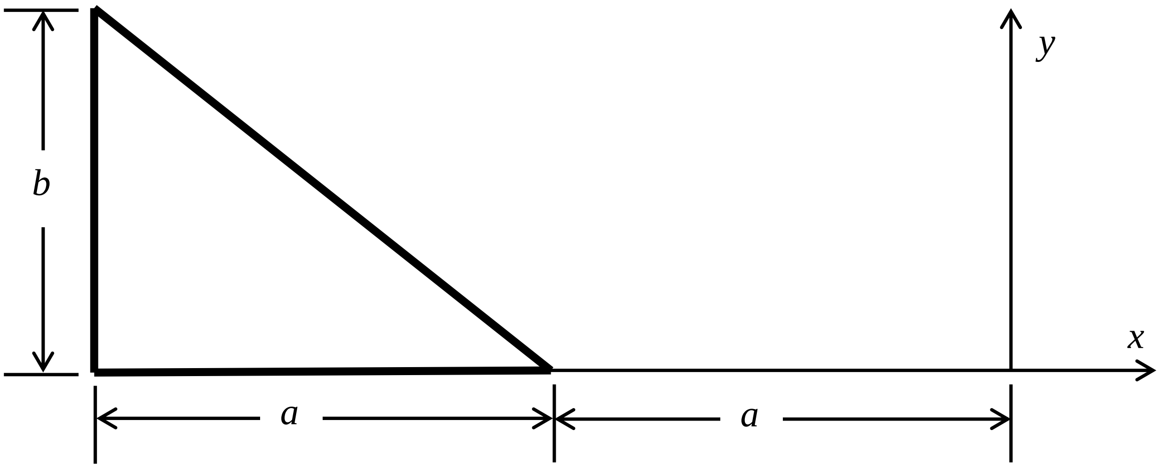
Wright State University
Department of Mechanical and Materials Engineering

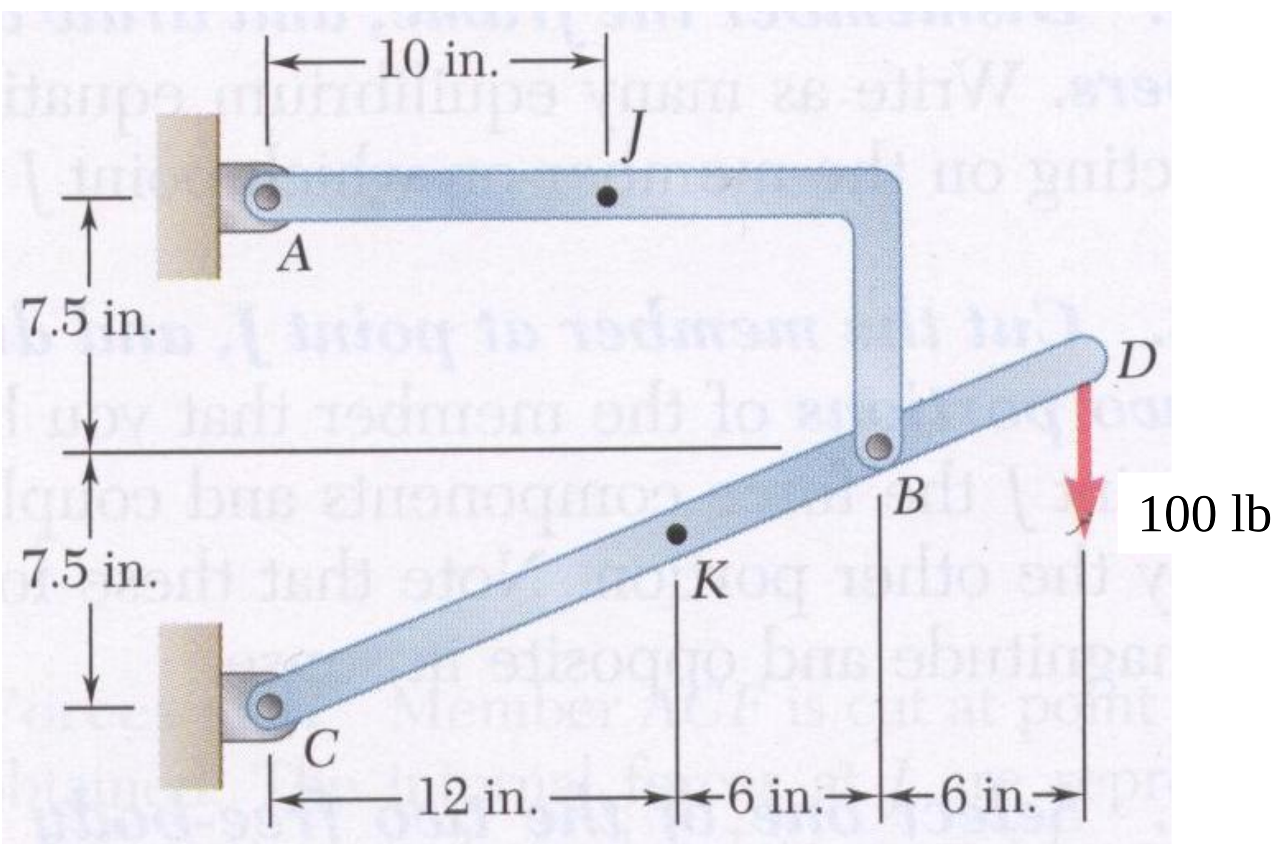
ME 2120 Statics, Fall 2014

FINAL EXAM
Open Book, Closed Notes, Do not write on this sheet, Show all work

Problem 1: (35 points) By direct integration, derive expressions for the area and the location of the centroid of the area shown from the *y*-axis.

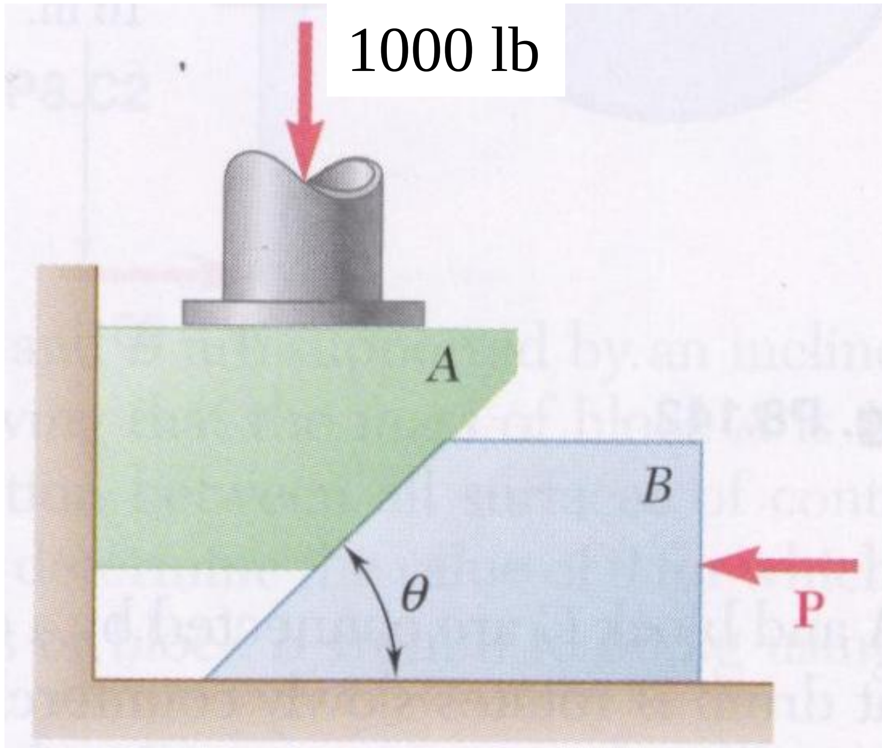


Problem 2: (25 points) Determine the internal forces at point *K* of the structure shown.

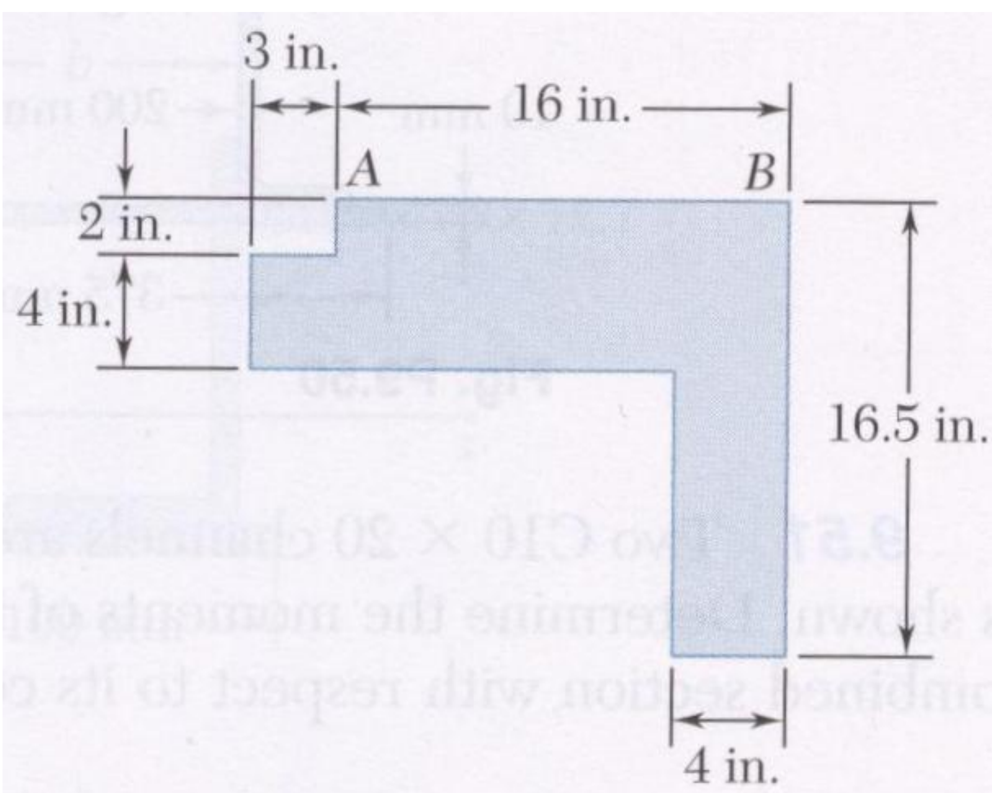


See back of this page for Problems 3 and 4.

Problem 3: (20 points) Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

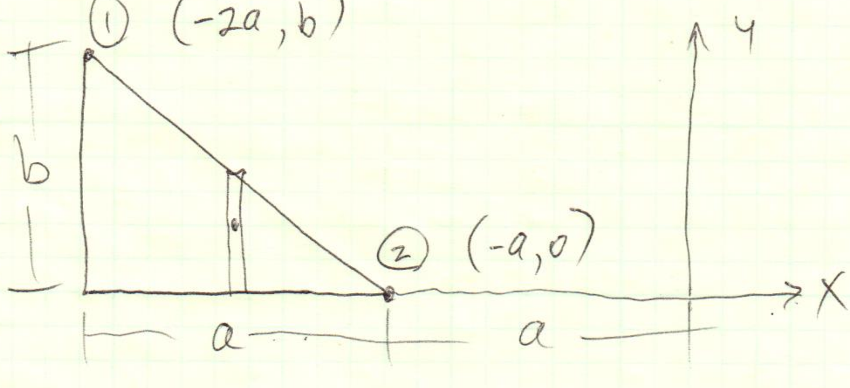


Problem 4: (20 points) Determine the moment of inertia of the area shown with respect to the centroidal axis parallel to side *AB*.



FINAL EXAM, ME2120, FALL 2014

PROB. 1



FIND $y = mx + g$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(0 - b)}{(-a) - (-2a)} = -\frac{b}{a}$$

$y = mx + g$: EVALUATE AT $x = -a$, $y = 0$

$$0 = -\left(\frac{b}{a}\right)(-a) + g$$

$$g = -b$$

$$\boxed{y = \left(-\frac{b}{a}\right) \cdot x - b}$$

$$A = \int dA = \int_{-2a}^{-a} y\,dx = \int_{-2a}^{-a} \left[-\left(\frac{b}{a}\right)x - b\right] dx$$

$$A = -b \int_{-2a}^{-a} \left(\frac{x}{a} + 1\right) dx$$

PROB. 1 CONT.

$$A = -b\left[\frac{x^2}{2a} + x\right]_{-2a}^{-a}$$

$$= -b\left\{\left[\frac{(-a)^2}{2a} + (-a)\right] - \left[\frac{(-2a)^2}{2a} + (-2a)\right]\right\}$$

$$= -b\left(\frac{a^2}{2a} - a - \frac{4a^2}{2a} + 2a\right)$$

$$= -ab\left(\frac{1}{2} - 1 - 2 + 2\right)$$

$$\boxed{A = \frac{1}{2}ab}$$

$$\bar{x} = \frac{1}{A}\int x_{el}\, dA \ , \quad x_{el} = x, \quad dA = y\,dx$$

$$\bar{x} = \frac{1}{A}\int_{-2a}^{-a} x\left[-\left(\frac{b}{a}\right)\cdot x - b\right]dx$$

$$= -\frac{b}{A}\int_{-2a}^{a}\left(\frac{x^2}{a} + x\right)dx$$

$$= -\frac{b}{A}\left[\frac{x^3}{3a} + \frac{x^2}{2}\right]_{-2a}^{-a}$$

$$= -\frac{b}{A}\left\{\left[\frac{(-a)^3}{3a} + \frac{(-a)^2}{2}\right] - \left[\frac{(-2a)^3}{3a} + \frac{(-2a)^2}{2}\right]\right\}$$

$$= -\frac{b}{A}\left(-\frac{a^3}{3a} + \frac{a^2}{2} + \frac{8a^3}{3a} - \frac{4a^2}{2}\right)$$

PROB. 1 CONT.

$$\bar{X} = -\frac{ba^2}{A}\left(-\frac{1}{3} + \frac{1}{2} + \frac{8}{3} - 2\right)$$

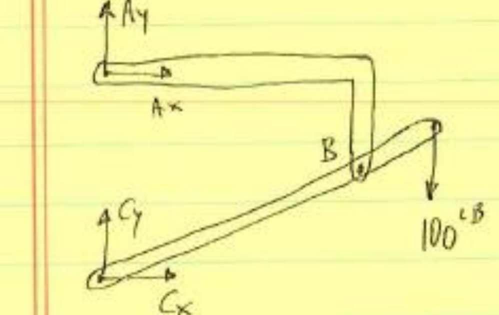
$$= \frac{-ba^2}{\left(\frac{1}{2}ab\right)} \cdot \left(\frac{-2 + 3 + 16 - 12}{6}\right)$$

$$= -2a\left(\frac{5}{6}\right)$$

$$\boxed{\bar{X} = -\frac{5}{3}a}$$

PROB. 7.6

FBD



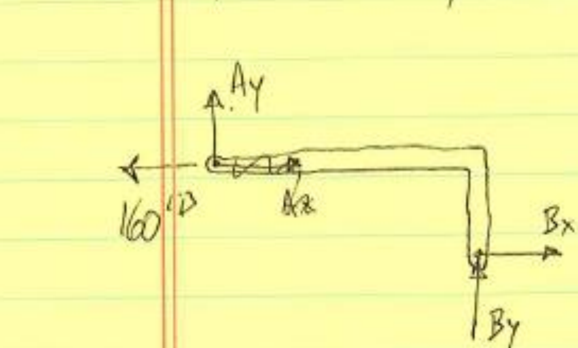
$\Sigma F_x = 0: \; A_x + C_x = 0$

$\Sigma F_y = 0: \; A_y + C_y - 100 = 0$

$\Sigma M_A = 0$ ↺:

$-(24^{IN})(100^{LB}) + (15^{IN})C_x = 0$

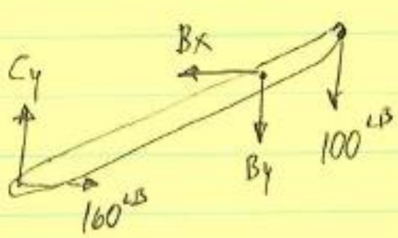
$C_x = 160^{LB}, \; A_x = -160^{LB}$



$\Sigma F_x = 0: \; B_x - 160 = 0, \; B_x = 160^{LB}$

$\Sigma F_y = 0: \; A_y + B_y = 0$

$\Sigma M_A = 0$ ↺: $(18^{IN})B_y + (7.5^{IN})(160) = 0$



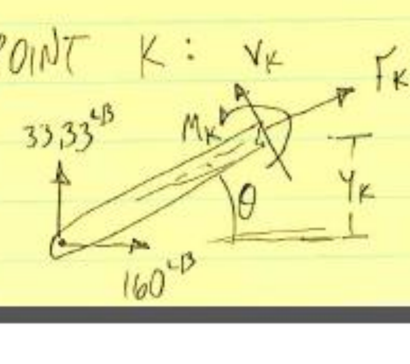
$B_y = -66.67^{LB}, \; A_y = 66.67^{LB}$

$C_y - (-66.67) - 100 = 0$

$C_y = 33.33^{LB}$

$\theta = TAN^{-1}\left(\frac{7.5}{18}\right) = 22.62^{\circ}$

CUT POINT K:



$TAN\,\theta = \frac{Y_K}{12}$

$Y_K = 12\,TAN\,22.62^{\circ} = 5.0^{IN}$

$\vec{F}_K = (F_K \cos 22.62°)\hat{i} + (F_K \sin 22.62)\hat{k}$ LB

$\vec{F}_K = (0.923\, F_K)\hat{i} + (0.385\, F_K)\hat{j}$ LB

$\vec{V}_K = (V_K \cos 112.6)\hat{i} + (V_K \sin 112.6)\hat{j}$ LB

$\vec{V}_K = (-0.385\, V_K)\hat{i} + (0.923\, V_K)\hat{j}$ LB

$\Sigma F_x = 0: \; 160 + 0.923\, F_K - 0.385\, V_K = 0$

$V_K = 415.6 + 2.397\, F_K$ 1

$\Sigma F_y = 0: \; 33.33 + 0.385\, F_K + 0.923\, V_K = 0$

$33.33 + 0.385\, F_K + 0.923\,(415.6 + 2.397\, F_K) = 0$

$(0.385 + 2.212)\, F_K = -33.33 - 0.923 \cdot 415.6$

$\boxed{F_K = -160 \text{ LB}}$

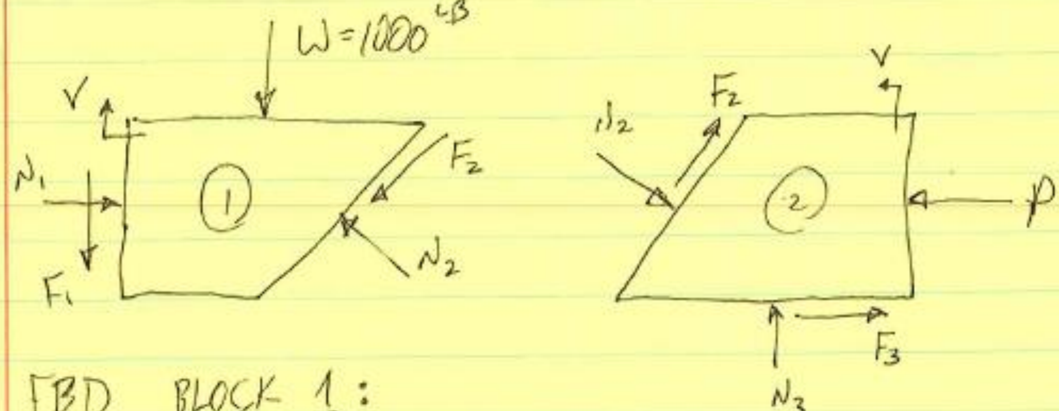
$V_K = 415.6 + 2.397(-160)$

$\boxed{V_K = 32.1 \text{ LB}}$

$\Sigma M_K = 0 \circlearrowleft: \; M_K + (5.0^{IN})(160^{LB}) - (12^{IN})(33.33^{LB}) = 0$

$\boxed{M_K = -400^{IN\text{-}LB}}$

PROB. 8.139



FBD BLOCK 1:

$\Sigma F_x = 0: \; N_1 - 0.707 F_2 - 0.707 N_2 = 0$

$F_2 = \mu_s N_2 = 0.25 N_2$

$N_1 = 0.707(0.25 N_2) + 0.707 N_2$

$N_1 = 0.884 N_2$ (1)

$\Sigma F_y = 0: \; -1000 - F_1 - 0.707 F_2 + 0.707 N_2 = 0$

$-1000 - 0.25 N_1 - 0.707(0.25 N_2) + 0.707 N_2 = 0$

$-1000 - 0.25(0.884 N_2) + 0.530 N_2 = 0$

$N_2 = 3236^{LB}$

(4)

GCRCC.NET

PROB. 8.139 CONT.

FBD BLOCK 2:

$\Sigma F_y = 0: \; -0.707 N_2 + 0.707 F_2 + N_3 = 0$

$-0.707 N_2 + 0.707 (0.25 N_2) + N_3 = 0$

$N_3 = 0.707(3236) - 0.707(0.25)(3236)$

$N_3 = 1716$ LB

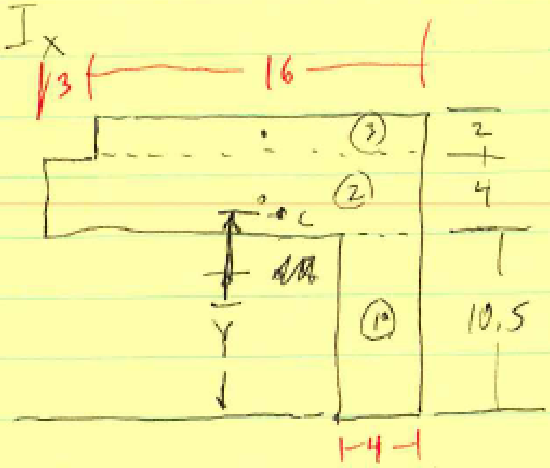
$\Sigma F_x = 0: \; 0.707 N_2 + 0.707 F_2 + F_3 - P = 0$

$P = 0.707(3236) + 0.707(0.25)(3236) + 0.25(1716)$

$\boxed{P = 3289 \text{ LB}}$

PROB. ~~[illegible]~~
9.43

FIND $\bar{I}_x$



AREA 1: $A_1 = (4)(10.5)$

$A_1 = 42 \text{ IN}^2$, $\bar{Y}_1 = \frac{1}{2}(10.5) = 5.25 \text{ IN}$, $\bar{Y}_1 A_1 = (42)(5.25)$

$\bar{Y}_1 A_1 = 220.5 \text{ IN}^3$

AREA 2: $A_2 = (4 \text{ IN})(19 \text{ IN}) = 76 \text{ IN}^2$, $\bar{Y}_2 = 10.5 + \frac{1}{2}(4) = 12.5 \text{ IN}$

$\bar{Y}_2 A_2 = 950 \text{ IN}^3$

AREA 3: $A_3 = (2 \text{ IN})(16 \text{ IN}) = 32 \text{ IN}^2$, $\bar{Y}_3 = 14.5 + \frac{1}{2}(2) = 15.5 \text{ IN}$

$\bar{Y}_3 A_3 = (32)(15.5) = 496 \text{ IN}^3$

$\bar{Y} = (220.5 + 950 + 496 \text{ IN}^3) / (42 + 76 + 32 \text{ IN}^2)$

$\bar{Y} = 11.11 \text{ IN}$ (5.39 IN)

PROB. 9.43 CONT.

$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$(I_x)_1 = \tfrac{1}{12} bh^3 + Ad^2 = \tfrac{1}{12}(4^{in})(10.5^{in})^3 + (4)(10.5)\left(11.11 - \tfrac{1}{2}(10.5)\right)^2$$

$$(I_x)_1 = 1828^{\,in^4}$$

$$(I_x)_2 = \tfrac{1}{12}(19^{in})(4^{in})^3 + (19)(4)(\cancel{16} - 11.11 - 2)^2 \quad 14.5$$

$$(I_x)_2 = 248^{\,in^4}$$

$$(I_x)_3 = \tfrac{1}{12}(16^{in})(2^{in})^3 + (16)(2)(16.5 - 11.11 - 1)^2$$

$$(I_x)_3 = 627^{\,in^4}$$

$$\bar{I}_x = 1828 + 248 + 627$$

$$\boxed{\bar{I}_x = 2703\ in^4}$$