Wright State University
Department of Mechanical and Materials Engineering

ME 2120: Statics, Fall 2015

FINAL EXAM

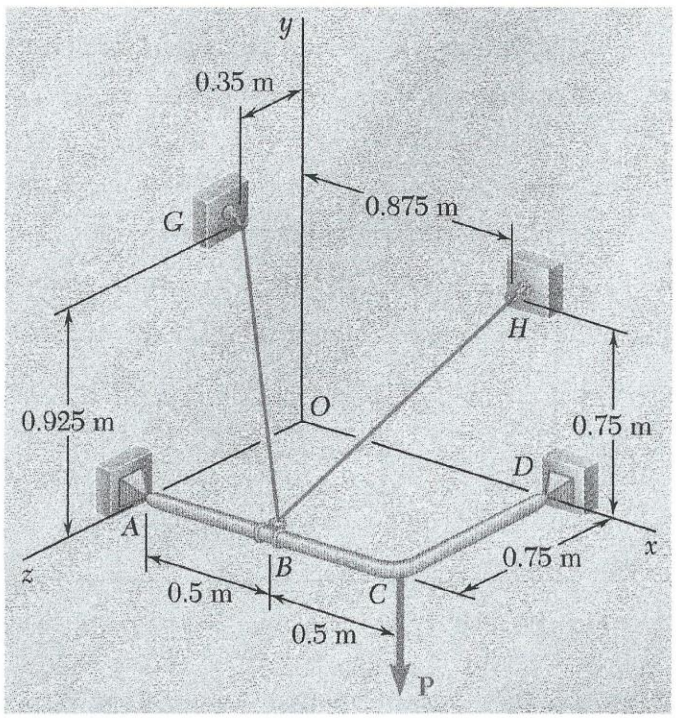
Open book, Closed notes, Do not write on this sheet, Show all work

Problem 1: (10 points) The frame *ACD* is supported by ball-and-socket joints at *A* and *D* and by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the frame supports a load of magnitude $P = 268$ N at point *C*, determine the tension in the cable. The tension forces in the cables are:

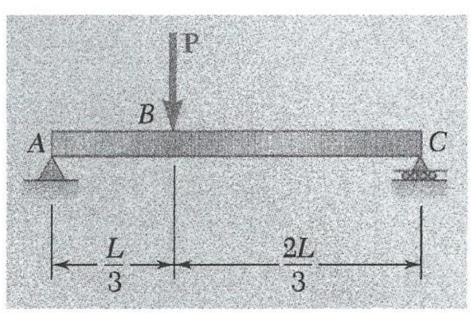
$$\vec{F}_{BG} = (-0.4444T)\hat{\imath} + (0.8222T)\hat{\jmath} + (-0.3556T)\hat{k}$$

$$\vec{F}_{BH} = (0.3333T)\hat{\imath} + (0.6667T)\hat{\jmath} + (-0.6667T)\hat{k}$$

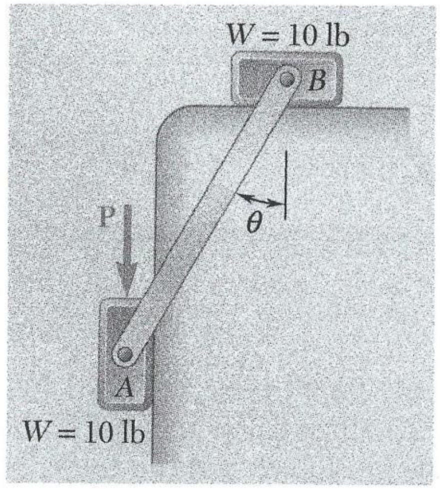
(Answer: $T = 360$ N)



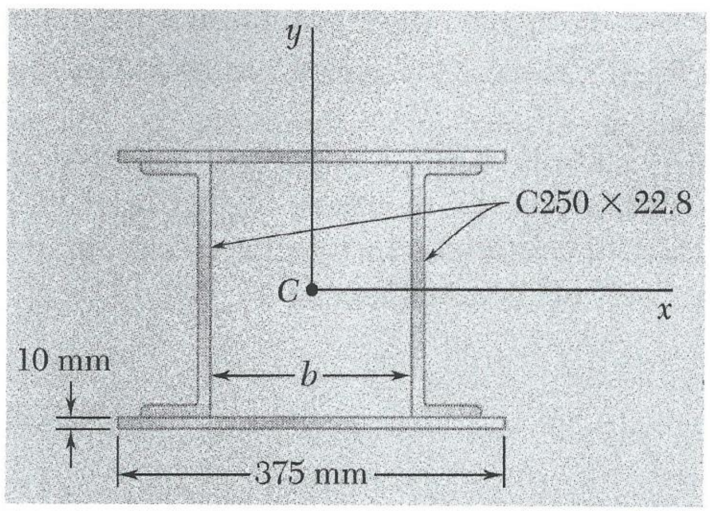
Problem 2: (10 points) Determine the maximum absolute values of the shear and bending moment. (Answers: $M_{max} = 2PL/9$, $V_{max} = 2P/3$)



Problem 3: (10 points) Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$ with the vertical. Determine the largest value of P for which equilibrium is maintained. (Answer: $P = 2.69$ lb)



Problem 4: (5 points) Two channels and two plates are used to form the column section shown. For $b = 200$ mm, determine the moment of inertia of the combined section with respect to the centroidal x axis. (Answer: $I_x = 1.867\text{E}8\ \text{mm}^4$)



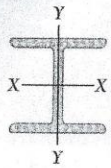
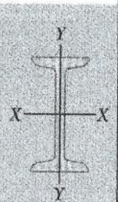
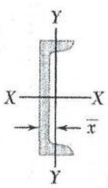
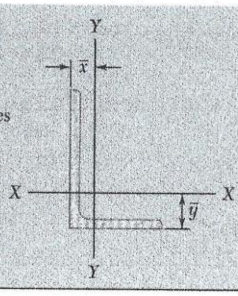
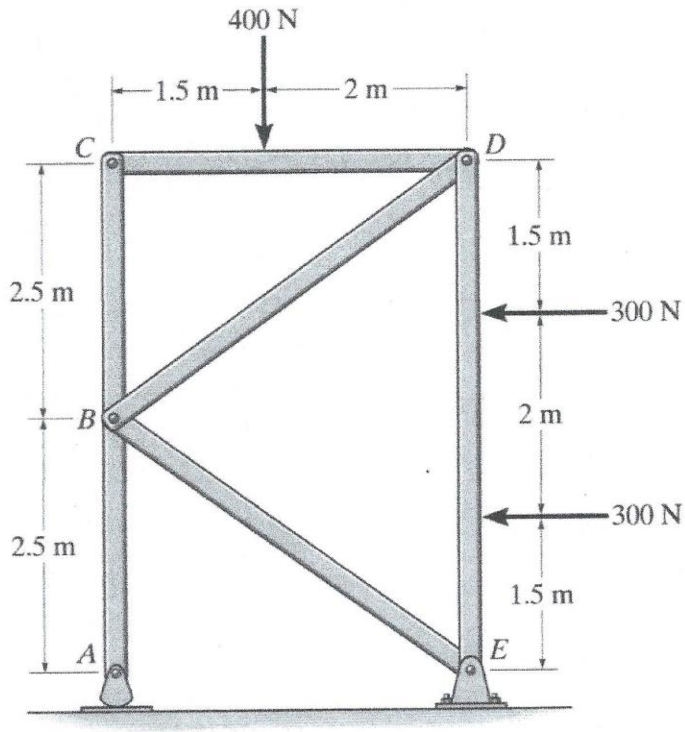
		Designation	Area mm²	Depth mm	Width mm	Axis X–X: $\bar{I}_x$ 10^6 mm⁴	Axis X–X: $\bar{k}_x$ mm	Axis X–X: $\bar{y}$ mm	Axis Y–Y: $\bar{I}_y$ 10^6 mm⁴	Axis Y–Y: $\bar{k}_y$ mm	Axis Y–Y: $\bar{x}$ mm
W Shapes (Wide-Flange Shapes)		W460 × 113†	14 400	462	279	554	196		63.3	66.3	
		W410 × 85	10 800	417	181	316	171		17.9	40.6	
		W360 × 57.8	7230	358	172	160	149		11.1	39.4	
		W200 × 46.1	5880	203	203	45.8	88.1		15.4	51.3	
S Shapes (American Standard Shapes)		S460 × 81.4†	10 300	457	152	333	180		8.62	29.0	
		S310 × 47.3	6010	305	127	90.3	123		3.88	25.4	
		S250 × 37.8	4810	254	118	51.2	103		2.80	24.1	
		S150 × 18.6	2360	152	84.6	9.16	62.2		0.749	17.8	
C Shapes (American Standard Channels)		C310 × 30.8†	3920	305	74.7	53.7	117		1.61	20.2	17.7
		C250 × 22.8	2890	254	66.0	28.0	98.3		0.945	18.1	16.1
		C200 × 17.1	2170	203	57.4	13.5	79.0		0.545	15.8	14.5
		C150 × 12.2	1540	152	48.8	5.45	59.4		0.286	13.6	13.0
Angles		L152 × 152 × 25.4‡	7100			14.7	45.5	47.2	14.7	45.5	47.2
		L102 × 102 × 12.7	2420			2.30	30.7	30.0	2.30	30.7	30.0
		L76 × 76 × 6.4	929			0.512	23.5	21.2	0.512	23.5	21.2
		L152 × 102 × 12.7	3060			7.20	48.5	50.3	2.59	29.0	24.9
		L127 × 76 × 12.7	2420			3.93	40.1	44.2	1.06	20.9	18.9
		L76 × 51 × 6.4	768			0.454	24.2	24.9	0.162	14.5	12.4

Fig. 9.13B Properties of rolled-steel shapes (SI units).

†Nominal depth in millimeters and mass in kilograms per meter

‡Depth, width, and thickness in millimeters

Extra Credit Problem: (5 points) Draw the free-body diagram(s) for the following situation. Do not solve!



Determine the horizontal and vertical components of force which the pins at *A*, *B*, and *C* exert on member *ABC* of the frame.

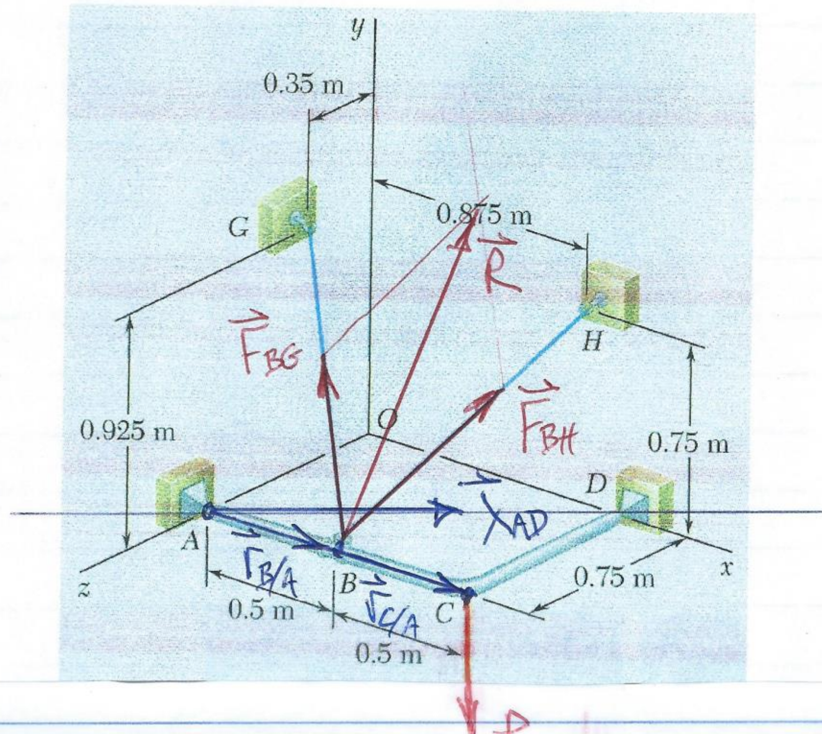
(10) PROB. 4.133

Problem 1: (points) The frame *ACD* is supported by ball-and-socket joints at *A* and *D* and by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the frame supports a load of magnitude $P = 268$ N at point *C*, determine the tension in the cable. The tension forces in the cables are:

$$\vec{F}_{BG} = (-0.4444T)\hat{\imath} + (0.8222T)\hat{\jmath} + (-0.3556T)\hat{k}$$

$$\vec{F}_{BH} = (0.3333T)\hat{\imath} + (0.6667T)\hat{\jmath} + (-0.6667T)\hat{k}$$

(Answer: $T = 360$ N)



POINTS:

$A(0, 0, 0.75)^m$ $\quad$ $G(0, 0.925, 0.35)^m$

$B(0.5, 0, 0.75)^m$ $\quad$ $H(0.875, 0.75, 0)^m$

$C(1.0, 0, 0.75)^m$

$D(1.0, 0, 0)^m$

FIND $\vec{F}_{BG}$:

$$dx = X_G - X_B = 0 - 0.5 = -0.5^{m}$$

$$dy = Y_G - Y_B = 0.925 - 0 = 0.925^{m}$$

$$dz = Z_G - Z_B = 0.35 - 0.75 = -0.40^{m}$$

$$d = \sqrt{0.5^2 + 0.925^2 + 0.4^2} = 1.125^{m}$$

$$F_x = T \cdot \frac{dx}{d} = T\left(\frac{-0.5}{1.125}\right) = -0.4444 \cdot T$$

$$F_y = T \cdot \frac{dy}{d} = T \cdot \left(\frac{0.925}{1.125}\right) = 0.8222 \cdot T$$

$$F_z = T \cdot \frac{dz}{d} = T\left(\frac{-0.4}{1.125}\right) = -0.3556 \cdot T$$

$$\boxed{\vec{F}_{BG} = (-0.4444 \cdot T)\hat{i} + (0.8222 \cdot T)\hat{j} + (-0.3556 \cdot T)\hat{k}^{\ N}}$$

FIND $\vec{F}_{BH}$:

$$dx = X_H - X_B = 0.875 - 0.5 = 0.375^{m}$$

$$dy = Y_H - Y_B = 0.75 - 0 = 0.75^{m}$$

$$dz = Z_H - Z_B = 0 - 0.75 = -0.75^{m}$$

$\cancel{F_x = T \cdot \frac{dx}{d}}$ $d = \sqrt{0.375^2 + 0.75^2 + 0.75^2} = 1.125^m$

$$F_x = T \cdot \frac{dx}{d} = T\left(\frac{0.375}{1.125}\right) = 0.3333 \cdot T$$

$$F_y = T \cdot \frac{dy}{d} = T\left(\frac{0.75}{1.125}\right) = 0.6667 \cdot T$$

$$F_z = T \cdot \frac{dz}{d} = T\left(\frac{-0.75}{1.125}\right) = -0.6667 \cdot T$$

$$\boxed{\vec{F}_{BH} = (0.3333 \cdot T)\hat{i} + (0.6667 \cdot T)\hat{j} + (-0.6667 \cdot T)\hat{k}^{\,N}}$$

$$\boxed{\vec{P} = (-268)\hat{j}^{\,N}}$$

FIND $\vec{r}_{B/A}$:

$dx = X_B - X_A = 0.5^m$, $dy = 0$, $dz = 0$

$$\boxed{\vec{r}_{B/A} = (0.5)\hat{i}^{\,m}}$$

$$\boxed{\vec{r}_{C/A} = (1.0)\hat{i}^{\,m}}$$

$$\vec{R} = \vec{F}_{BG} + \vec{F}_{BH}$$

$$\vec{R} = (-0.4444 \cdot T + 0.3333 \cdot T)\hat{i} + (0.8222 \cdot T + 0.6667 \cdot T)\hat{j}$$

$$+ (-0.3556 \cdot T - 0.6667 \cdot T)\hat{k} \ ^{N}$$

$$\boxed{\vec{R} = (-0.1111 \cdot T)\hat{i} + (1.489 \cdot T)\hat{j} + (-1.022 \cdot T)\hat{k} \ ^{N}}$$

FIND $\vec{\lambda}_{AD}$:

$$dx = X_D - X_A = 1.0 - 0 = 1.0^{m}$$

$$dy = Y_D - Y_A = 0$$

$$dz = Z_D - Z_A = 0 - 0.75 = -0.75^{m}$$

$$d = \sqrt{1^2 + 0.75^2} = 1.25^{m}$$

$$\vec{\lambda}_{AD} = \left(\frac{1}{1.25}\right)\hat{i} + (0)\hat{j} + \left(\frac{-0.75}{1.25}\right)\hat{k}$$

$$\boxed{\vec{\lambda}_{AD} = (0.8)\hat{i} + (0)\hat{j} + (-0.6)\hat{k}}$$

$$\vec{M}_1 = \vec{r}_{B/A} \times \vec{R} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0 & 0 \\ -0.1111 \cdot T & 1.489 \cdot T & -1.022 \cdot T \end{vmatrix}$$

$$\vec{M}_1 = (0)\hat{i} - [(0.5)(-1.022 \cdot T) - 0]\hat{j} + [(0.5)(1.489 \cdot T) - 0]\hat{k} \; N\text{-}m$$

$$\boxed{\vec{M}_1 = (0)\hat{i} + (0.511 \cdot T)\hat{j} + (0.7445 \cdot T)\hat{k} \; N\text{-}m}$$

$$\vec{M}_2 = \vec{r}_{C/A} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.0 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix}$$

$$\vec{M}_2 = (0)\hat{i} - (0)\hat{j} + [(1.0)(-268) - 0]\hat{k} \; N\cdot m$$

$$\boxed{\vec{M}_2 = (0)\hat{i} + (0)\hat{j} + (-268)\hat{k} \; N\cdot m}$$

$$\vec{M}_A = \vec{M}_1 + \vec{M}_2$$

$$\boxed{M_A = (0)\hat{i} + (0.511 \cdot T)\hat{j} + (0.7445 \cdot T - 268)\hat{k} \; N\cdot m}$$

$$\vec{M}_A \cdot \vec{\lambda}_{AD} = 0 :$$

$$(0)(0.8) + (0.511 \cdot T)(0) + (0.7445 \cdot T - 268)(-0.6) = 0$$

$$\boxed{T = 360 \, N}$$

$$\vec{r}_B \times \vec{T}_{BG} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0 & 0 \\ -0.4444T & 0.8222T & -0.3556T \end{vmatrix}$$

$$= [0]\hat{i} - [(0.5)(-0.3556T)]\hat{j} + [(0.5)(0.8222T)]\hat{k} \ ^{N \cdot m}$$

$$\vec{M}_1 = (0)\hat{i} + (0.1778T)\hat{j} + (0.4111T)\hat{k} \ ^{N \cdot m}$$

$$\vec{M}_2 = \vec{r}_B \times \vec{F}_{BH} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0 & 0 \\ \frac{1}{3}T & \frac{2}{3}T & -\frac{2}{3}T \end{vmatrix}$$

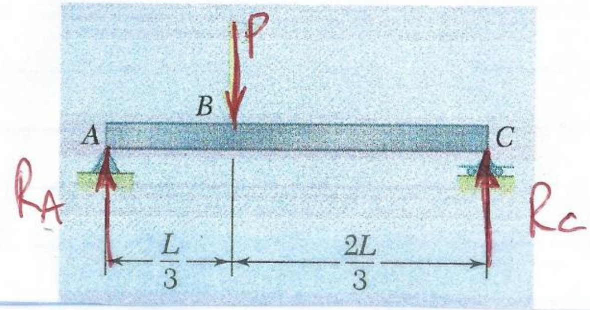
$$\vec{M}_2 = [0]\hat{i} - [(\tfrac{1}{2})(-\tfrac{2}{3}T)]\hat{j} + [(\tfrac{1}{2})(\tfrac{2}{3}T)]\hat{k} \ ^{N \cdot m}$$

$$\vec{M}_2 = (0)\hat{i} + (\tfrac{1}{3}T)\hat{j} + (\tfrac{1}{3}T)\hat{k} \ ^{N \cdot m}$$

M_3A

10 PROB. 7-31

Problem 2: (points) Determine the maximum absolute values of the shear and bending moment. (Answer: $M_{max} = 2PL/9$, $V_{max} = 2P/3$)

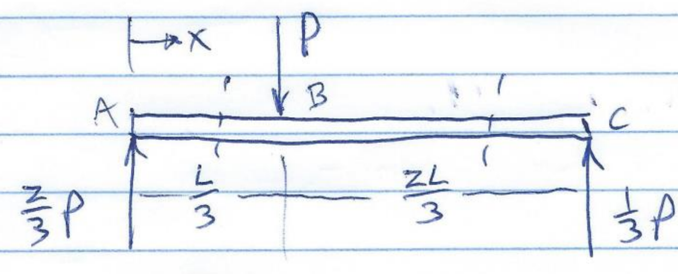


$\Sigma F_y = 0: \; R_A + R_C - P = 0$

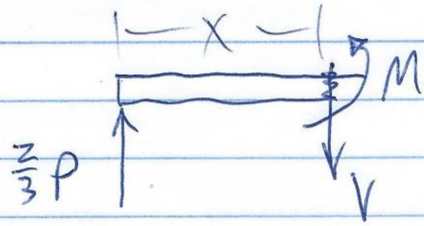
$\Sigma M_A = 0: \circlearrowleft^{+} \; -\left(\frac{L}{3}\right)P + (L)R_C = 0$

$\boxed{R_C = \frac{1}{3}P}$

$\boxed{R_A} = P - R_C = P - \frac{1}{3}P = \boxed{\frac{2}{3}P}$



FBD: $0 \le x \le \frac{L}{3}^{-}$:



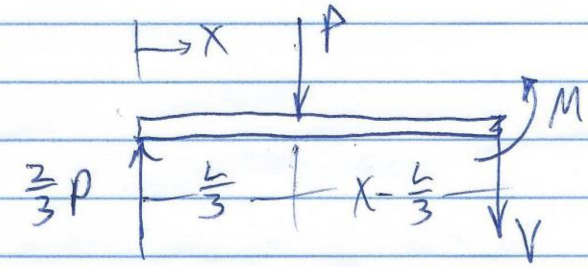
$\Sigma F_y = 0: \; \frac{2}{3}P - V = 0$

$\boxed{V = \frac{2}{3}P}$

$\Sigma M_{CUT} = 0 \; \circlearrowleft^{+}: \; M - (x)\left(\frac{2}{3}P\right) = 0$

$$\boxed{M = \frac{2}{3}Px}$$

FBD: $\frac{L}{3}^{+} \le x \le L$:



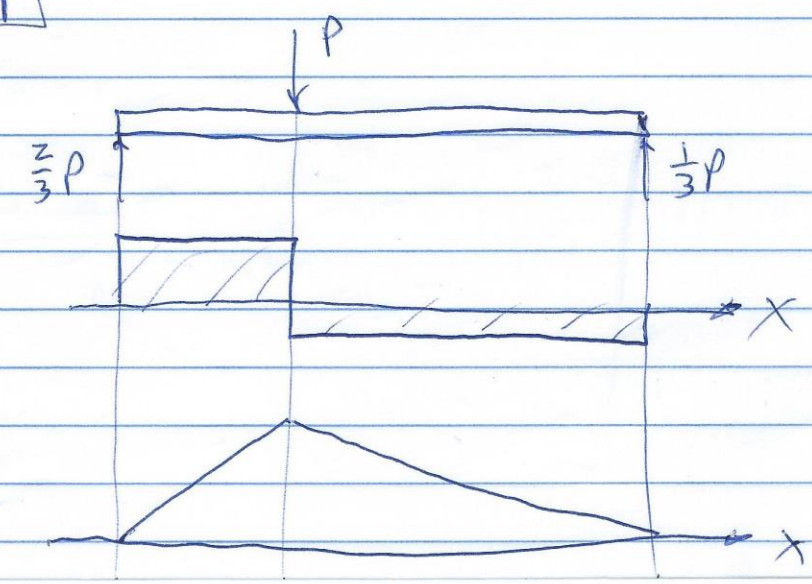
$\Sigma F_y = 0: \quad \frac{2}{3}P - P - V = 0 \quad \boxed{V = -\frac{1}{3}P}$

$\Sigma M_{cut} = 0 \circlearrowleft: \quad M - (x)\left(\frac{2}{3}P\right) + \left(x - \frac{L}{3}\right)(P) = 0$

$$\boxed{M = \frac{1}{3}P(L-x)}$$

$\boxed{M_{MAX}} = M\left(x = \frac{1}{3}L\right) = \frac{2}{3}P\left(\frac{1}{3}L\right) = \boxed{\frac{2}{9}PL}$

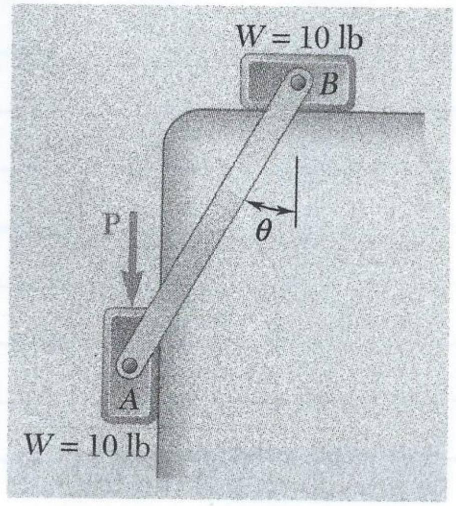
$$\boxed{V_{max} = \frac{2}{3}P}$$



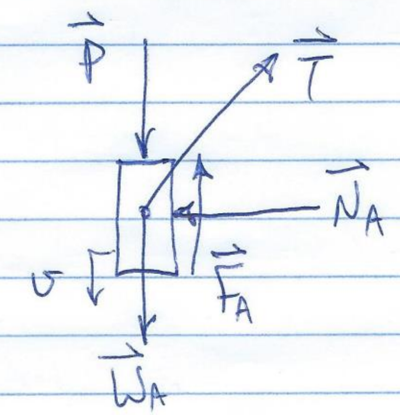
10 8-36

8

Problem 3: (points) Two 10-lb blocks *A* and *B* are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$ with the vertical. Determine the largest value of *P* for which equilibrium is maintained. (Answer: $P = 2.69$ lb)



FBD: BLOCK A



$$\vec{T} = (T \cdot \sin 30°)\,\hat{i} + (T \cdot \cos 30°)\,\hat{j} \text{ LB}$$

$$\vec{T} = (0.5 \cdot T)\,\hat{i} + (0.866 \cdot T)\,\hat{j} \text{ LB}$$

$\Sigma F_x = 0$: $0.5 \cdot T - N_A = 0$, $\boxed{N_A = 0.5 \cdot T}$

$\Sigma F_y = 0$: $F_A + 0.866T - P - W_A = 0$

$P = F_A + 0.866T - W_A$

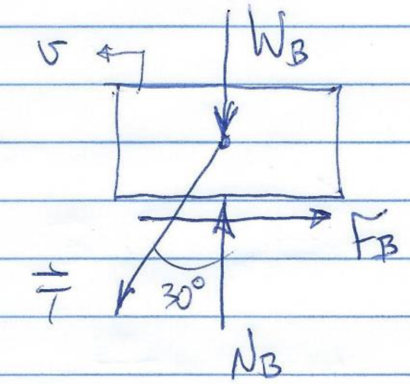
FOR MAXIMUM P, IMPENDING MOTION: $F_A = \mu_s N_A$

$P = \mu_s N_A + 0.866T - W_A$

$P = \mu_s (0.5T) + 0.866 \cdot T - W_A$

$\boxed{P = T\left(\frac{1}{2}\mu_s + 0.866\right) - W_A}$

FBD : BLOCK B



$\vec{T} = (-0.5 \cdot T)\hat{i} + (-0.866 \cdot T)\hat{j}$ LB

$\Sigma F_x = 0: \quad F_B - \frac{1}{2}T = 0$

IMPENDING MOTION: $F_B = \mu_s N_B$

$\mu_s N_B - \frac{1}{2}T = 0$

$\boxed{T = 2\mu_s N_B}$ EQN. 1

$\Sigma F_y = 0: \quad N_B - 0.866T - W_B = 0$

$N_B = 0.866T + W_B$ EQN. 2

EQN. 2 INTO 1:

$$T = 2\mu_s(0.866T + W_B)$$

$$T = 1.732\mu_s T + 2\mu_s W_B$$

$$T(1 - 1.732\mu_s) = 2\mu_s W_B$$

$$T = \frac{2\mu_s W_B}{(1 - 1.732\mu_s)} = \frac{2(0.3)(10^{LB})}{[1 - 1.732(0.3)]}$$

$$T = 12.49^{LB}$$

$$P = (12.49^{LB})\left[\tfrac{1}{2}(0.3) + 0.866\right] - (10^{LB})$$

$$\boxed{P = 2.689^{LB}}$$

9-49 (11)

(5)

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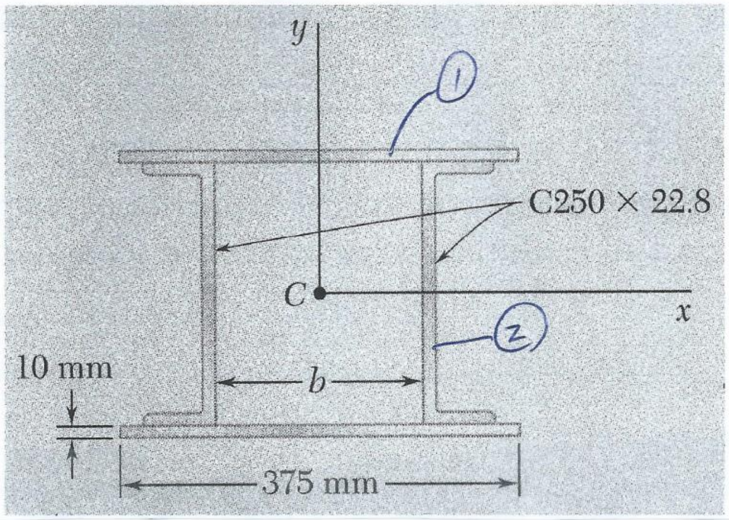

$$I_x = 2(I_x)_1 + 2(I_x)_2$$

AREA 1: RECTANGLE

$$(I_x)_1 = \bar{I}_x + A_1 d_1^2$$

$$= \frac{1}{12} b h^3 + bh \cdot d_1^2$$

132

$$= \frac{1}{12}(375)(10)^3 + (375)(10)\left[\frac{1}{2}(254) + \frac{1}{2}(10)\right]^2$$

$$(I_x)_1 = 6.537 \times 10^7 \text{ mm}^4$$

0

AREA 2: C SHAPE

$$(I_x)_2 = \bar{I}_x + A_2 d_2^2$$ 90

$$(I_x)_2 = 28.0 \times 10^6 \text{ mm}^4$$

$$I_x = 2(6.537 \times 10^7) + 2(28.0 \times 10^6)$$

$$\boxed{I_x = 1.867 \times 10^8 \text{ mm}^4}$$

EXTRA CREDIT 5 POINTS