ME 1020 Engineering Programming with MATLAB

Problem 9.27:

27. The equation for the voltage y across the capacitor of an RC circuit is

$$RC\frac{dy}{dt} + y = v(t)$$

where $v(t)$ is the applied voltage. Suppose that $RC = 0.2$ s and that the capacitor voltage is initially 2 V. Suppose also that the applied voltage is $v(t) = 10[2 - e^{-t}\sin(5\pi t)]$ V. Plot the voltage $y(t)$ for $0 \le t \le 5$ s.

Problem setup:

Use the Predictor-Corrector method to solve this problem. Solve the differential equation for dv/dt:

$$\frac{dy}{dt} = \frac{1}{RC}(v(t) - y) = g(t, y)$$

Approximate the derivative as follows:

$$\frac{dy}{dt} \approx \frac{\big(y(t + \Delta t) - y(t)\big)}{\Delta t} = g(t, y)$$

where Δt is the time step size. Solve for $y(t + \Delta t)$:

$$y(t + \Delta t) = y(t) + g(t, y)\Delta t$$

Put this equation into a form appropriate for computer solution. This is called the Difference Equation:

$$y(t_{k+1}) = y(t_k) + g[t_k, y(t_k)]\Delta t; \quad \text{where} \quad t_{k+1} = t_k + \Delta t$$

Rewrite this equation as follows. This is the Euler Predictor Difference Equation:

$$x_{k+1} = y_k + g(t_k, y_k)\Delta t$$

This represents a preliminary estimate for y_{k+1}, which is designated as x_{k+1} to avoid confusion. This estimate is then corrected using the Trapezoidal Corrector Difference Equation:

$$y_{k+1} = y_k + \frac{\Delta t}{2}[g(t_k, y_k) + g(t_{k+1}, x_{k+1})]$$

Substitute the expressions for $v(t)$ and $g(t, y)$ into the difference equations:

$$v(t) = 10[2 - e^{-t}\sin(5\pi t)]$$

$$g(t, y) = \frac{1}{RC}[v(t) - y]$$

$$v(t_k) = 10[2 - e^{-t_k}\sin(5\pi t_k)]$$

$$v(t_{k+1}) = 10[2 - e^{-t_{k+1}}\sin(5\pi t_{k+1})] \qquad (\text{where } \; t_{k+1} = t_k + \Delta t)$$

$$g(t_k, y_k) = \frac{1}{RC}[v(t_k) - y_k]$$

$$g(t_{k+1}, x_{k+1}) = \frac{1}{RC}[v(t_{k+1}) - x_{k+1}]$$

Predictor:

$$x_{k+1} = y_k + g(t_k, y_k)\Delta t$$

Corrector:

$$y_{k+1} = y_k + \frac{\Delta t}{2}[g(t_k, y_k) + g(t_{k+1}, x_{k+1})]$$

In the problem statement, the initial condition is $y(0) = 2.0\text{ V}$, and $RC = 0.2$ seconds. Let $\Delta t = 0.001$. The order in which calculations are made is important, as shown below. When performing calculations, make sure that your calculator is in Radians Mode.

<u>For $k = 1$:</u>

$$y_1 = 2.0$$

$$t_1 = 0.0$$

$$v(t_1) = 10[2 - e^{-t_1}\sin(5\pi t_1)] = 10[2 - e^{-0}\sin(5\pi \cdot 0)] = 20.0$$

$$g(t_1, y_1) = \frac{1}{RC}[v(t_1) - y_1] = \frac{1}{0.2}[(20.0) - (2.0)] = 90.0$$

Predictor:

$$x_2 = y_1 + g(t_1, y_1)\Delta t = (2.0) + (90.0)(0.001) = 2.09$$

Corrector:

$$t_2 = t_1 + \Delta t = 0.0 + 0.001 = 0.001$$

$$v(t_2) = 10[2 - e^{-t_2}\sin(5\pi t_2)] = 10[2 - e^{-0.001}\sin(5\pi \cdot 0.001)] = 19.84308$$

$$g(t_2, x_2) = \frac{1}{RC}[v(t_2) - x_2] = \frac{1}{(0.2)}[(19.84308) - (2.09)] = 88.76540$$

$$y_2 = y_1 + \frac{\Delta t}{2}[g(t_1, y_1) + g(t_2, x_2)] = (2.0) + \frac{0.001}{2}[(90.0) + (88.7654)] = 2.08938$$

<u>For $k = 2$:</u>

$$g(t_2, y_2) = \frac{1}{RC}[v(t_2) - y_2] = \frac{1}{0.2}[(19.84308) - (2.08938)] = 88.76849$$

Predictor:

$$x_3 = y_2 + g(t_2, y_2)\Delta t = (2.08938) + (88.76849)(0.001) = 2.17815$$

Corrector:

$$t_3 = t_2 + \Delta t = 0.001 + 0.001 = 0.002$$

$$v(t_3) = 10[2 - e^{-t_3}\sin(5\pi t_3)] = 10[2 - e^{-0.002}\sin(5\pi \cdot 0.002)] = 19.68652$$

$$g(t_3, x_3) = \frac{1}{RC}[v(t_3) - x_3] = \frac{1}{(0.2)}[(19.68652) - (2.17815)] = 87.54185$$

$$y_3 = y_2 + \frac{\Delta t}{2}[g(t_2, y_2) + g(t_3, x_3)] = (2.08938) + \frac{0.001}{2}[(88.76849) + (87.54185)] = 2.17753$$

<u>For $k = 3$:</u>

$$g(t_3, y_3) = \frac{1}{RC}[v(t_3) - y_3] = \frac{1}{0.2}[(19.68652) - (2.17753)] = 87.54492$$

Predictor:

$$x_4 = y_3 + g(t_3, y_3)\Delta t = (2.17753) + (87.54492)(0.001) = 2.26507$$

Corrector:

$$t_4 = t_3 + \Delta t = 0.002 + 0.001 = 0.003$$

$$v(t_4) = 10[2 - e^{-t_4}\sin(5\pi t_4)] = 10[2 - e^{-0.003}\sin(5\pi \cdot 0.003)] = 19.53035$$

$$g(t_4, x_4) = \frac{1}{RC}[v(t_4) - x_4] = \frac{1}{(0.2)}[(19.53035) - (2.26507)] = 86.32637$$

$$y_4 = y_3 + \frac{\Delta t}{2}[g(t_3, y_3) + g(t_4, x_4)] = (2.17753) + \frac{0.001}{2}[(87.54492) + (86.32637)] = 2.26446$$

```
% Problem 9.27
clear
clc
disp('Problem 9.27: Scott Thomas')

RC = 0.2;% seconds

N = 3;
delta_t = 0.001;
y = zeros(1,N);
t = zeros(1,N);
y(1) = 2.0;
for k = 1:N
    v(k) = 10*(2.0 - exp(-t(k)).*sin(5*pi*t(k)));
    g(k) = 1/RC*(v(k) - y(k));
%    disp('Predictor')
    x(k+1) = y(k) + g(k)*delta_t;
    t(k+1) = t(k) + delta_t;
    v(k+1) = 10*(2.0 - exp(-t(k+1)).*sin(5*pi*t(k+1)));
    g(k+1) = 1/RC*(v(k+1) - x(k+1));
%    disp('Corrector')
    y(k+1) = y(k) + delta_t/2*(g(k) + g(k+1));
%    disp('End of Process')
end
t
y
```

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t =

         0    0.0010    0.0020    0.0030


y =

    2.0000    2.0894    2.1775    2.2645

>>
```

Time-Step independence study:

```
% Problem 9.27
clear
clc
disp('Problem 9.27: Scott Thomas')

RC = 0.2;% seconds

N1 = 50;
delta_t1 = 0.1;
y1 = zeros(1,N1);
t1 = zeros(1,N1);
y1(1) = 2.0;
for k = 1:N1
    v1(k) = 10*(2.0 - exp(-t1(k)).*sin(5*pi*t1(k)));
    g1(k) = 1/RC*(v1(k) - y1(k));
%    disp('Predictor')
    x1(k+1) = y1(k) + g1(k)*delta_t1;
    t1(k+1) = t1(k) + delta_t1;
```

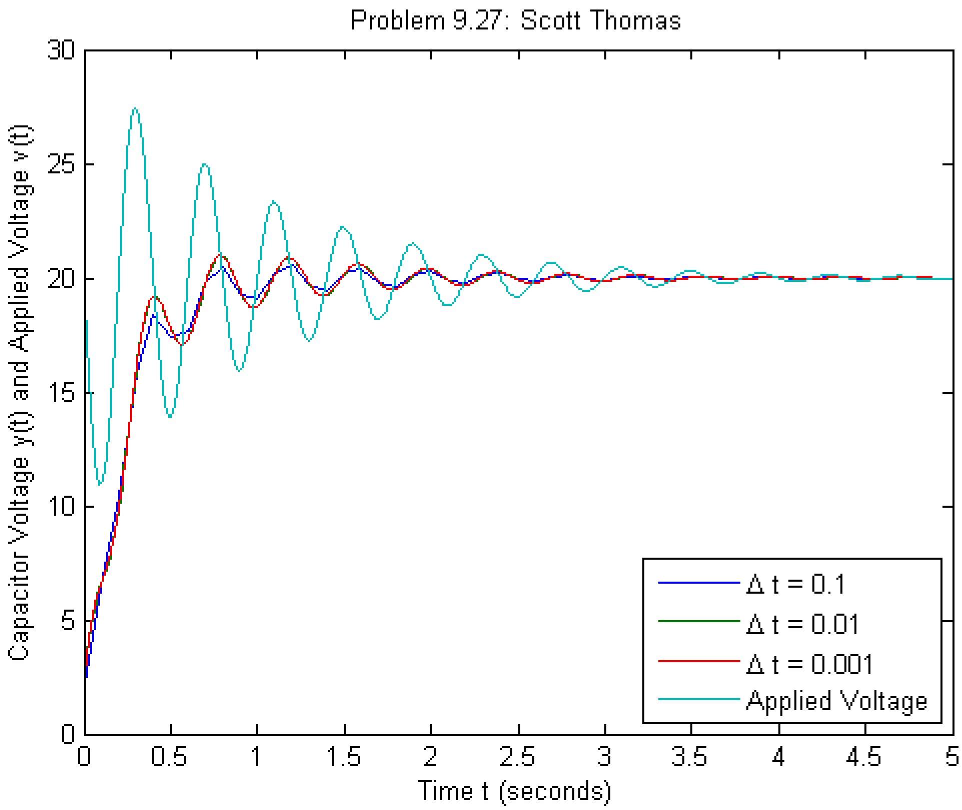
```
    v1(k+1) = 10*(2.0 - exp(-t1(k+1)).*sin(5*pi*t1(k+1)));
    g1(k+1) = 1/RC*(v1(k+1) - x1(k+1));
%    disp('Corrector')
    y1(k+1) = y1(k) + delta_t1/2*(g1(k) + g1(k+1));
%    disp('End of Process')
end

N2 = 500;
delta_t2 = 0.01;
y2 = zeros(1,N2);
t2 = zeros(1,N2);
y2(1) = 2.0;
for k = 1:N2
    v2(k) = 10*(2.0 - exp(-t2(k)).*sin(5*pi*t2(k)));
    g2(k) = 1/RC*(v2(k) - y2(k));
%    disp('Predictor')
    x2(k+1) = y2(k) + g2(k)*delta_t2;
    t2(k+1) = t2(k) + delta_t2;
    v2(k+1) = 10*(2.0 - exp(-t2(k+1)).*sin(5*pi*t2(k+1)));
    g2(k+1) = 1/RC*(v2(k+1) - x2(k+1));
%    disp('Corrector')
    y2(k+1) = y2(k) + delta_t2/2*(g2(k) + g2(k+1));
%    disp('End of Process')
end

N3 = 5000;
delta_t3 = 0.001;
y3 = zeros(1,N3);
t3 = zeros(1,N3);
y3(1) = 2.0;
for k = 1:N3
    v3(k) = 10*(2.0 - exp(-t3(k)).*sin(5*pi*t3(k)));
    g3(k) = 1/RC*(v3(k) - y3(k));
%    disp('Predictor')
    x3(k+1) = y3(k) + g3(k)*delta_t3;
    t3(k+1) = t3(k) + delta_t3;
    v3(k+1) = 10*(2.0 - exp(-t3(k+1)).*sin(5*pi*t3(k+1)));
    g3(k+1) = 1/RC*(v3(k+1) - x3(k+1));
%    disp('Corrector')
    y3(k+1) = y3(k) + delta_t3/2*(g3(k) + g3(k+1));
%    disp('End of Process')
end

plot(t1,y1,t2,y2,t3,y3, t3,v3)
xlabel('Time t (seconds)'), ylabel('Capacitor Voltage y(t) and Applied Voltage v(t)')
title('Problem 9.27: Scott Thomas')
legend('\Delta t = 0.1','\Delta t = 0.01', '\Delta t = 0.001','Applied Voltage','Location', 'SouthEast')
axis([0 5 0 30])
```

Problem 9.27: Scott Thomas



Problem 9.27: Scott Thomas
Capacitor Voltage y(t) and Applied Voltage v(t)
Time t (seconds)
0
5
10
15
20
25
30
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Δ t = 0.1
Δ t = 0.01
Δ t = 0.001
Applied Voltage