ME 1020 Engineering Programming with MATLAB

Problem 9.10:

10.* A rocket's mass decreases as it burns fuel. The equation of motion for a rocket in vertical flight can be obtained from Newton's law, and it is

$$m(t)\frac{dv}{dt} = T - m(t)g$$

where T is the rocket's thrust and its mass as a function of time is given by $m(t) = m_0(1 - rt/b)$. The rocket's initial mass is m_0, the burn time is b, and r is the fraction of the total mass accounted for by the fuel.

Use the values $T = 48{,}000$ N, $m_0 = 2200$ kg, $r = 0.8$, $g = 9.81$ m/s^2, and $b = 40$ s. Determine the rocket's velocity at burnout.

Problem setup:

$$F = ma$$

$$T - mg = ma$$

$$a(t) = \frac{T}{m(t)} - g$$

$$a(t) = \frac{T}{\left[m_0\left(1 - \frac{rt}{b}\right)\right]} - g$$

Integrate the acceleration to find the velocity.

$$v(t) = \int_0^b a(t)dt + v(0) = \int_0^b \left\{\frac{T}{\left[m_0\left(1 - \frac{rt}{b}\right)\right]} - g\right\} dt + v(0)$$

The initial velocity for this problem is $v(0) = 0$. Use the Substitution Method to integrate the acceleration.

$$\text{Let } \; w = m_0\left(1 - \frac{rt}{b}\right), \;\; dw = -\left(\frac{m_0 r}{b}\right)dt, \;\; dt = -\left(\frac{b}{m_0 r}\right)dw$$

$$\text{Limits: } \; @\, t = 0, \;\; w = m_0; \quad @\, t = b, \;\; w = m_0(1 - r)$$

$$v = \int_{m_0}^{m_0(1-r)} \left(\frac{T}{w} - g\right)\left[-\left(\frac{b}{m_0 r}\right)dw\right]$$

$$v = -\left(\frac{b}{m_0 r}\right)\int_{m_0}^{m_0(1-r)} \left(\frac{T}{w} - g\right)dw$$

$$v = -\left(\frac{b}{m_0 r}\right)(T\ln w - gw)\Big|_{m_0}^{m_0(1-r)}$$

$$v = -\left(\frac{b}{m_0 r}\right)\{[T\ln(m_0(1-r)) - g(m_0(1-r))] - [T\ln(m_0) - g(m_0)]\}$$

$$v = -\left(\frac{b}{m_0 r}\right)\left\{T\ln\left[\frac{m_0(1-r)}{m_0}\right] - g[m_0(1-r) - m_0]\right\}$$

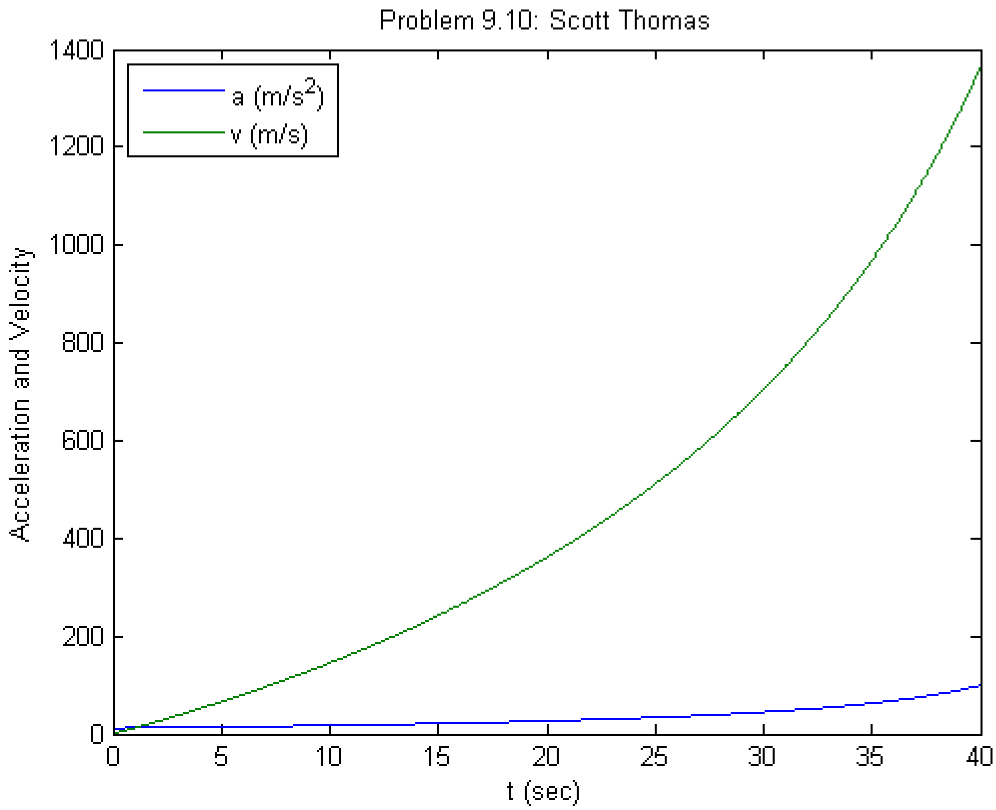
$$v = -\left(\frac{b}{m_0 r}\right)[T\ln(1-r) + g m_0 r]$$

Evaluate the velocity at burnout:

$$v = -\left(\frac{(40)}{(2200)(0.8)}\right)[(48{,}000)\ln(1-0.8) + (9.81)(2200)(0.8)] = 1363.3\text{ m/s}$$

```
% Problem 9.10
clear
clc
disp('Problem 9.10: Scott Thomas')

T = 48000; %N
m0 = 2200; %kg
r = 0.8;
g = 9.81; %m/s^2
b = 40; %sec
% Create an anonymous function for a(t) = T/(m0*(1 - (r*t/b))) - g
aoft = @(t) (T./(m0*(1 - (r*t/b))) - g);
N = 1001;
t = linspace(0,b,N);
a = aoft(t);
v(1) = 0.0;
for k = 1:N-1
v(k+1) = v(k) + 0.5*(t(k+1) - t(k))*(a(k) + a(k+1));
end
disp('Acceleration at burnout = ')
a(N)
disp('Velocity at burnout = ')
v(N)

plot(t,a, t,v), xlabel('t (sec)')
ylabel('Acceleration and Velocity')
title('Problem 9.10: Scott Thomas')
legend('a (m/s^2)', 'v (m/s)','Location', 'NorthWest')
```

```
Problem 9.10: Scott Thomas
Acceleration at burnout =

ans =

   9.9281e+01

Velocity at burnout =

ans =

   1.3634e+03
```


Problem 9.10: Scott Thomas
a (m/s2)
v (m/s)
Acceleration and Velocity
t (sec)
0
200
400
600
800
1000
1200
1400
5
10
15
20
25
30
35
40