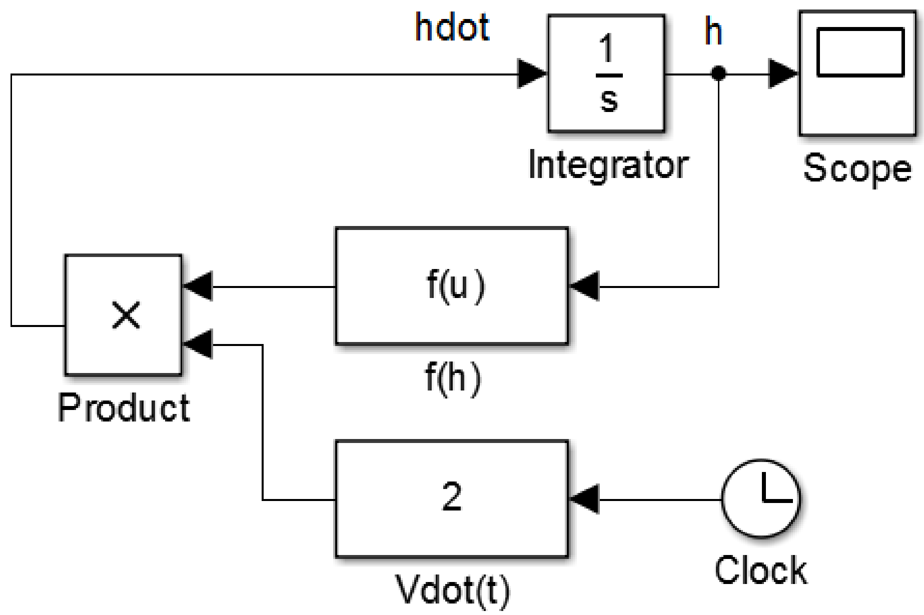
b. If the flow rate from the fountain into the cup is given by $2(1 - e^{-2t})$ in.^{3}/sec, use Simulink to determine how long will it take to fill the cup to the brim.

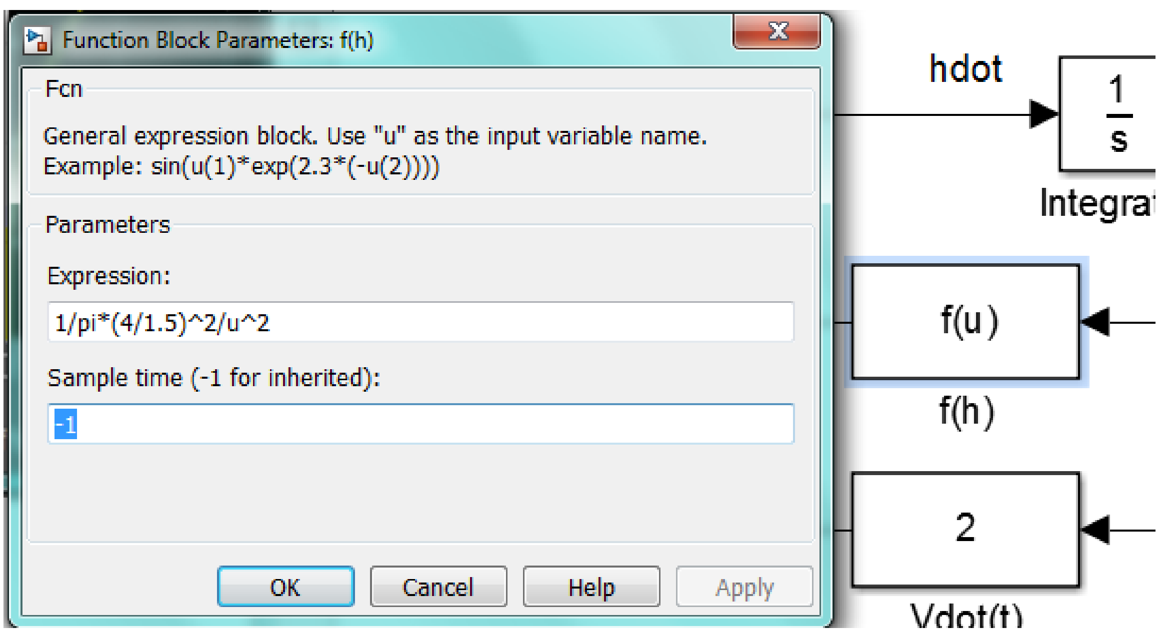
$$V = \frac{\pi}{3}\left(\frac{R}{H}\right)^2 h^3$$

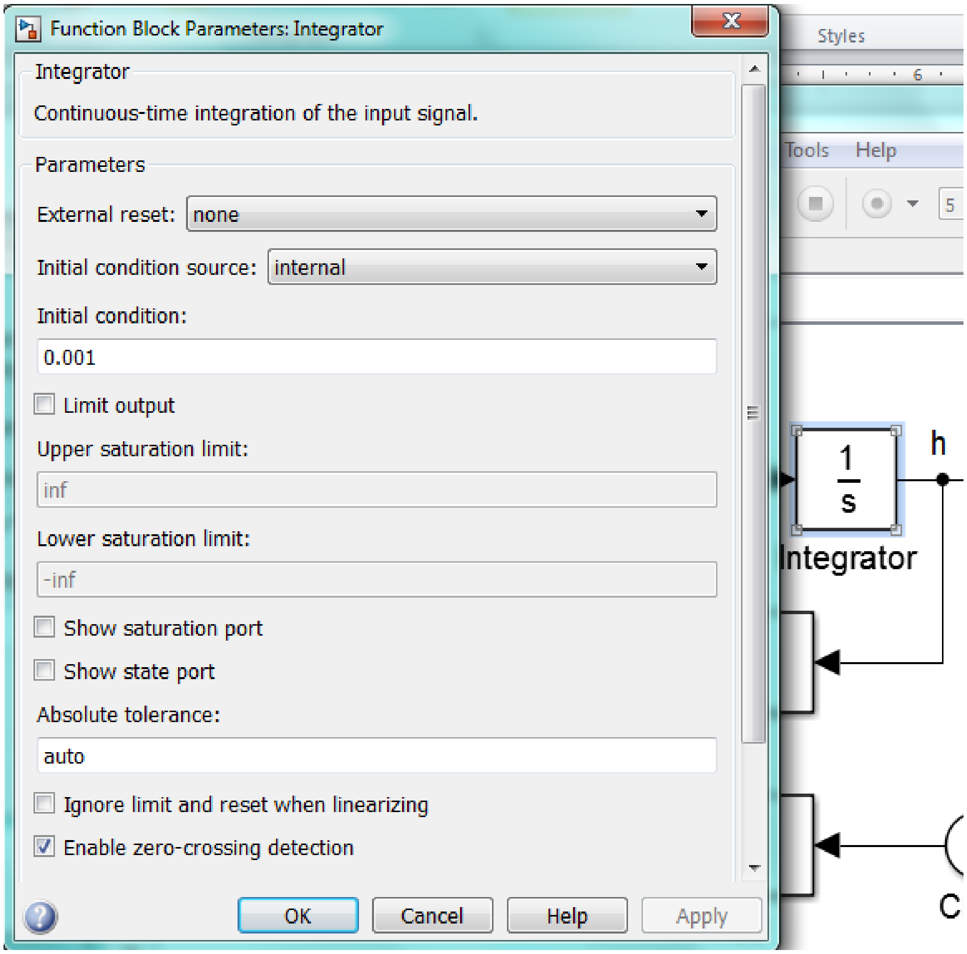
Take the derivative of both sides of the equation with respect to *t*:

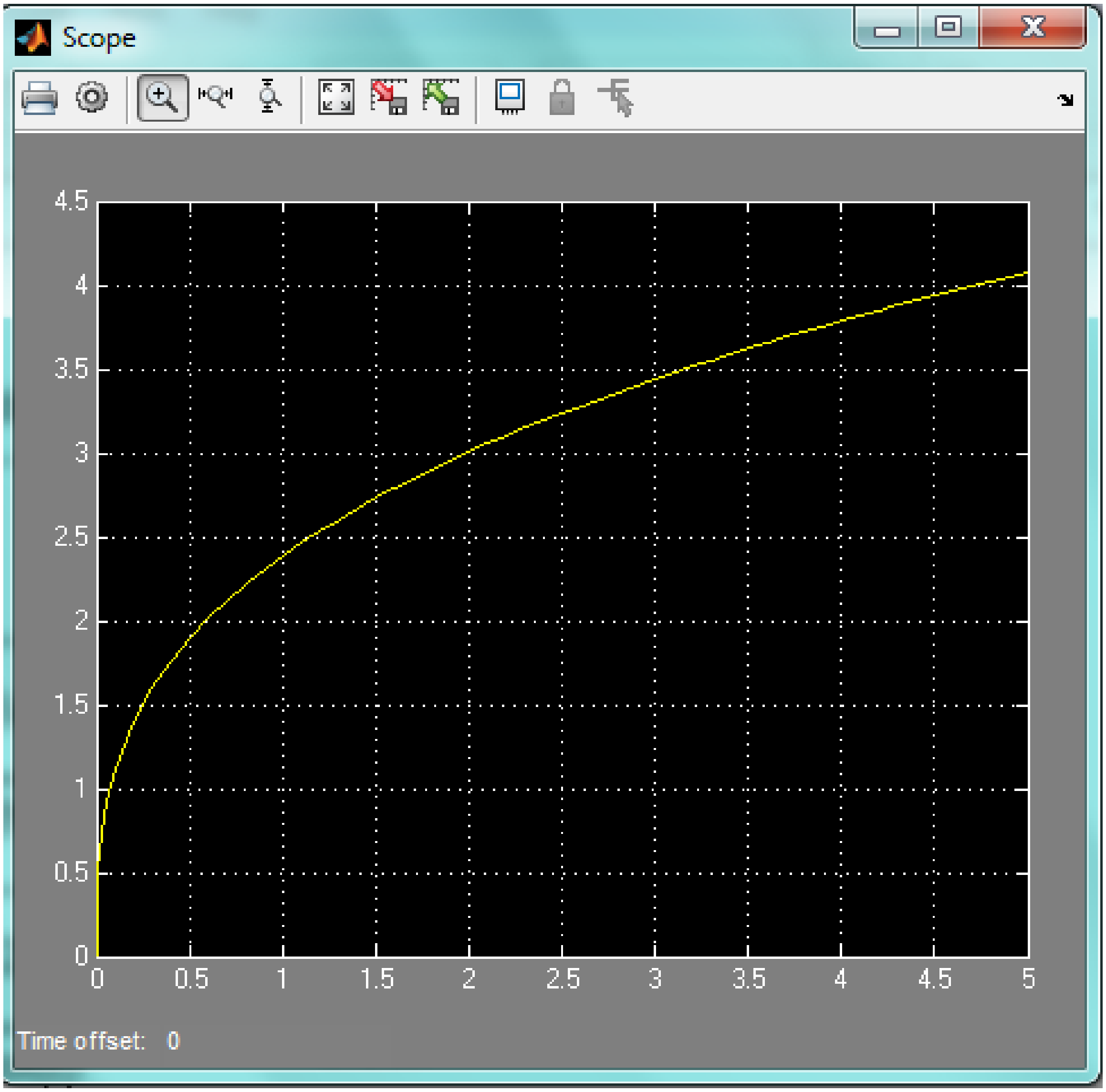
$$\frac{dV}{dt} = \frac{\pi}{3}\left(\frac{R}{H}\right)^2 3h^2 \frac{dh}{dt}$$

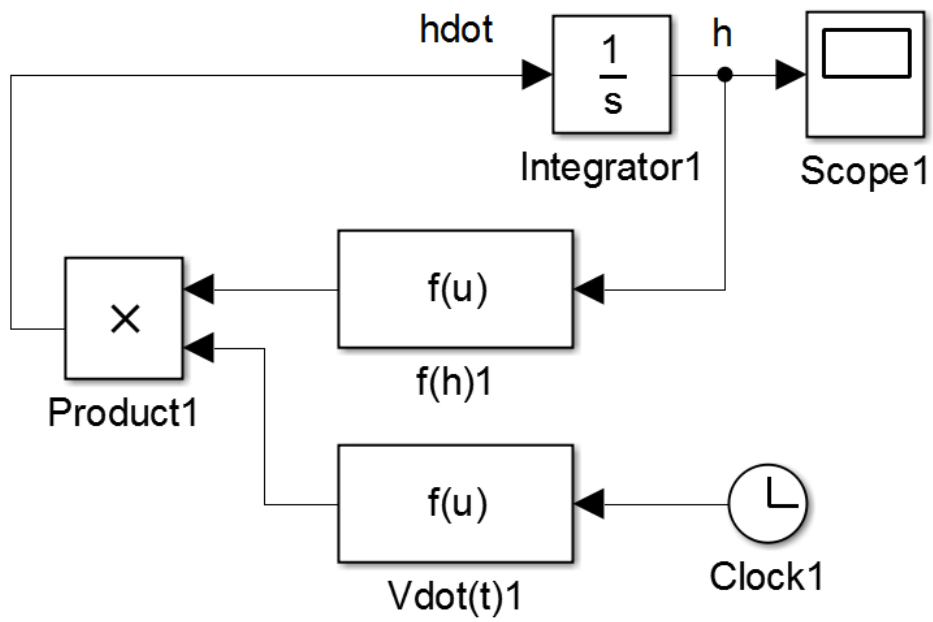
$$\dot{h} = \frac{1}{\pi}\left(\frac{H}{R}\right)^2 \frac{\dot{V}}{h^2}$$

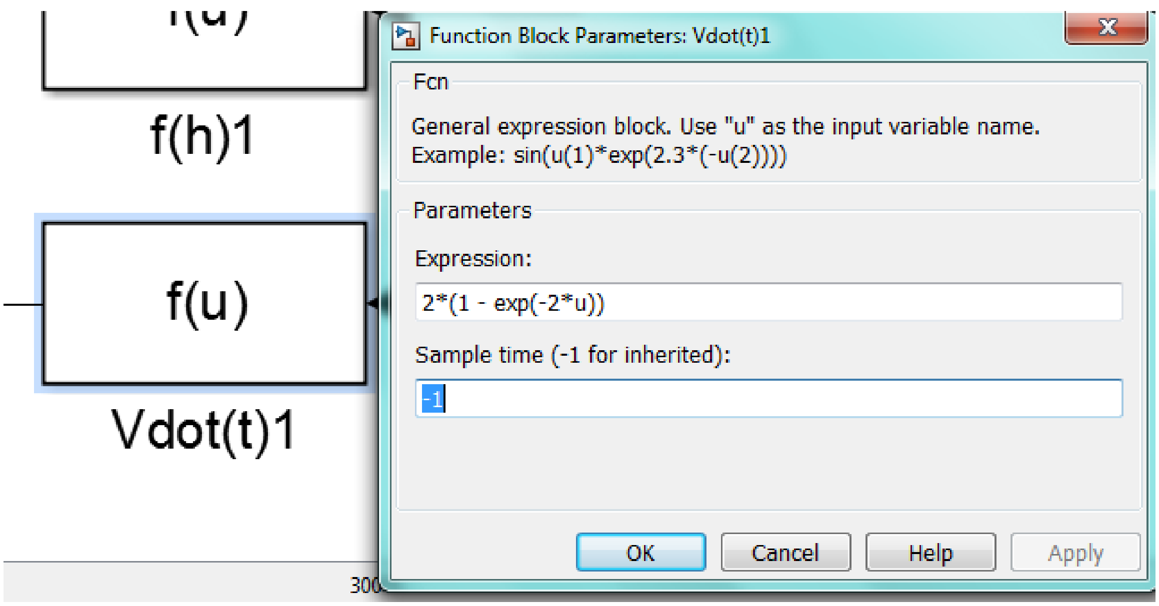

hdot
1
s
h
Integrator
Scope
f(u)
f(h)
×
Product
2
Vdot(t)
Clock

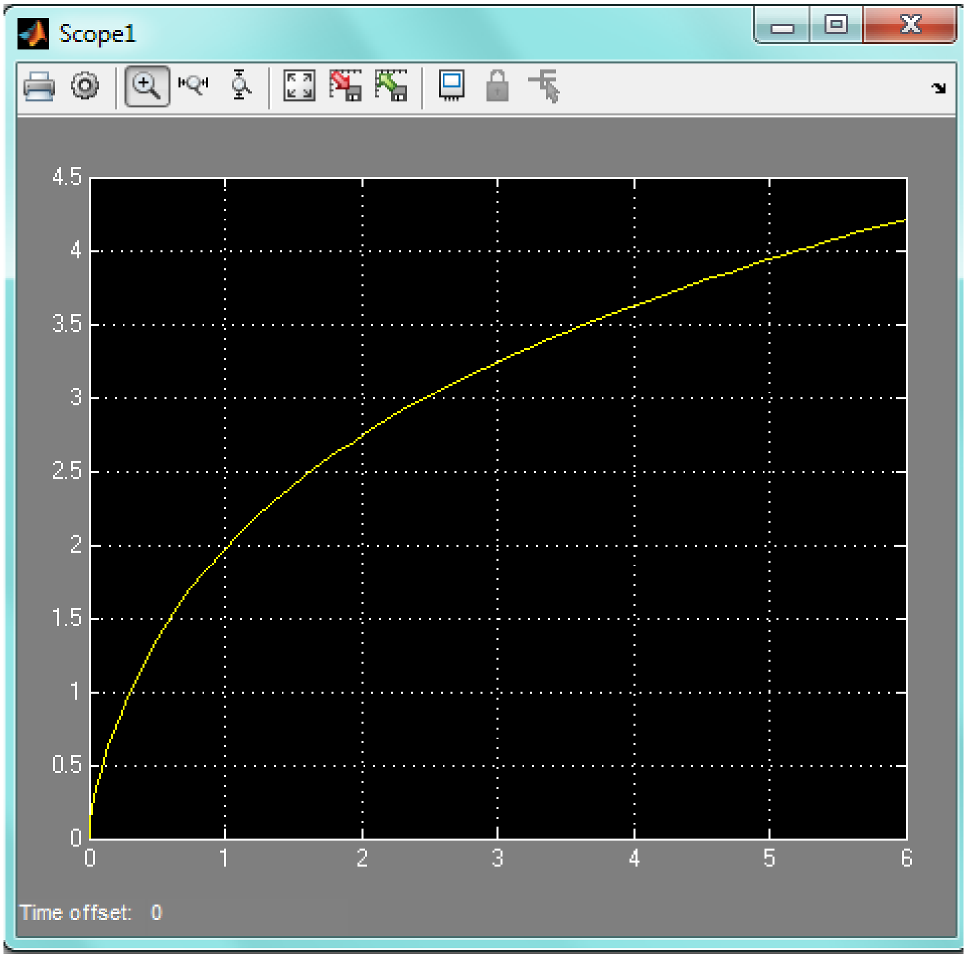

Function Block Parameters: f(h)
Fcn
General expression block. Use "u" as the input variable name.
Example: sin(u(1)*exp(2.3*(-u(2))))
Parameters
Expression:
1/pi*(4/1.5)^2/u^2
Sample time (-1 for inherited):
-1
OK
Cancel
Help
Apply
hdot
1
s
f(u)
f(h)
2


Function Block Parameters: Integrator
Integrator
Continuous-time integration of the input signal.
Parameters
External reset: none
Initial condition source: internal
Initial condition:
0.001
Limit output
Upper saturation limit:
inf
Lower saturation limit:
-inf
Show saturation port
Show state port
Absolute tolerance:
auto
Ignore limit and reset when linearizing
Enable zero-crossing detection
OK
Cancel
Help
Apply
Styles
Tools
Help
1
s
h
Integrator
C


Scope
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time offset: 0


hdot
1
s
h
Integrator1
Scope1
f(u)
×
Product1
f(h)1
f(u)
Vdot(t)1
Clock1


f(h)1
f(u)
Vdot(t)1
Function Block Parameters: Vdot(t)1
Fcn
General expression block. Use "u" as the input variable name.
Example: sin(u(1)*exp(2.3*(-u(2))))
Parameters
Expression:
2*(1 - exp(-2*u))
Sample time (-1 for inherited):
-1
OK
Cancel
Help
Apply



Scope1
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
Time offset: 0