ME 1020 Engineering Programming with MATLAB

Problem 8.6:

6. Fluid flows in pipe networks can be analyzed in a manner similar to that used for electric resistance networks. Figure P6 shows a network with three pipes. The volume flow rates in the pipes are q_1, q_2, and q_3. The pressures at the pipe ends are p_a, p_b, and p_c. The pressure at the junction is p_1. Under certain conditions, the pressure–flow rate relation in a pipe has the same form as the voltage-current relation in a resistor. Thus, for the three pipes, we have

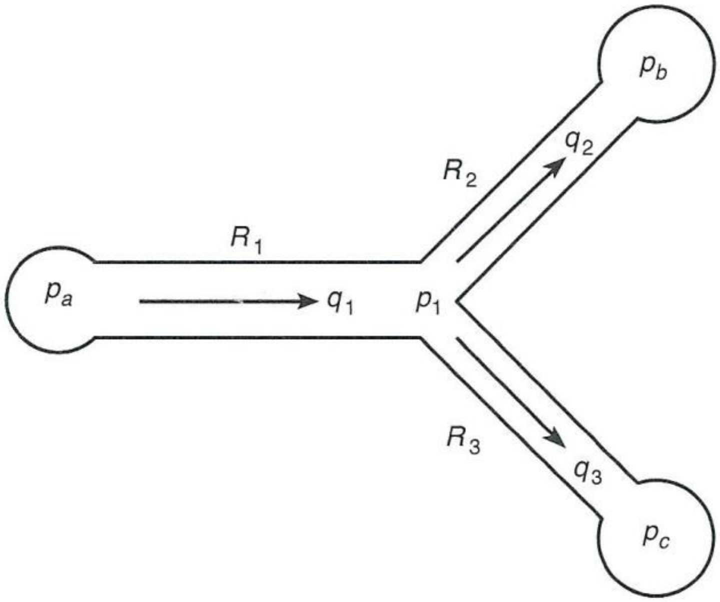
$$q_1 = \frac{1}{R_1}(p_a - p_1)$$

$$q_2 = \frac{1}{R_2}(p_1 - p_b)$$

$$q_3 = \frac{1}{R_3}(p_1 - p_c)$$

where the R_i are the pipe resistances. From conservation of mass, $q_1 = q_2 + q_3$.

a. Set up these equations in a matrix form $\mathbf{Ax} = \mathbf{b}$ suitable for solving for the three flow rates q_1, q_2, and q_3 and the pressure p_1, given the values of pressures p_a, p_b, and p_c and the values of resistances R_1, R_2, and R_3. Find the expressions for **A** and **b.**



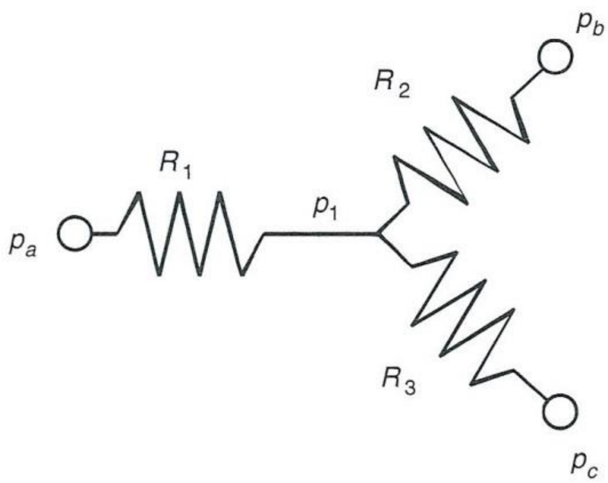


Figure P6

b. Use MATLAB to solve the matrix equations obtained in part *a* for the case where $p_a = 4320$ lb/ft^2, $p_b = 3600$ lb/ft^2, and $p_c = 2880$ lb/ft^2. These correspond to 30, 25, and 20 psi, respectively (1 psi = 1 lb/in^2, and atmospheric pressure is 14.7 psi). Use the resistance values $R_1 = 10{,}000$; $R_2 = R_3 = 14{,}000$ lb sec/ft^5. These values correspond to fuel oil flowing through pipes 2 ft long, with 2- and 1.4-in. diameters, respectively. The units of the answers are ft^3/sec for the flow rates and lb/ft^2 for pressure.

Problem setup:

$$(1)p_1 + (R_1)q_1 + (0)q_2 + (0)q_3 = p_a$$

$$(-1)p_1 + (0)q_1 + (R_2)q_2 + (0)q_3 = -p_b$$

$$(-1)p_1 + (0)q_1 + (0)q_2 + (R_3)q_3 = -p_c$$

$$(0)p_1 + (1)q_1 + (-1)q_2 + (-1)q_3 = 0$$

$$x^T = [p_1 \quad q_1 \quad q_2 \quad q_3]$$

$$R_1 = 10{,}000; \quad R_2 = 14{,}000; \quad R_3 = 14{,}000$$

$$p_a = 4320; \quad p_b = 3600; \quad p_c = 2880$$

```
% Problem 8.6
clear
clc
disp('Problem 8.6: Scott Thomas')

R1 = 10000;
R2 = 14000;
R3 = 14000;
pa = 4320;
pb = 3600;
pc = 2880;

A = [ 1    R1    0    0; ...
     -1    0     R2   0; ...
     -1    0     0    R3;...
      0    1    -1   -1]
b = [pa; -pb; -pc; 0]

format ShortE
%x = inv(A)*b
x = A\b
```

Problem 8.6: Scott Thomas

A =

1.0000e+000	10.0000e+003	0.0000e+000	0.0000e+000
-1.0000e+000	0.0000e+000	14.0000e+003	0.0000e+000
-1.0000e+000	0.0000e+000	0.0000e+000	14.0000e+003
0.0000e+000	1.0000e+000	-1.0000e+000	-1.0000e+000

b =

4.3200e+003

```
  -3.6000e+003
  -2.8800e+003
   0.0000e+000


x =

   3.6847e+03
   6.3529e-02
   6.0504e-03
   5.7479e-02
```