Problem 9.25:

25.* The equation of motion of a rocket-propelled sled is, from Newton's law,

$$m\dot{v} = f - cv$$

where m is the sled mass, f is the rocket thrust, and c is an air resistance coefficient. Suppose that $m = 1000$ kg and $c = 500$ N · s/m. Suppose also that $v(0) = 0$ and $f = 75{,}000$ N for $t \geq 0$. Determine the speed of the sled at $t = 10$ s.

Problem setup:

Use the Predictor-Corrector method to solve this problem. Solve the differential equation for dv/dt:

$$\frac{dv}{dt} = \frac{1}{m}(f - cv) = g(t, v)$$

Approximate the derivative as follows:

$$\frac{dv}{dt} \approx \frac{\big(v(t + \Delta t) - v(t)\big)}{\Delta t} = g(t, v)$$

where Δt is the time step size. Solve for $v(t + \Delta t)$:

$$v(t + \Delta t) = v(t) + g(t, v)\Delta t$$

Put this equation into a form appropriate for computer solution. This is called the Difference Equation:

$$v(t_{k+1}) = v(t_k) + g[t_k, v(t_k)]\Delta t; \quad \text{where} \quad t_{k+1} = t_k + \Delta t$$

Rewrite this equation as follows. This is the Euler Predictor Difference Equation:

$$x_{k+1} = v_k + g(t_k, v_k)\Delta t$$

This represents a preliminary estimate for v_{k+1}, which is designated as x_{k+1} to avoid confusion. This estimate is then corrected using the Trapezoidal Corrector Difference Equation:

$$v_{k+1} = v_k + \frac{\Delta t}{2}[g(t_k, v_k) + g(t_{k+1}, x_{k+1})]$$

Substitute the expression for $g(t, v)$ into the difference equations:

$$x_{k+1} = v_k + \left[\frac{1}{m}(f - cv_k)\right]\Delta t = v_k + \frac{\Delta t}{m}(f - cv_k)$$

$$v_{k+1} = v_k + \frac{\Delta t}{2}\left[\left(\frac{1}{m}(f - cv_k)\right) + \left(\frac{1}{m}(f - cx_{k+1})\right)\right]$$

$$= v_k + \frac{\Delta t}{2m}[(f - cv_k) + (f - cx_{k+1})]$$

$$v_{k+1} = v_k + \frac{\Delta t}{2m}[2f - c(v_k + x_{k+1})]$$

In the problem statement, the initial condition is
$v(0) = 0,\ m = 1000\text{ kg},\ f = 75{,}000\text{ N},\ \text{and}\ c = 500\ \frac{N\cdot s}{m}$. Let $\Delta t = 0.1$.

Predictor:

$$x_{k+1} = v_k + \frac{\Delta t}{m}(f - cv_k) = v_k + \frac{0.1}{1000}(75{,}000 - 500v_k)$$

Corrector:

$$v_{k+1} = v_k + \frac{\Delta t}{2m}[2f - c(v_k + x_{k+1})] = v_k + \frac{0.1}{2000}[150{,}000 - 500(v_k + x_{k+1})]$$

<u>For $k = 1$:</u>

Predictor:

$$x_2 = v_1 + \frac{0.1}{1000}(75{,}000 - 500v_1) = (0.0) + \frac{0.1}{1000}(75{,}000 - 500(0.0)) = 7.5$$

Corrector:

$$v_2 = v_1 + \frac{0.1}{2000}[150{,}000 - 500(v_1 + x_2)] = (0.0) + \frac{0.1}{2000}\big[150{,}000 - 500\big((0.0) + (7.5)\big)\big]$$
$$= 7.3125$$

<u>For $k = 2$:</u>

Predictor:

$$x_3 = v_2 + \frac{0.1}{1000}(75{,}000 - 500v_2) = (7.3125) + \frac{0.1}{1000}\big(75{,}000 - 500(7.3125)\big) = 14.4469$$

Corrector:

$$v_3 = v_2 + \frac{0.1}{2000}[150{,}000 - 500(v_2 + x_3)]$$
$$= (7.3125) + \frac{0.1}{2000}\left[150{,}000 - 500\big((7.3125) + (14.4469)\big)\right] = 14.2685$$

<u>For $k = 3$:</u>

Predictor:

$$x_4 = v_3 + \frac{0.1}{1000}(75{,}000 - 500v_3) = (14.2685) + \frac{0.1}{1000}\big(75{,}000 - 500(14.2685)\big)$$
$$= 21.0551$$

Corrector:

$$v_4 = v_3 + \frac{0.1}{2000}[150{,}000 - 500(v_3 + x_4)]$$
$$= (14.2685) + \frac{0.1}{2000}\left[150{,}000 - 500\big((14.2685) + (21.0551)\big)\right] = 20.8854$$

```
% Problem 9.25
clear
clc
disp('Problem 9.25: Scott Thomas')

m = 1000;% kg
c = 500;% N-s/m
f = 75000;% N

N = 10;
delta_t = 0.1;
v = zeros(1,N);
t = zeros(1,N);
v(1) = 0.0;
for k = 1:N
    x(k+1) = v(k) + delta_t/m*(f - c*v(k));
    v(k+1) = v(k) + delta_t/(2*m)*(2*f - c*(v(k)+ x(k+1)));
    t(k+1) = t(k) + delta_t;
end
x
v
t
```

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Problem 9.25: Scott Thomas

x =

         0    7.5000   14.4469   21.0551   27.3412   33.3208   39.0089   44.4197   49.5667   54.4629   59.1203


v =

         0    7.3125   14.2685   20.8854   27.1798   33.1672   38.8628   44.2808   49.4346   54.3372   59.0007


t =

         0    0.1000    0.2000    0.3000    0.4000    0.5000    0.6000    0.7000    0.8000    0.9000    1.0000

>>
```

Time-Step independence study:

```
% Problem 9.25
clear
clc
disp('Problem 9.25: Scott Thomas')

m = 1000;% kg
c = 500;% N-s/m
f = 75000;% N

N = 100;
delta_t = 0.1;
v = zeros(1,N);
t = zeros(1,N);
v(1) = 0.0;
for k = 1:N
    x(k+1) = v(k) + delta_t/m*(f - c*v(k));
    v(k+1) = v(k) + delta_t/(2*m)*(2*f - c*(v(k)+ x(k+1)));
    t(k+1) = t(k) + delta_t;
end

N2 = 10;
delta_t2 = 1.0;
v2 = zeros(1,N2);
t2 = zeros(1,N2);
v2(1) = 0.0;
for k = 1:N2
    x2(k+1) = v2(k) + delta_t2/m*(f - c*v2(k));
    v2(k+1) = v2(k) + delta_t2/(2*m)*(2*f - c*(v2(k)+ x2(k+1)));
    t2(k+1) = t2(k) + delta_t2;
end

N3 = 1000;
delta_t3 = 0.01;
v3 = zeros(1,N3);
```

```
t3 = zeros(1,N3);
v3(1) = 0.0;
for k = 1:N3
    x3(k+1) = v3(k) + delta_t3/m*(f - c*v3(k));
    v3(k+1) = v3(k) + delta_t3/(2*m)*(2*f - c*(v3(k)+ x3(k+1)));
    t3(k+1) = t3(k) + delta_t3;
end

plot(t3, v3, t,v, t2, v2)
xlabel('Time t (seconds)'), ylabel('Rocket Sled Speed v(t) (m/s)')
title('Problem 9.25: Scott Thomas')
legend('\Delta t = 0.01','\Delta t = 0.1','\Delta t = 1.0','Location', 'SouthEast')
%axis([0 50 5 25])
```

Problem 9.25: Scott Thomas