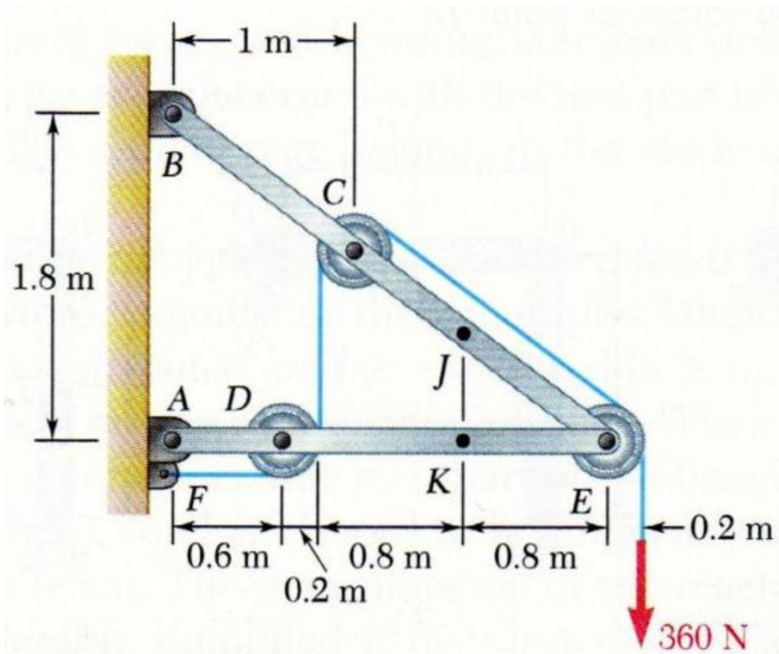
Wright State University
Department of Mechanical and Materials Engineering

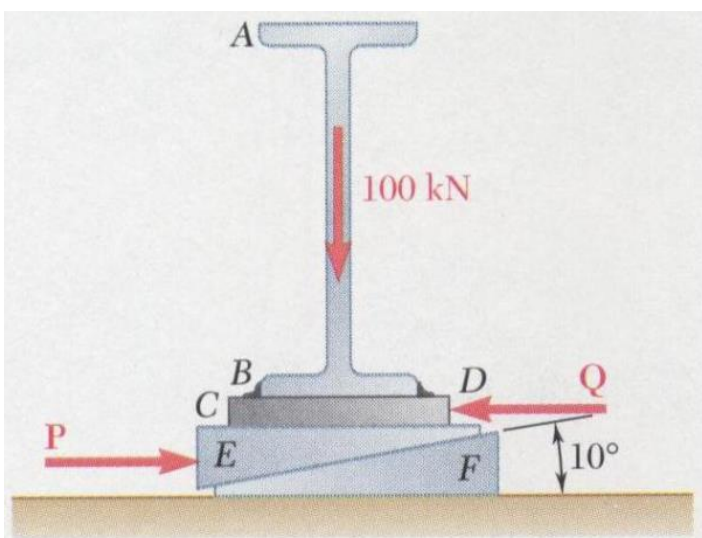
Spring 2015

ME 2120 STATICS: FINAL EXAM
Open Book, Closed Notes, DO NOT Write on this Sheet
Show All Work for Partial Credit

Problem 1 (50 points): Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at point *J* of the frame shown.

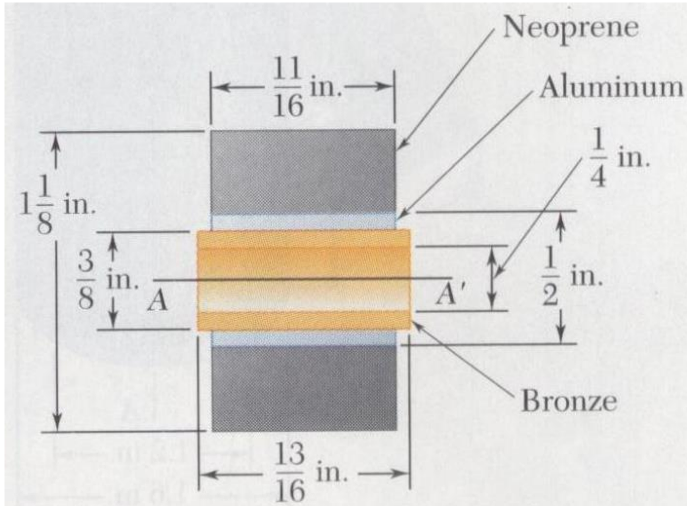


Problem 2 (20 points): The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine the force **P** required to raise the beam, and the corresponding force **Q**.



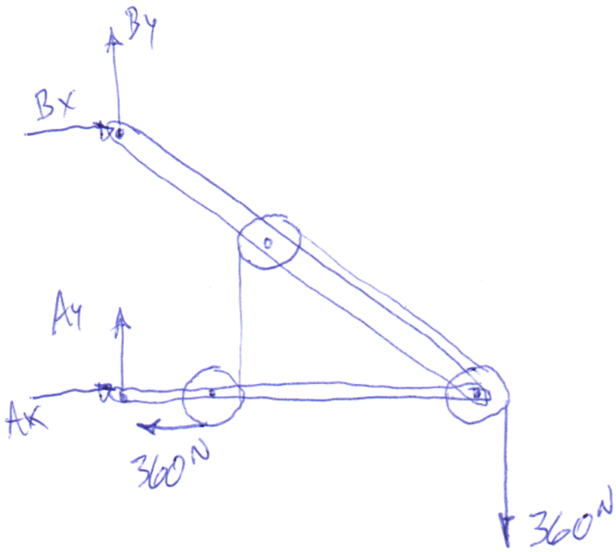
Next problem on the back of this page.

Problem 3 (30 points): Shown is the cross section of an idler roller. Determine its mass moment of inertia and its radius of gyration with respect to the axis *AA'*. The specific weight of bronze is 0.310 lb/in^3; of aluminum, 0.100 lb/in^3; and of neoprene, 0.0452 lb/in^3.



STATICS PROB. 1

FBD : ENTIRE STRUCTURE



$$\Sigma F_x = 0: \quad A_x + B_x - 360 = 0$$

$$\Sigma F_y = 0: \quad A_y + B_y - 360 = 0$$

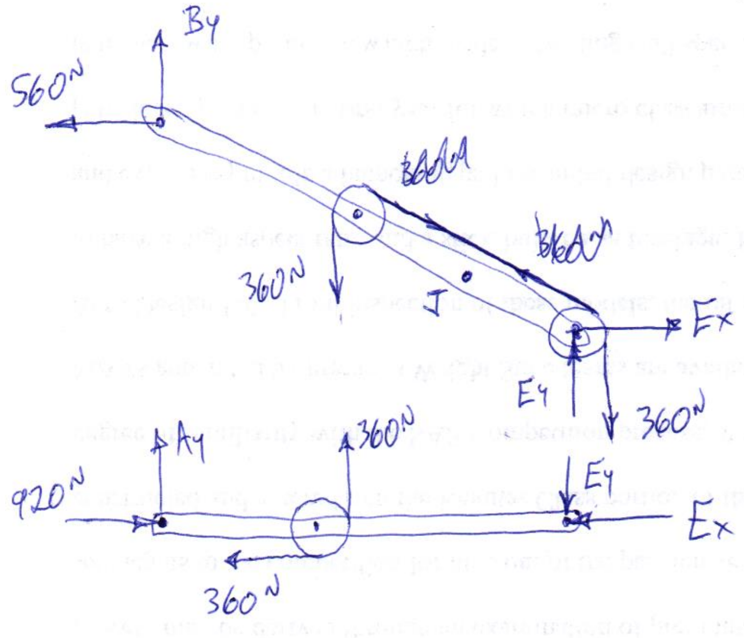
$\Sigma M_A = 0$ $+\circlearrowleft$:

$$-(1.8^m)B_x - (0.2^m)(360^N) - (2.6^m)(360^N) = 0$$

$$\boxed{B_x = -560^N}$$

$$\boxed{A_x} = 360 - B_x = 360 - (-560) = \boxed{920^N}$$

BREAK STRUCTURE APART:



FBD: UPPER PART

$\Sigma F_x = 0: \ -560 + E_x = 0 \quad \boxed{E_x = 560^N}$

$\Sigma F_y = 0: \ B_y - 360 + E_y - 360 = 0$

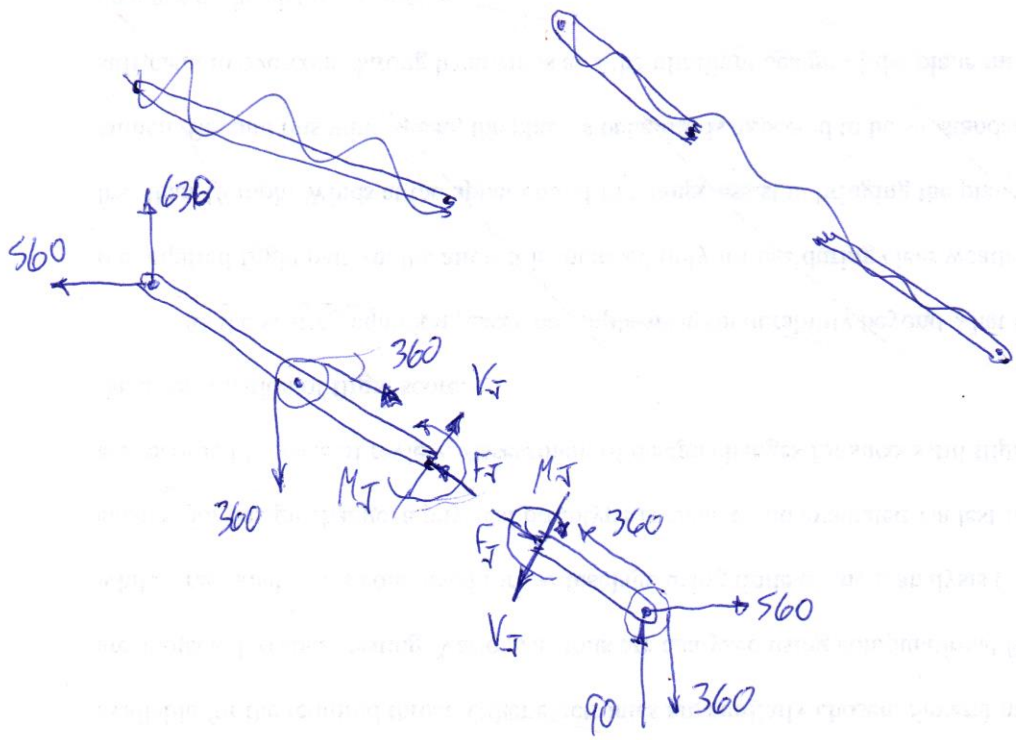
$\Sigma M_E = 0 \ \circlearrowleft+:$

$(1.8^m)(560^N) - (2.4^m)(B_y) + (1.6^m)(360^N) - (0.2^m)(360^N) = 0$

$\boxed{B_y = 630^N}$

$\boxed{E_y} = 360 + 360 - B_y = 720 - 630 = \boxed{90^N}$

BREAK BCE:

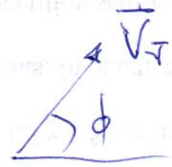


FIND $\vec{F_J}$: $\theta = TAN^{-1}\left(\frac{1.8}{2.4}\right) = 36.9°$

$$\vec{F_J} = \left(-F_J \cdot COS\,36.9°\right)\hat{i} + \left(F_J \cdot SIN\,36.9°\right)\hat{j}$$

$$\vec{F_J} = \left(-0.8\,F_J\right)\hat{i} + \left(0.6\,F_J\right)\hat{j}\ N$$

FIND $\vec{V_J}$: $\phi = 90 - 36.9° = 53.1°$



$$\vec{V_J} = \left(V_J \cdot COS\,53.1°\right)\hat{i} + \left(V_J \cdot SIN\,53.1°\right)\hat{j}$$

$$\vec{V_J} = \left(0.6\,V_J\right)\hat{i} + \left(0.8\,V_J\right)\hat{j}\ N$$

FIND $\overrightarrow{360}$:

$$\overrightarrow{360} = (360 \cdot \cos 36.9^\circ)\hat{i} + (-360 \cdot \sin 36.9^\circ)\hat{j}$$

$$\overrightarrow{360} = (288)\hat{i} + (-216)\hat{j} \ ^N$$

$$\Sigma F_x = 0: \ -560 + 288 - 0.8 F_J + 0.6 V_J = 0$$

$$V_J = \frac{1}{0.6}(560 - 288 + 0.8 F_J)$$

$$\boxed{V_J = 453 + 1.33 F_J}$$

$$\Sigma F_y = 0: \ 630 - 360 - 216 + 0.6 F_J + 0.8 V_J = 0$$

$$V_J = \frac{1}{0.8}(-630 + 360 + 216 - 0.6 F_J)$$

$$\boxed{V_J = -67.5 - 0.75 F_J}$$

$$453 + 1.33 F_J = -67.5 - 0.75 F_J$$

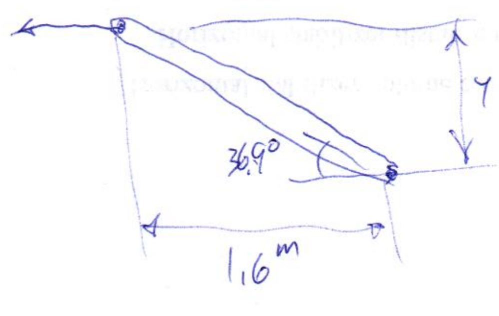
$$(1.33 + 0.75) F_J = -67.5 - 453$$

$$\frac{Y}{1.6} = \tan 36.9^\circ$$

$$\boxed{F_J = -250 \ ^N}$$

$$V_J = -67.5 - 0.75(-250)$$

$$\boxed{V_J = -255 \ ^N}$$



$$\Sigma M_J = 0 \ \circlearrowleft^{+}:$$

$$Y = 1.6 \tan 36.9^\circ = 1.20^m$$

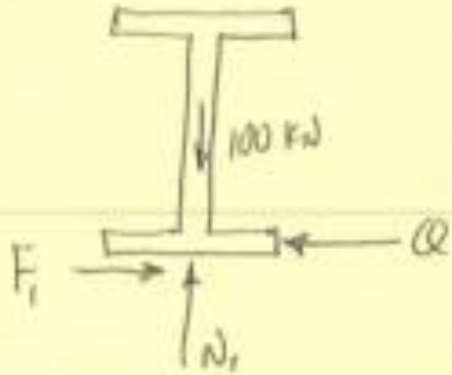
5

$$M_J - (0.2^m)(360^N) + (0.9^m)(360^N) - (1.6^m)(630^N) + (1.2^m)(560^N) = 0$$

$$\boxed{M_J = 120^{N\text{-}m}}$$

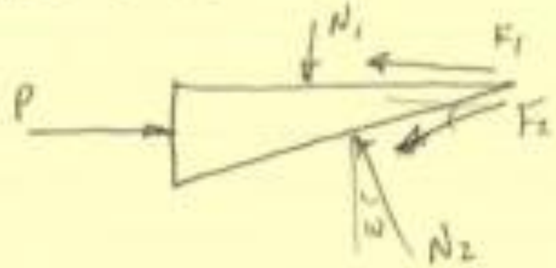
PROB. 8.66

FBD) I-BEAM



$N_1 = 100$ kN, $Q = F_1 = 0.3 N_1 = 0.3(100 \text{ kN}) = 30$ kN

FBD) WEDGE C:



$\Sigma F_x = 0: \; P - F_1 - F_2 \cos 10° - N_2 \sin 10° = 0$

$P = 30 + [(0.3)(\cos 10°) + \sin 10°] N_2$

$P = 30 + 0.469 N_2$

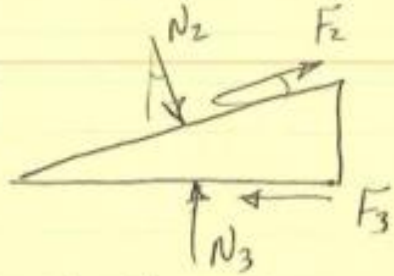
$\Sigma F_y = 0: \; -N_1 + N_2 \cos 10° - F_2 \sin 10° = 0$

$N_2 [\cos 10° - (0.3) \sin 10°] = 100$

$N_2 = 107$ kN, $P = 80.3$ kN

CHECK TO SEE IF WEDGE F WILL SLIDE OVER CONCRETE:

FBD WEDGE F:



$$\Sigma F_x = 0: \quad N_2 \sin 10^\circ + F_2 \cos 10^\circ - F_3 = 0$$

$$F_3 = (107)\sin 10^\circ + (0.3)(107)\cos 10^\circ = 50.2 \text{ kN}$$

$$\Sigma F_y = 0: \quad -N_2 \cos 10^\circ + F_2 \sin 10^\circ + N_3 = 0$$

$$N_3 = (107)\cos 10^\circ - (0.3)(107)\sin 10^\circ = 99.8 \text{ kN}$$

(SHOULD BE 100 kN)

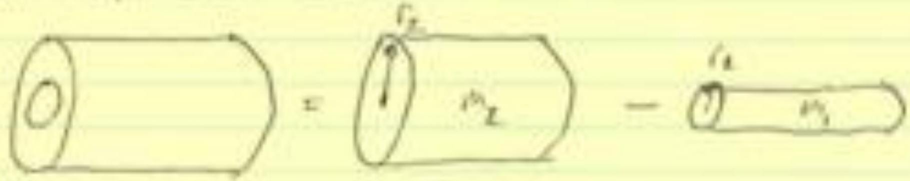
$$F_{max} = \mu_s N_3 = (0.6)(100 \text{ kN}) = 60 \text{ kN}$$

SINCE $F_3 < F_{max}$, WEDGE IS IN EQUILIBRIUM AND WILL NOT MOVE.

EX. PROB. 9.128

$\gamma_B = 0.310 \frac{LBF}{IN^3}, \quad \gamma_A = 0.100 \frac{LBF}{IN^3}, \quad \gamma_N = 0.0452 \frac{LBF}{IN^3}$

$I_{AA} = I_1 + I_2 + I_3$



$$I_1 = \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2 \qquad W = mg, \; m = \frac{W}{g}$$

$$= \frac{1}{2}\left(\gamma_B V_2 r_2^2 - \gamma_B V_1 r_1^2\right) \cdot \frac{1}{g}$$

$$= \frac{\frac{1}{2}\gamma_B}{g}\left(\pi r_2^2 L_1 \cdot r_2^2 - \pi r_1^2 L_1 \cdot r_1^2\right)$$

$$I_1 = \frac{\pi \gamma_B L_1}{2g}\left(r_2^4 - r_1^4\right)$$

$$= \frac{\pi}{2} \frac{\left(0.310 \frac{LBF}{IN^3}\right)\left(\frac{13}{16} IN\right)}{\left(32.2 \frac{FT}{S^2}\right)}\left[\left(\frac{3/8}{2} IN\right)^4 - \left(\frac{1/4}{2} IN\right)^4\right]\left(\frac{FT}{12 \, IN}\right)$$

$$I_1 = 1.015 \times 10^{-6} \; LBF \cdot IN \cdot S^2$$

$$I_2 = \frac{\pi \gamma_A L_2}{2g}\left(r_3^4 - r_2^4\right)$$

$$= \frac{\pi}{2} \frac{(0.100)\left(\frac{11}{16}\right)}{(32.2)}\left[\left(\frac{1/2}{2}\right)^4 - \left(\frac{3/8}{2}\right)^4\right]\left(\frac{1}{12}\right)$$

$$I_2 = 7.463 \times 10^{-7} \; LBF \cdot IN \cdot S^2$$

9.128 CONT.

$$I_3 = \frac{\pi \gamma_N L_3}{2g}\left(r_4^4 - r_3^4\right)$$

$$= \frac{\pi}{2}\,\frac{(0.0452)\left(\frac{11}{16}\right)}{(32.2)}\left[\left(\frac{1.125}{2}\right)^4 - \left(\frac{1/2}{2}\right)^4\right]\left(\frac{1}{12}\right)$$

$$I_3 = 1.215 \times 10^{-5} \text{ LBF}\cdot\text{IN}\cdot\text{s}^2$$

$$\boxed{I_{AA} = 1.391 \times 10^{-5} \text{ LBF}\cdot\text{IN}\cdot\text{s}^2 = 1.16 \times 10^{-6} \text{ LBF}\cdot\text{FT}\cdot\text{s}^2}$$

RADIUS OF GYRATION:

$$K = \sqrt{\frac{I}{m}} \qquad I = mK^2$$

$$m = m_1 + m_2 + m_3$$

$$m_1 = \frac{\gamma_B}{g}\left(\pi r_2^2 L_1 - \pi r_1^2 L_1\right)$$

$$= \frac{\pi \gamma_B L_1}{g}\left(r_2^2 - r_1^2\right)$$

$$= \frac{\pi\left(0.310 \frac{\text{LBF}}{\text{IN}^3}\right)\left(\frac{13}{16}\text{ IN}\right)}{\left(32.2 \frac{\text{FT}}{\text{s}^2}\right)}\left[\left(\frac{3/8}{2}\right)^2 - \left(\frac{1/4\text{ IN}}{2}\right)^2\right]\left(\frac{\text{FT}}{12\text{ IN}}\right)$$

$$m_1 = 4.00 \times 10^{-5} \ \frac{\text{LBF}\cdot\text{s}^2}{\text{IN}}$$

$$m_2 = \frac{\pi \gamma_A L_2}{g}\left(r_3^2 - r_2^2\right)$$

$$= \frac{\pi (0.1)\left(\frac{11}{16}\right)}{(32.2)}\left[\left(\frac{1/2}{2}\right)^2 - \left(\frac{3/8}{2}\right)^2\right]\left(\frac{1}{12}\right)$$

$$m_2 = 1.528 \times 10^{-5} \ \frac{\text{LBF}\cdot\text{s}^2}{\text{IN}}$$

9.178 CONT

$$M_3 = \frac{\pi \gamma_A L_2}{g}\left(r_4^2 - r_3^2\right)$$

$$= \frac{\pi (0.0452)\left(\frac{11}{16}\right)}{(32.2)}\left[\left(\frac{1.125}{2}\right)^2 - \left(\frac{1/2}{2}\right)^2\right]\left(\frac{1}{12}\right)$$

$$M_3 = 6.415 \times 10^{-5} \ \frac{LBF \cdot s^2}{IN}$$

$$M = 1.194 \times 10^{-4} \ \frac{LBF \cdot s^2}{IN}$$

$$K = \sqrt{\frac{\left(1.391 \times 10^{-5} \ LBF \cdot IN \cdot s^2\right)}{\left(1.194 \times 10^{-4} \ \frac{LBF \cdot s^2}{IN}\right)}}$$

$$\boxed{K = 0.3413 \ IN}$$