ME 1020 Engineering Programming with MATLAB

Problem 4.38:

38. Many applications require us to know the temperature distribution in an object. For example, this information is important for controlling the material properties, such as hardness, when cooling an object formed from molten metal. In a heat-transfer course, the following description of the temperature distribution in a flat, rectangular metal plate is often derived. The temperature is held constant at T_1 on three sides and at T_2 on the fourth side (see Figure P38). The temperature $T(x, y)$ as a function of the xy coordinates shown is given by

$$T(x, y) = (T_2 - T_1)w(x, y) + T_1$$

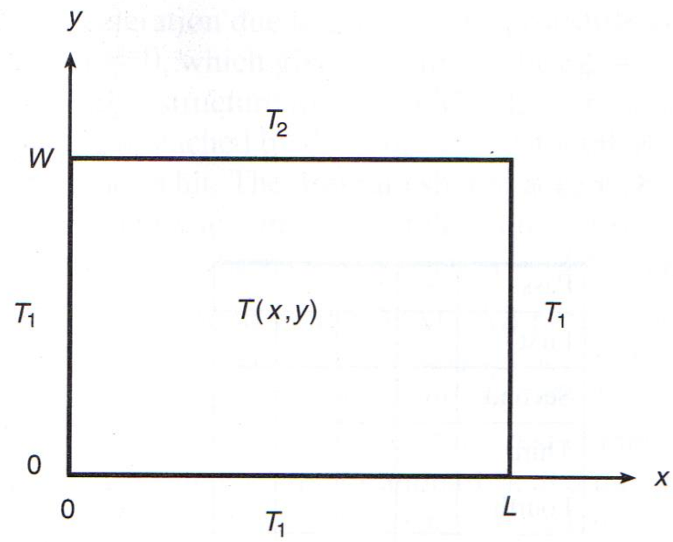

Figure P38

where

$$w(x, y) = \frac{2}{\pi} \sum_{n \text{ odd}}^{\infty} \frac{2}{n} \sin\left(\frac{n\pi x}{L}\right) \frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}$$

Use the following data: $T_1 = 70°\text{F}$, $T_2 = 200°\text{F}$, and $W = L = 2$ ft.

a. The terms in the preceding series become smaller in magnitude as n increases. Write a MATLAB program to verify this fact for $n = 1, \ldots, 19$ for the center of the plate ($x = y = 1$).

b. Using $x = y = 1$, write a MATLAB program to determine how many terms are required in the series to produce a temperature calculation that is accurate to within 1 percent. (That is, for what value of n will the addition of the next term in the series produce a change in T of less than 1 percent?) Use your physical insight to determine whether this answer gives the correct temperature at the center of the plate.

c. Modify the program from part b to compute the temperatures in the plate; use a spacing of 0.2 for both x and y.

```
% Problem 4.38a
clear
clc
disp('Problem 4.38a: Scott Thomas')

T1 = 70;%degrees F
T2 = 200;%degrees F
W = 2;%ft
L = 2;%ft

disp('Part a): terms in series become smaller')

x = 1;% m
y = 1;% m
```

```
for n = 1:2:19
    n;
    wterms(n) = 2/n*sin(n*pi*x/L)*sinh(n*pi*y/L)/sinh(n*pi*W/L);
end
for n = 1:9
w(n) = wterms(2*n-1);
end
format shortE
w'
```

```
Problem 4.38a: Scott Thomas
Part a): terms in series become smaller

ans =

   3.9854e-01
  -5.9884e-03
   1.5528e-04
  -4.7931e-06
   1.6110e-07
  -5.6960e-09
   2.0828e-10
  -7.8004e-12
   2.9743e-13
```

```
% Problem 4.38b
clear
clc
disp('Problem 4.38b: Scott Thomas')

T1 = 70;%degrees F
T2 = 200;%degrees F
W = 2;%ft
L = 2;%ft

disp('Part b): how many terms for percent difference = 1')

x = 1;% m
y = 1;% m
dT = 10;
Told = 0;
Tnew = 0;
n = 1;
wsum = 0;
while dT > 0.01
    wterm = 2/n*sin(n*pi*x/L)*sinh(n*pi*y/L)/sinh(n*pi*W/L);
    wsum = wsum + wterm;
    w = wsum*2/pi;
    Tnew = (T2 - T1)*w + T1;
    dT = abs((Tnew - Told)/Tnew);
    Told = Tnew;
    n = n + 2;
end

n
Tnew
```

```
Problem 4.38b: Scott Thomas
Part b): how many terms for percent difference = 1

n =

     5


Tnew =

   1.0249e+02
```

```
% Problem 4.38c
clear
clc
disp('Problem 4.38c: Scott Thomas')

format short
T1 = 70;%degrees F
T2 = 200;%degrees F
W = 2;%ft
L = 2;%ft

disp('Part c): calculate temperatures')

x = 0:0.2:L;% m
y = 0:0.2:W;% m
mmaxx = length(x);
mmaxy = length(y);
Tnew = zeros(mmaxx,mmaxy);

for my = 1:mmaxy

    for mx = 1:mmaxx

wsum = 0;
for n = 1:2:101
    wterm = 2/n*sin(n*pi*x(mx)/L)*sinh(n*pi*y(my)/L)/sinh(n*pi*W/L);
    wsum = wsum + wterm;
end
    w = wsum*2/pi;
    Tnew(my,mx) = (T2 - T1)*w + T1;

    end
end

figure;
contour(x,y,(Tnew),15);
axis square;
Tnew
```

```
Problem 4.38c: Scott Thomas
Part c): calculate temperatures

Tnew =

  Columns 1 through 7
```

70.0000	70.0000	70.0000	70.0000	70.0000	70.0000	70.0000
70.0000	71.4223	72.6996	73.7059	74.3474	74.5674	74.3474
70.0000	72.9928	75.6757	77.7831	79.1225	79.5811	79.1225
70.0000	74.8814	79.2399	82.6420	84.7909	85.5240	84.7909
70.0000	77.3094	83.7846	88.7772	91.8935	92.9491	91.8935
70.0000	80.6065	89.8580	96.8243	101.0777	102.5000	101.0777
70.0000	85.3423	98.3083	107.6354	113.1065	114.8954	113.1065
70.0000	92.6878	110.5862	122.3580	128.7964	130.8273	128.7964
70.0000	105.6135	129.3243	142.3908	148.7846	150.7030	148.7846
70.0000	133.5777	158.6941	168.7249	173.0010	174.2196	173.0010
70.0000	197.8335	199.5559	200.3031	200.6885	200.8113	200.6885

Columns 8 through 11

70.0000	70.0000	70.0000	70.0000
73.7059	72.6996	71.4223	70.0000
77.7831	75.6757	72.9928	70.0000
82.6420	79.2399	74.8814	70.0000
88.7772	83.7846	77.3094	70.0000
96.8243	89.8580	80.6065	70.0000
107.6354	98.3083	85.3423	70.0000
122.3580	110.5862	92.6878	70.0000
142.3908	129.3243	105.6135	70.0000
168.7249	158.6941	133.5777	70.0000
200.3031	199.5559	197.8335	70.0000