Problem 9.30:

30. The following equation describes the motion of a certain mass connected to a spring, with viscous friction on the surface

$$3\ddot{y} + 18\dot{y} + 102y = f(t)$$

where $f(t)$ is an applied force. Suppose that $f(t) = 0$ for $t < 0$ and $f(t) = 10$ for $t \geq 0$.

a. Plot $y(t)$ for $y(0) = \dot{y}(0) = 0$.

b. Plot $y(t)$ for $y(0) = 0$ and $\dot{y}(0) = 10$. Discuss the effect of the nonzero initial velocity.

This is a second-order ordinary differential equation. Rewrite the equation by solving for the second derivative.

$$\ddot{y} = -\frac{18}{3}\dot{y} - \frac{102}{3}y + 10/3 = -6\dot{y} - 34y + 10/3$$

$$\text{Let } \ x_1 = y \ \text{ and } \ x_2 = \dot{y}$$

Taking the derivative of the first equation gives

$$\dot{x}_1 = \dot{y} = x_2 \ \text{ or } \ \dot{x}_1 = x_2$$

Taking the derivative of the second equation gives

$$\dot{x}_2 = \ddot{y} = -6x_2 - 34x_1 + 10/3$$

The original second order ordinary differential equation is now converted into two first order ordinary differential equations that are coupled.

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -6x_2 - 34x_1 + 10/3, \quad x_1(0) = 0, \ x_2(0) = 0$$

Use the Euler method for this problem. The system of equations can be discretized as follows:

$$x_{1,k+1} = x_{1,k} + \Delta t \cdot x_{2,k}$$

$$x_{2,k+1} = x_{2,k} + \Delta t\left[-6x_{2,k} - 34x_{1,k} + 10/3\right]$$

In the problem statement, the initial conditions are $y(0) = x_1(0) = 0$ and $\dot{y}(0) = x_2(0) = 0$. Let $\Delta t = 0.01$.

For $k = 1$:

$$x_{1,2} = x_{1,1} + \Delta t \cdot x_{2,1} = (0.0) + (0.01)(0.0) = 0.0$$

$$x_{2,2} = x_{2,1} + \Delta t\left[-6x_{2,1} - 34x_{1,1} + 10/3\right]$$

$$x_{2,2} = (0.0) + (0.01)[-(6)(0.0) - (34)(0.0) + 10/3] = 0.0\overline{3}$$

For $k = 2$:

$$x_{1,3} = x_{1,2} + \Delta t \cdot x_{2,2} = (0.0) + (0.01)(0.0\overline{3}) = 0.000\overline{3}$$

$$x_{2,3} = x_{2,2} + \Delta t\left[-6x_{2,2} - 34x_{1,2} + 10/3\right]$$

$$x_{2,3} = (0.0\overline{3}) + (0.01)[-(6)(0.0\overline{3}) - (34)(0.0) + 10/3] = 0.064\overline{6}$$

For $k = 3$:

$$x_{1,4} = x_{1,3} + \Delta t \cdot x_{2,3} = (0.000\overline{3}) + (0.01)(0.064\overline{6}) = 0.0098$$

$$x_{2,4} = x_{2,3} + \Delta t\left[-6x_{2,3} - 34x_{1,3} + 10/3\right]$$

$$x_{2,4} = (0.064\overline{6}) + (0.01)[-(6)(0.064\overline{6}) - (34)(0.000\overline{3}) + 10/3] = 0.09400\overline{6}$$

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% Problem 9.30
clear
clc
disp('Problem 9.30: Scott Thomas')

Na = 10;
delta_ta = 0.01;
x1a = zeros(1,Na);
x2a = zeros(1,Na);

ta = zeros(1,Na);
x1a(1) = 0.0;
x2a(1) = 0.0;

for k = 1:Na
    x1a(k+1) = x1a(k) + x2a(k)*delta_ta;
    x2a(k+1) = x2a(k) + (-6*x2a(k) - 34*x1a(k) + 10/3)*delta_ta;
    ta(k+1) = ta(k) + delta_ta;
end
x1a
x2a
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x1a =

  Columns 1 through 5

                 0                   0   0.000333333333333   0.000980000000000   0.001920066666667

  Columns 6 through 10

   0.003133730666667   0.004601379933333   0.006303648893067   0.008221470356776   0.010336123459760

  Column 11

   0.012629277710685


x2a =

  Columns 1 through 5

                 0   0.033333333333333   0.064666666666667   0.094006666666667   0.121366400000000

  Columns 6 through 10

   0.146764926666667   0.170226895973333   0.191782146370933   0.211465310298368   0.229315425092495

  Column 11

   0.245375550943961

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% Problem 9.30
clear
clc
disp('Problem 9.30: Scott Thomas')

Na = 3000;
delta_ta = 0.001;
x1a = zeros(1,Na);
x2a = zeros(1,Na);

ta = zeros(1,Na);
x1a(1) = 0.0;
x2a(1) = 0.0;

for k = 1:Na
    x1a(k+1) = x1a(k) + x2a(k)*delta_ta;
    x2a(k+1) = x2a(k) + (-6*x2a(k) - 34*x1a(k) + 10/3)*delta_ta;
    ta(k+1) = ta(k) + delta_ta;
end

Nb = 3000;
delta_tb = 0.001;
x1b = zeros(1,Nb);
x2b = zeros(1,Nb);

tb = zeros(1,Nb);
x1b(1) = 0.0;
x2b(1) = 10.0;
```

```
for k = 1:Nb
    x1b(k+1) = x1b(k) + x2b(k)*delta_tb;
    x2b(k+1) = x2b(k) + (-6*x2b(k) - 34*x1b(k) + 10/3)*delta_tb;
    tb(k+1) = tb(k) + delta_tb;
end

plot(ta, x1a, tb, x1b)
xlabel('Time t'), ylabel('Function y(t)')
title('Problem 9.30: Scott Thomas')
legend('Part a: dy/dt(0) = 0', 'Part b: dy/dt(0) = 10','Location', 'NorthEast')
%axis([0 50 5 25])
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