ME 1020 Engineering Programming with MATLAB

Problem 9.32:

32. The following equation describes the motion of a certain mass connected to a spring, with no friction

$$3\ddot{y} + 75y = f(t)$$

where $f(t)$ is an applied force. Suppose the applied force is sinusoidal with a frequency of ω rad/s and an amplitude of 10 N: $f(t) = 10\sin(\omega t)$. Suppose that the initial conditions are $y(0) = \dot{y}(0) = 0$. Plot $y(t)$ for $0 \le t \le 20$ s. Do this for the following three cases. Compare the results of each case.

a. $\omega = 1$ rad/s
b. $\omega = 5$ rad/s
c. $\omega = 10$ rad/s

This is a second-order ordinary differential equation. Rewrite the equation by solving for the second derivative.

$$\ddot{y} = -\frac{75}{3}y + \frac{10}{3}\sin(\omega t) = -25y + \frac{10}{3}\sin(\omega t)$$

$$\text{Let } \ x_1 = y \ \text{ and } \ x_2 = \dot{y}$$

Taking the derivative of the first equation gives

$$\dot{x}_1 = \dot{y} = x_2 \quad \text{or} \quad \dot{x}_1 = x_2$$

Taking the derivative of the second equation gives

$$\dot{x}_2 = \ddot{y} = -25x_1 + \frac{10}{3}\sin(\omega t)$$

The original second order ordinary differential equation is now converted into two first order ordinary differential equations that are coupled.

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -25x_1 + \frac{10}{3}\sin(\omega t), \quad x_1(0) = 0, \ \ x_2(0) = 0$$

Use the Euler method for this problem. The system of equations can be discretized as follows:

$$x_{1,k+1} = x_{1,k} + \Delta t \cdot x_{2,k}$$

$$x_{2,k+1} = x_{2,k} + \Delta t\left[-25x_{1,k} + \frac{10}{3}\sin(\omega t_k)\right]$$

In the problem statement, the initial conditions are $y(0) = x_1(0) = 0$ and $\dot{y}(0) = x_2(0) = 0$. Let $\Delta t = 0.1$ and $\omega = 1\ \text{rad/sec}$.

For $k = 1$:

$$x_{1,2} = x_{1,1} + \Delta t \cdot x_{2,1} = (0.0) + (0.1)(0.0) = 0.0$$

$$x_{2,2} = x_{2,1} + \Delta t \left[-25x_{1,1} + \frac{10}{3}\sin(\omega t_1) \right]$$

$$x_{2,2} = (0.0) + (0.1)\left[-(25)(0.0) + \frac{10}{3}\sin[(1.0)(0.0)] \right] = 0.0$$

$$t_2 = t_1 + \Delta t = 0.0 + 0.1 = 0.1$$

For $k = 2$:

$$x_{1,3} = x_{1,2} + \Delta t \cdot x_{2,2} = (0.0) + (0.1)(0.0) = 0.0$$

$$x_{2,3} = x_{2,2} + \Delta t \left[-25x_{1,2} + \frac{10}{3}\sin(\omega t_2) \right]$$

$$x_{2,3} = (0.0) + (0.1)\left[-(25)(0.0) + \frac{10}{3}\sin[(1.0)(0.1)] \right] = 0.0332778$$

$$t_3 = t_2 + \Delta t = 0.1 + 0.1 = 0.2$$

For $k = 3$:

$$x_{1,4} = x_{1,3} + \Delta t \cdot x_{2,3} = (0.0) + (0.1)(0.0332778) = 0.00332778$$

$$x_{2,4} = x_{2,3} + \Delta t \left[-25x_{1,3} + \frac{10}{3}\sin(\omega t_3) \right]$$

$$x_{2,4} = (0.0332778) + (0.1)\left[-(25)(0.0) + \frac{10}{3}\sin[(1.0)(0.2)] \right] = 0.0995009$$

$$t_4 = t_3 + \Delta t = 0.2 + 0.1 = 0.3$$

```
% Problem 9.32
clear
clc
disp('Problem 9.32: Scott Thomas')

format short
omegaa = 1.0;
omegab = 5.0;
omegac = 10.0;

Na = 10;
delta_ta = 0.1;
x1a = zeros(1,Na);
x2a = zeros(1,Na);

ta = zeros(1,Na);
x1a(1) = 0.0;
x2a(1) = 0.0;

for k = 1:Na
    x1a(k+1) = x1a(k) + x2a(k)*delta_ta;
    x2a(k+1) = x2a(k) + (-25*x1a(k) + 10/3*sin(omegaa*ta(k)))*delta_ta;
    ta(k+1) = ta(k) + delta_ta;
end
ta;
x1a
x2a
```

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Problem 9.32: Scott Thomas

x1a =

         0         0         0    0.0033    0.0133    0.0322    0.0609    0.0974    0.1376    0.1748    0.2016


x2a =

         0         0    0.0333    0.0995    0.1897    0.2863    0.3655    0.4015    0.3727    0.2679    0.0919

>>
```

```
% Problem 9.32
clear
clc
disp('Problem 9.32: Scott Thomas')

format short
omegaa = 1.0;
omegab = 5.0;
omegac = 10.0;

Na = 200000;
delta_ta = 0.0001;
x1a = zeros(1,Na);
x2a = zeros(1,Na);

ta = zeros(1,Na);
x1a(1) = 0.0;
x2a(1) = 0.0;

for k = 1:Na
    x1a(k+1) = x1a(k) + x2a(k)*delta_ta;
    x2a(k+1) = x2a(k) + (-25*x1a(k) + 10/3*sin(omegaa*ta(k)))*delta_ta;
    ta(k+1) = ta(k) + delta_ta;
end
ta;
x1a;
x2a;

Nb = 200000;
delta_tb = 0.0001;
x1b = zeros(1,Nb);
x2b = zeros(1,Nb);

tb = zeros(1,Nb);
x1b(1) = 0.0;
x2b(1) = 0.0;

for k = 1:Nb
    x1b(k+1) = x1b(k) + x2b(k)*delta_tb;
    x2b(k+1) = x2b(k) + (-25*x1b(k) + 10/3*sin(omegab*tb(k)))*delta_tb;
    tb(k+1) = tb(k) + delta_tb;
end

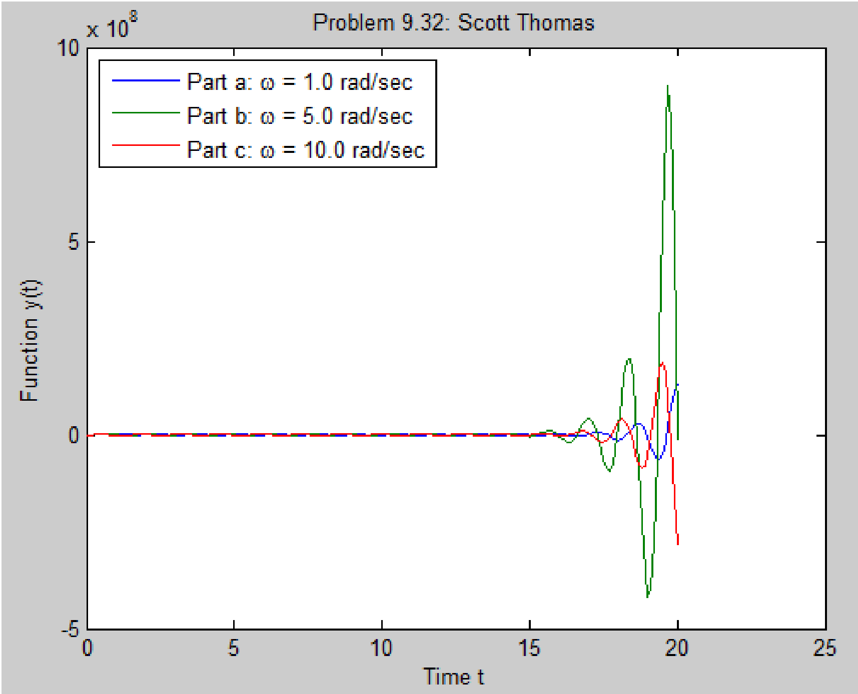
Nc = 200000;
delta_tc = 0.0001;
x1c = zeros(1,Nc);
x2c = zeros(1,Nc);

tc = zeros(1,Nc);
x1c(1) = 0.0;
x2c(1) = 0.0;

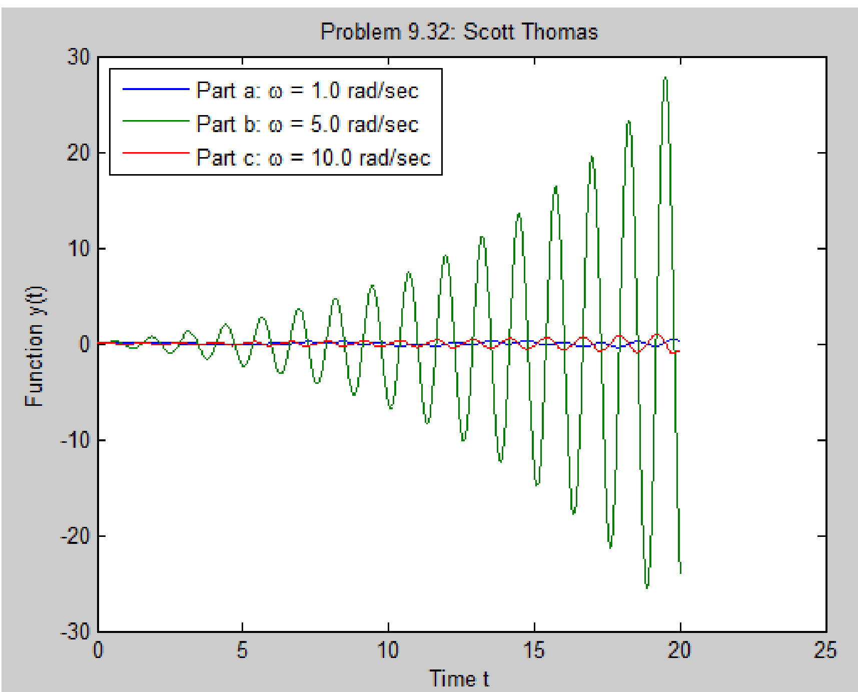
for k = 1:Nc
    x1c(k+1) = x1c(k) + x2c(k)*delta_tc;
    x2c(k+1) = x2c(k) + (-25*x1c(k) + 10/3*sin(omegac*tc(k)))*delta_tc;
    tc(k+1) = tc(k) + delta_tc;
end

%plot(ta, x1a)%, tb, x1b, tc, x1c)
%plot(tb, x1b)%, tb, x1b, tc, x1c)
plot(tc, x1c)%, tb, x1b, tc, x1c)
xlabel('Time t'), ylabel('Function y(t)')
title('Problem 9.32: Scott Thomas')
%legend('Part a: \omega = 1.0 rad/sec', 'Location', 'South')
%legend('Part b: \omega = 5.0 rad/sec', 'Location', 'South')
legend('Part c: \omega = 10.0 rad/sec', 'Location', 'South')
```

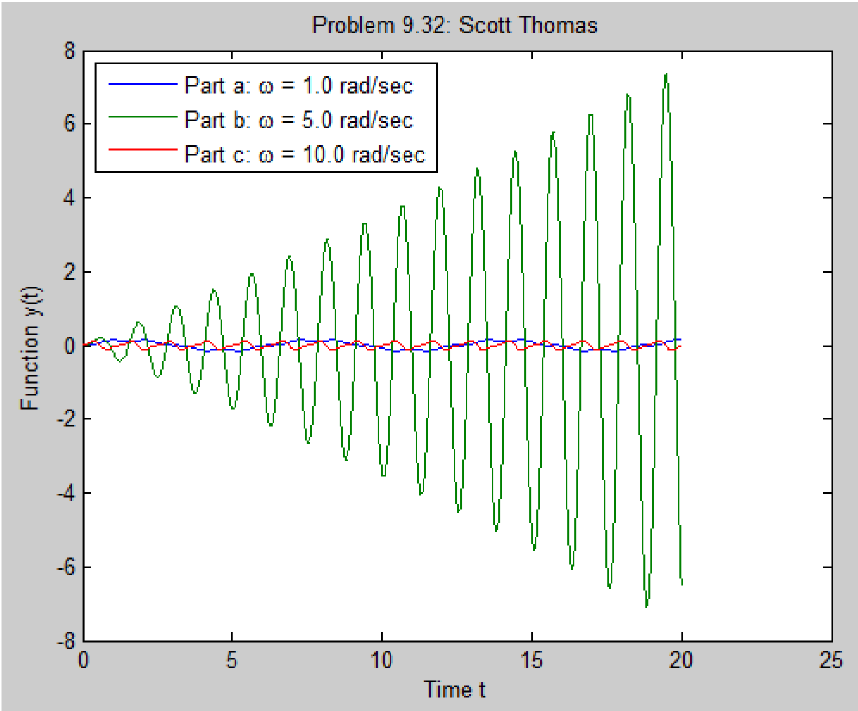
This problem is very sensitive to time step size. The following plot is for $\Delta t = 0.1$ second:



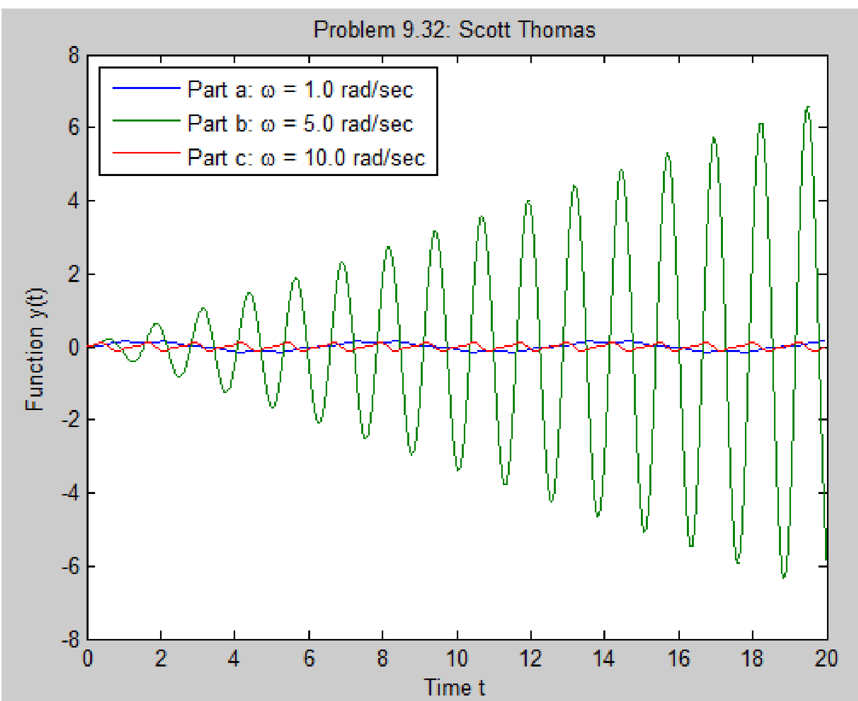
$\Delta t = 0.01$ second:

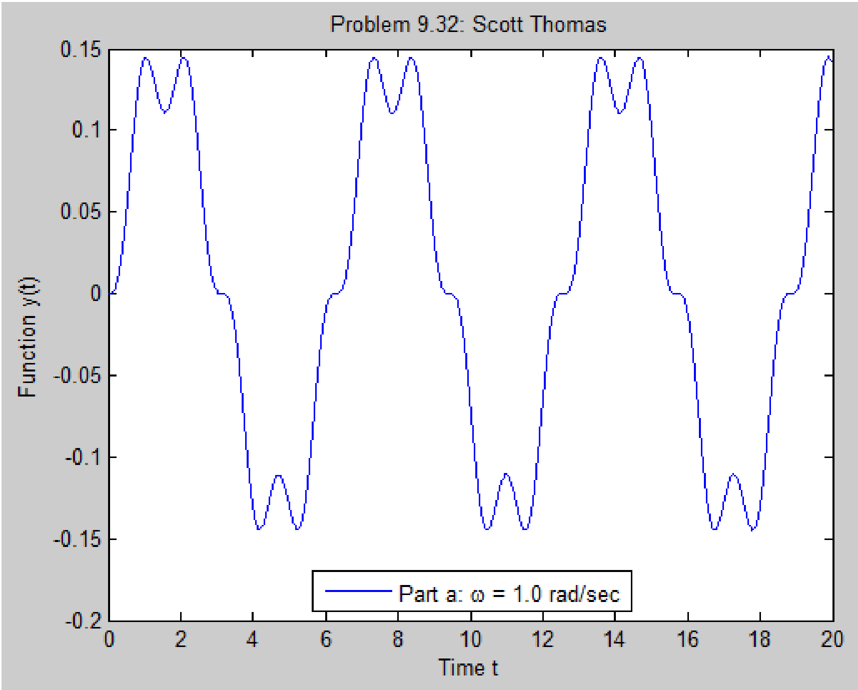


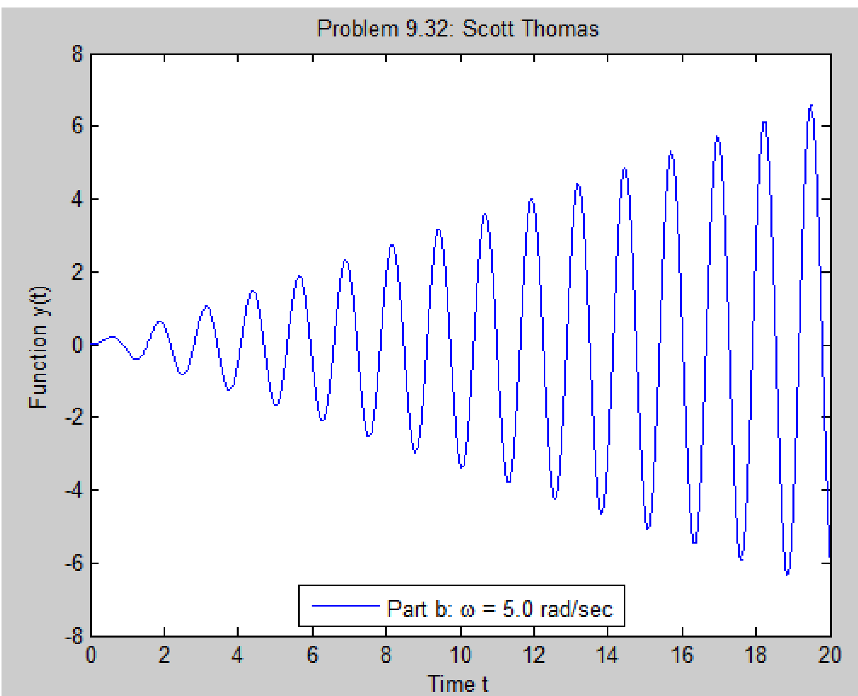
$\Delta t = 0.001$ second:



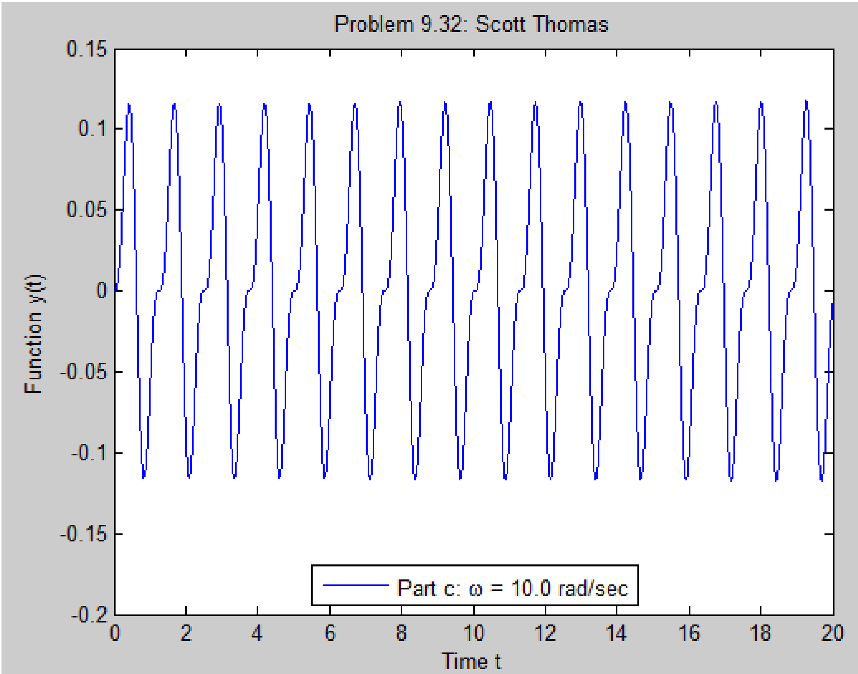
$\Delta t = 0.0001$ second: This seems to be sufficient.







This behavior is called “Resonance”.



Problem 9.32: Scott Thomas
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
Function y(t)
0
2
4
6
8
10
12
14
16
18
20
Time t
Part c: ω = 10.0 rad/sec