ME 1020 Engineering Programming with MATLAB

Problem 8.13:

13. See Figure P13. Assume that no vehicles stop within the network. A traffic engineer wants to know if the traffic flows $f_1, f_2, \ldots, f_7$ (in vehicles per hour) can be computed given the measured flows shown in the figure. If not, then determine how many more traffic sensors need to be installed, and obtain the expressions for the other traffic flows in terms of the measured quantities.

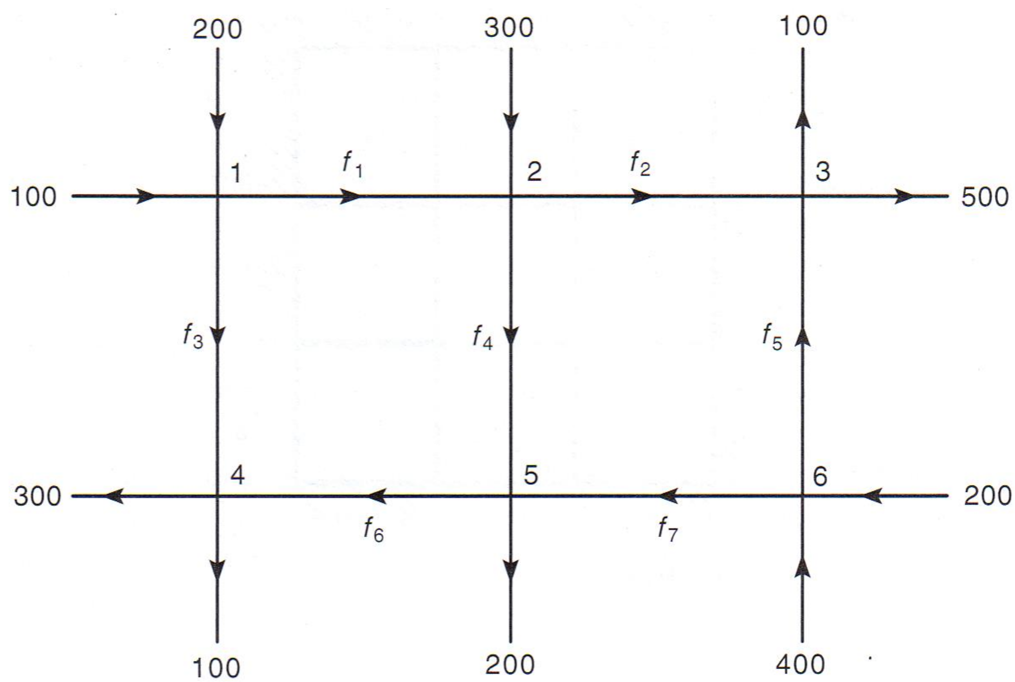

Figure P13

Problem setup:

$$100 + 200 = f_1 + f_3: \quad f_1 + f_3 = 300$$

$$f_1 + 300 = f_2 + f_4: \quad -f_1 + f_2 + f_4 = 300$$

$$f_2 + f_5 = 100 + 500: \quad f_2 + f_5 = 600$$

$$f_3 + f_6 = 100 + 300: \quad f_3 + f_6 = 400$$

$$f_4 + f_7 = 200 + f_6: \quad f_4 - f_6 + f_7 = 200$$

$$200 + 400 = f_5 + f_7: \quad f_5 + f_7 = 600$$

$$(1)f_1 + (0)f_2 + (1)f_3 + (0)f_4 + (0)f_5 + (0)f_6 + (0)f_7 = 300$$

$$(-1)f_1 + (1)f_2 + (0)f_3 + (1)f_4 + (0)f_5 + (0)f_6 + (0)f_7 = 300$$

$$(0)f_1 + (1)f_2 + (0)f_3 + (0)f_4 + (1)f_5 + (0)f_6 + (0)f_7 = 600$$

$$(0)f_1 + (0)f_2 + (1)f_3 + (0)f_4 + (0)f_5 + (1)f_6 + (0)f_7 = 400$$

$$(0)f_1 + (0)f_2 + (0)f_3 + (1)f_4 + (0)f_5 + (-1)f_6 + (1)f_7 = 200$$

$$(0)f_1 + (0)f_2 + (0)f_3 + (0)f_4 + (1)f_5 + (0)f_6 + (1)f_7 = 600$$

$$x = [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6 \quad f_7]^T$$

```
% Problem 8.13
clear
clc
disp('Problem 8.13: Scott Thomas')

A = [ 1.0   0.0   1.0   0.0   0.0   0.0   0.0; ...
     -1.0   1.0   0.0   1.0   0.0   0.0   0.0;...
      0.0   1.0   0.0   0.0   1.0   0.0   0.0;...
      0.0   0.0   1.0   0.0   0.0   1.0   0.0;...
      0.0   0.0   0.0   1.0   0.0  -1.0   1.0;...
      0.0   0.0   0.0   0.0   1.0   0.0   1.0;...
      ]
b = [300; 300; 600; 400; 200; 600]

%x1 = inv(A)*b
% Using inv(A)*b, the following error is given:
%Error using inv
%Matrix must be square.

x2 = A\b
% Using A\b, this warning appears:
% Warning: Rank deficient, rank = 5, tol =  7.691851e-16.

RankA = rank(A)
RankAb = rank([A b])

%since rank(A) = rank([A b])= 5, a solution exists, but the system is
% underdetermined. This means it has an infinite number of solutions.
% Use rref([A b]) to get the reduced equation set.
x3 = rref([A b])

x4 = pinv(A)*b
% Using pinv(A)*b gives one solution where the norm is minimized.
% x = [2.7255; 6.1074; 0.6432]
```

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Problem 8.13: Scott Thomas

A =
```

```
     1     0     1     0     0     0     0
    -1     1     0     1     0     0     0
     0     1     0     0     1     0     0
     0     0     1     0     0     1     0
     0     0     0     1     0    -1     1
     0     0     0     0     1     0     1


b =

   300
   300
   600
   400
   200
   600

Warning: Rank deficient, rank = 5, tol =  7.691851e-16.

x2 =

  -3.0000e+02
  -1.3862e-13
   6.0000e+02
            0
   6.0000e+02
  -2.0000e+02
            0


RankA =

     5


RankAb =

     5


x3 =

     1     0     0     0     0    -1     0  -100
     0     1     0     0     0     0    -1     0
     0     0     1     0     0     1     0   400
     0     0     0     1     0    -1     1   200
     0     0     0     0     1     0     1   600
     0     0     0     0     0     0     0     0


x4 =

   3.3333e+01
   2.3333e+02
   2.6667e+02
   1.0000e+02
   3.6667e+02
   1.3333e+02
   2.3333e+02
```

Discussion of results:

$$(1)f_1 + (0)f_2 + (0)f_3 + (0)f_4 + (0)f_5 + (-1)f_6 + (0)f_7 = -100$$

$$(0)f_1 + (1)f_2 + (0)f_3 + (0)f_4 + (0)f_5 + (0)f_6 + (-1)f_7 = 0$$

$$(0)f_1 + (0)f_2 + (1)f_3 + (0)f_4 + (0)f_5 + (1)f_6 + (0)f_7 = 400$$

$$(0)f_1 + (0)f_2 + (0)f_3 + (1)f_4 + (0)f_5 + (-1)f_6 + (1)f_7 = 200$$

$$(0)f_1 + (0)f_2 + (0)f_3 + (0)f_4 + (1)f_5 + (0)f_6 + (1)f_7 = 600$$

Solve for f_1, f_2, f_3, f_4 in terms of f_6 and f_7:

$$f_1 - f_6 = -100$$

$$f_2 - f_7 = 0$$

$$f_3 + f_6 = 400$$

$$f_4 - f_6 + f_7 = 200$$

$$f_5 + f_7 = 600$$

$$f_1 = f_6 - 100$$

$$f_2 = f_7$$

$$f_3 = -f_6 + 400$$

$$f_4 = f_6 - f_7 + 200$$

$$f_5 = -f_7 + 600$$

Need to monitor the traffic flow on streets 6 and 7 to fully model the intersections.