Chapter 2

Prob. 2.21

2.21 and 2.22 Determine the x and y components of each of the forces shown.

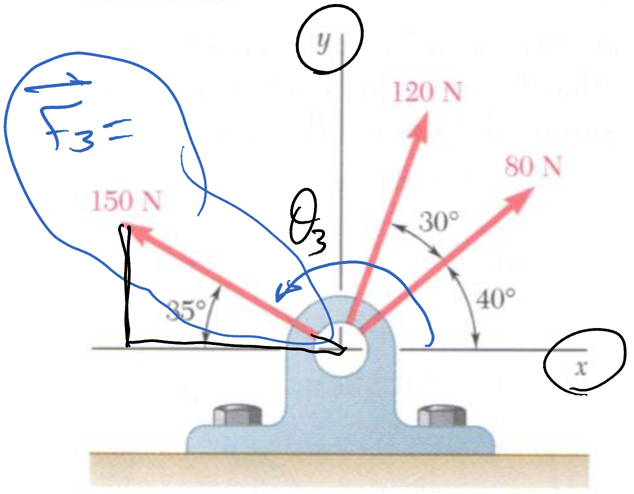


Fig. P2.21

$$\theta_3 = 180 - 35° = 145°$$

$$\vec{F_3} = (150 \cdot \cos 145°)\,\hat{i} + (150 \cdot \sin 145°)\,\hat{j} \text{ N}$$

$$\vec{F_3} = (-122.9)\,\hat{i} + (86.04)\,\hat{j} \text{ N}$$

$$\vec{F_3} = (-\cos 35° \cdot 150)\,\hat{i} + (\sin 35° \cdot 150)\,\hat{j} \text{ N}$$

Prob. 2.22

2.21 and 2.22 Determine the x and y components of each of the forces shown.

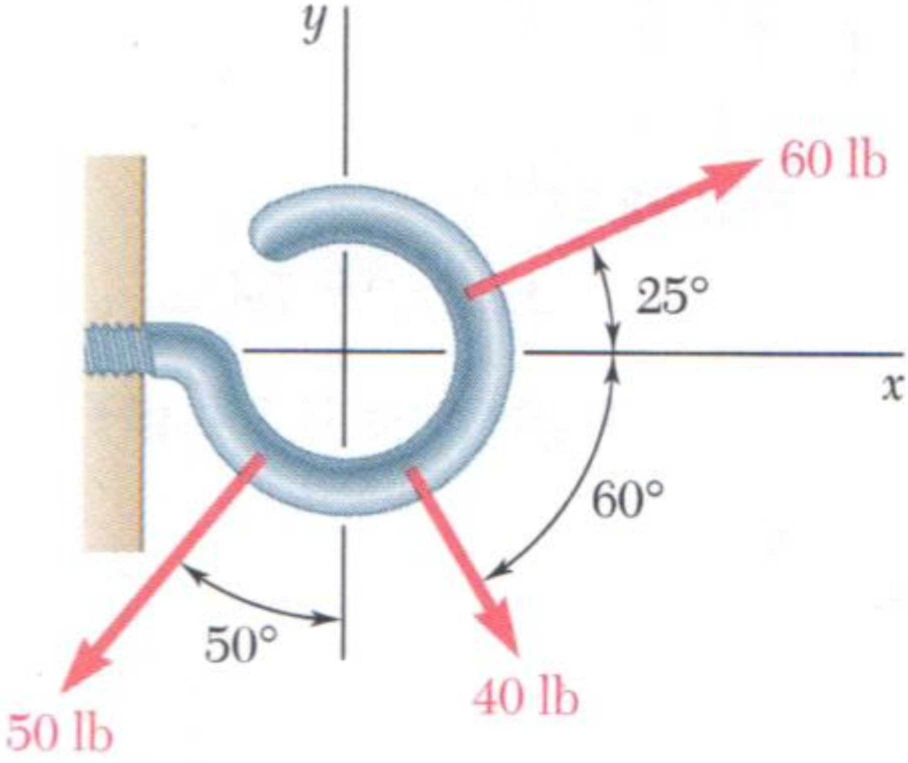


Fig. P2.22

Prob. 2.23

2.23 and *2.24* Determine the x and y components of each of the forces shown.

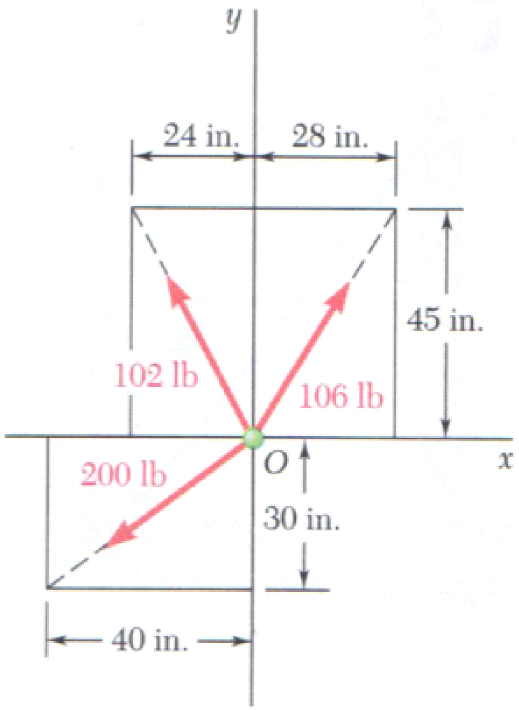


Fig. P2.23

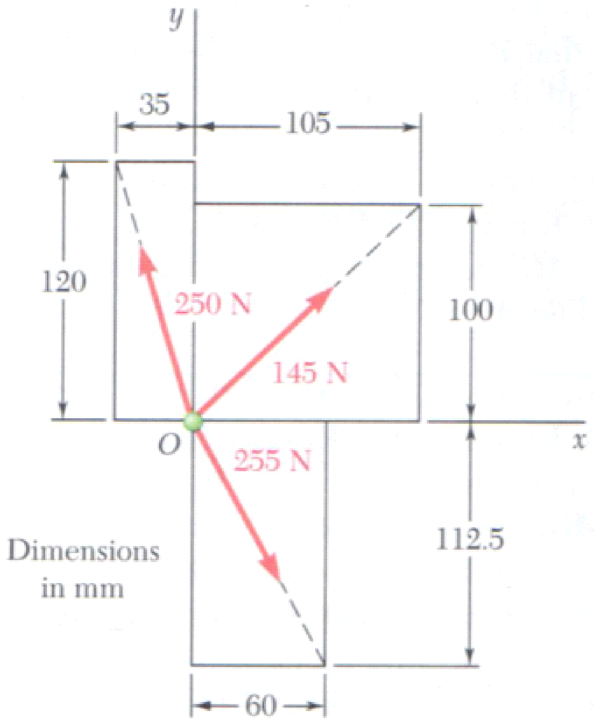


Fig. *P2.24*

Prob. 2.24

2.23 and *2.24* Determine the x and y components of each of the forces shown.

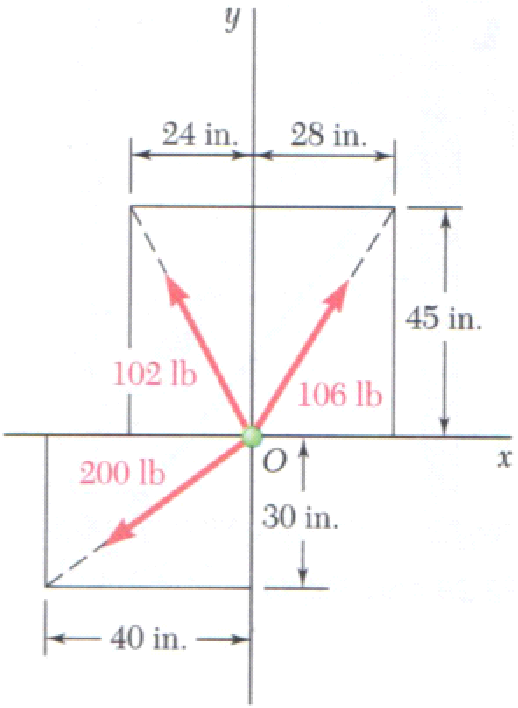


Fig. P2.23

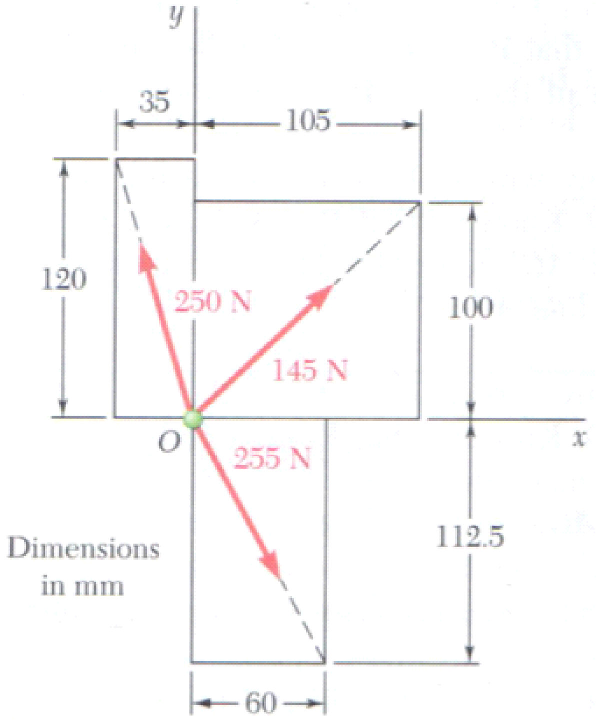


Fig. *P2.24*

Prob. 2.25

2.25 Member *CB* of the vise shown exerts on block *B* a force **P** directed along line *CB*. Knowing that **P** must have a 1200-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

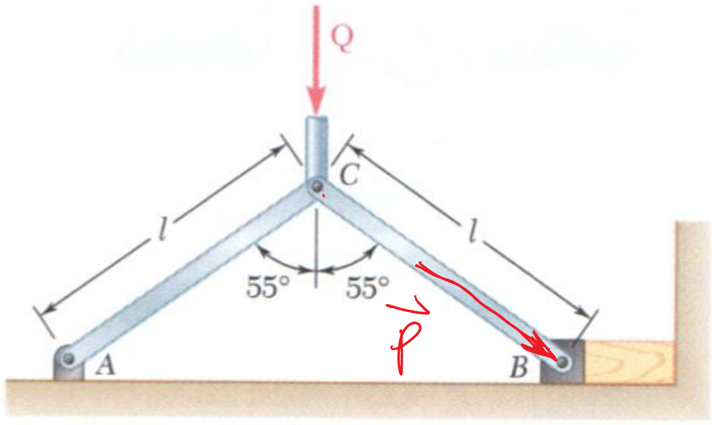
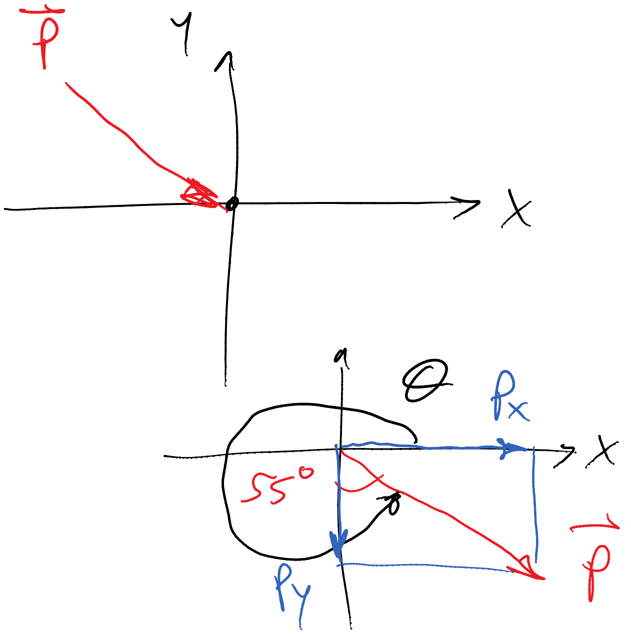


Fig. P2.25



$$\theta = 270 + 55 = 325°, \quad P_x = 1200^N$$

$$P_x = P \cdot \cos\theta, \quad \boxed{P =} \frac{P_x}{\cos\theta} = \frac{(1200^N)}{\cos 325°} = \boxed{1465^N}$$

$$\boxed{P_y} = P \cdot \sin\theta = (1465^N) \cdot \sin 325° = \boxed{-840^N}$$

Prob. 2.26

2.26 The hydraulic cylinder *BC* exerts on member *AB* a force **P** directed along line *BC*. Knowing that **P** must have a 600-N component perpendicular to member *AB*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AB*.

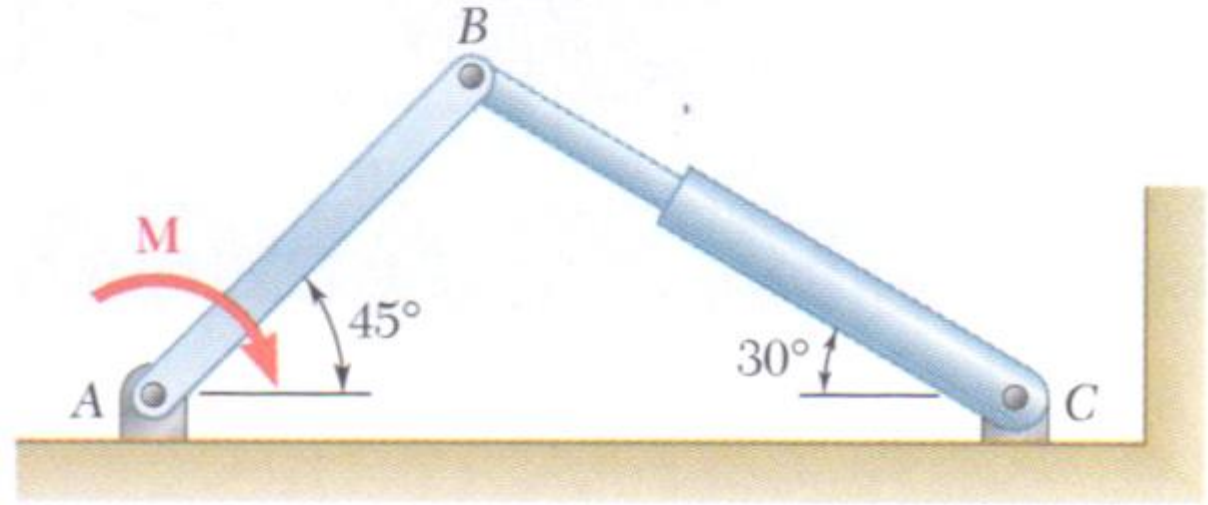


Fig. P2.26

Prob. 2.29

2.29 The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** must have a 120-N component perpendicular to the pole *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AC*.

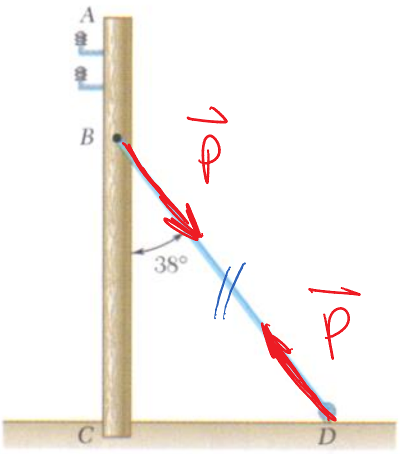
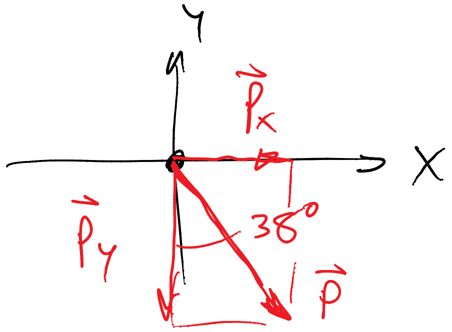


Fig. P2.29 and P2.30



$$\theta = 270 + 38 = 308^\circ$$

$$P_x = 120^N$$

$$P_x = P \cdot \cos\theta \,, \quad P = \frac{P_x}{\cos\theta}$$

$$\boxed{P =} \frac{120^N}{\cos 308} = \boxed{194.9^N}$$

$$\boxed{P_y} = P \cdot \sin\theta = (194.9^N) \cdot \sin 308^\circ = \boxed{-153.6^N}$$

NEWTON'S FIRST LAW OF MOTION:

A PARTICLE WILL REMAIN AT REST (IF ORIGINALLY AT REST) OR MOVE WITH CONSTANT SPEED IN A STRAIGHT LINE (IF ORIGINALLY IN MOTION) IF THE RESULTANT FORCE IS ZERO. (PARTICLE IS IN EQUILIBRIUM).

$$\vec{R} = \Sigma \vec{F} = 0 \quad \text{(STATICS PROBLEM!)}$$

IN TERMS OF RECTANGULAR COMPONENTS,

$$\Sigma \left(F_x \hat{i} + F_y \hat{j} \right) = 0 \quad \underline{\text{OR}}$$

$$\boxed{\Sigma F_x = 0 \quad \text{AND} \quad \Sigma F_y = 0}$$

TWO EQUATIONS, AT MOST TWO UNKNOWNS

THIS IS TRUE FOR A PARTICLE, A STRUCTURE, OR ANY <u>PART</u> OF A STRUCTURE.

WE NEED A METHOD TO DETERMINE THE FORCES ON A BODY, SOME OF WHICH ARE HIDDEN: FREE-BODY DIAGRAM.

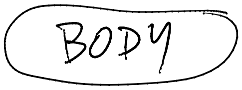
THE FBD IS A WAY TO ISOLATE A BODY FROM EVERYTHING IT IS IN CONTACT WITH AND REPLACE THE CONTACT WITH APPROPRIATE FORCES.

1. A SKETCH OF THE BODY.
2. BODY IS SHOWN COMPLETELY FREE OF ALL CONTACT.
3. FORCES REPLACE EACH CONTACT POINT.
4. THE DIRECTION OF THE FORCES ARE ASSUMED; THE ANALYSIS WILL AUTOMATICALLY

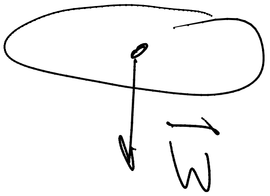
MAKE CORRECTIONS FOR YOU.

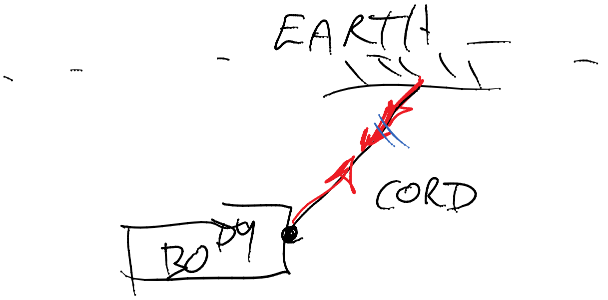
FREE-BODY COMPONENTS

ONE OR MORE OF THESE COMPONENTS MAY BE PRESENT IN A PROBLEM.

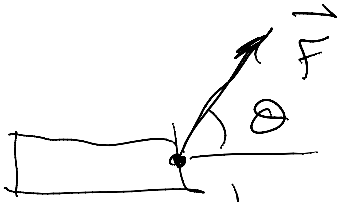


REMOVE EARTH

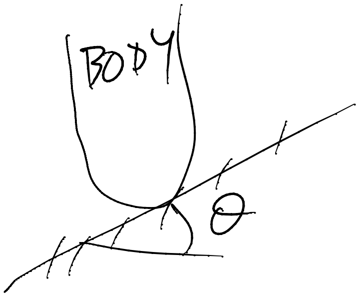




REMOVE CORD

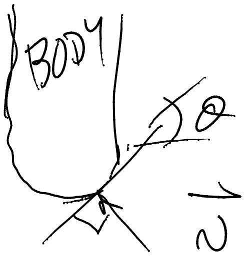


TENSION
LINE OF ACTION
IS KNOWN.

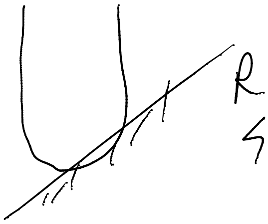


SMOOTH SURFACE

REMOVE SURFACE

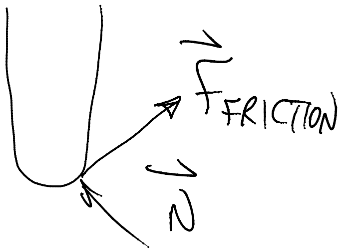


NORMAL FORCE



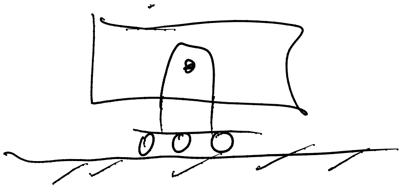
ROUGH SURFACE

REMOVE SURFACE

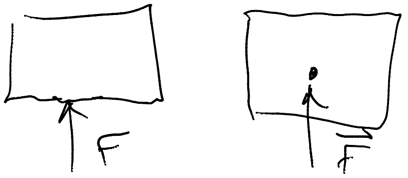


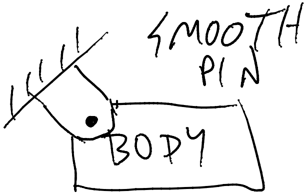
FRICTION FORCE
PARALLEL TO
SURFACE
(2 UNKNOWNS)

REMOVE ROLLER



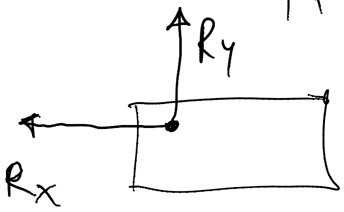
REMOVE ROLLER



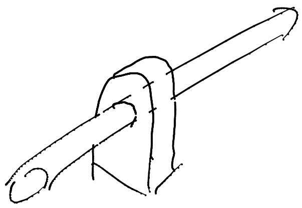


SMOOTH PIN

REMOVE SMOOTH PIN

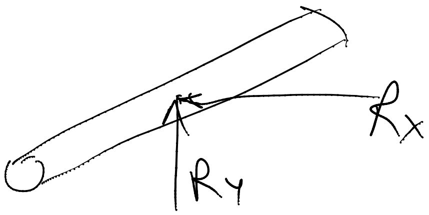


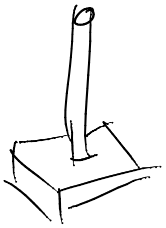
2 UNKNOWNS



SMOOTH BEARING ON SHAFT

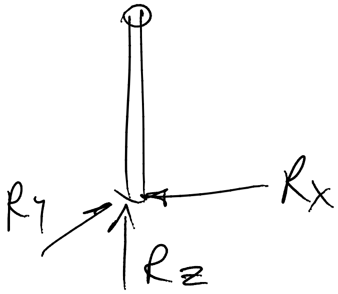
REMOVE BEARING





THRUST BEARING

REMOVE BEARING

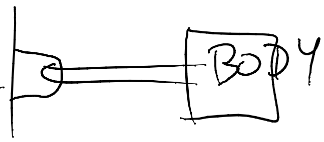


RUNNER IN A SLOT

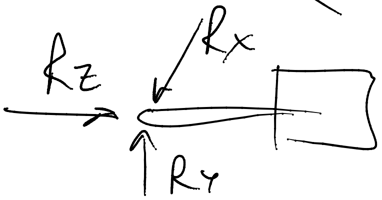
REMOVE SLOT

FORCE NORMAL TO SLOT

BALL AND SOCKET

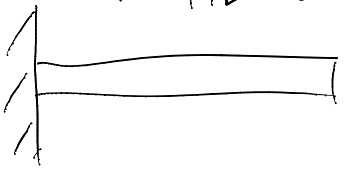


REMOVE SOCKET

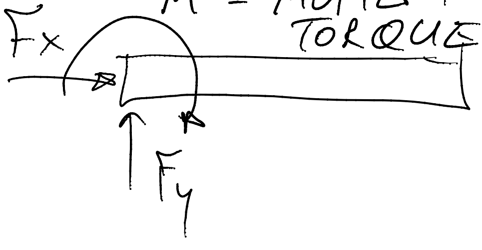


3 FORCE COMPONENTS

CANTILEVERED BEAM



REMOVE WALL

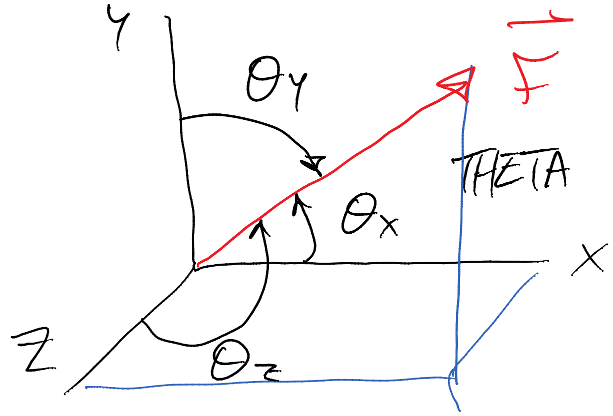


$\vec{M}$ = MOMENT OR TORQUE

VECTORS IN 3-D SPACE

WE NEED A METHOD TO APPLY NEWTON'S LAW

TO 3-D SITUATIONS.



$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$\vec{F} = F\cos\theta_x\,\hat{i} + F\cos\theta_y\,\hat{j} + F\cos\theta_z\,\hat{k}$$

$$\vec{F} = F(\cos\theta_x\,\hat{i} + \cos\theta_y\,\hat{j} + \cos\theta_z\,\hat{k})$$

$$\vec{F} = F\cdot\vec{\lambda}$$ LAMBDA

$\vec{\lambda}$ = UNIT VECTOR DIRECTED ALONG $\vec{F}$

$$\vec{\lambda} = (\cos\theta_x)\hat{i} + (\cos\theta_y)\hat{j} + (\cos\theta_z)\hat{k}$$

θ_x, θ_y, θ_z ARE THE DIRECTION ANGLES

$\cos\theta_x$: DIRECTION COSINE

SINCE THE MAGNITUDE OF $\vec{\lambda}$ IS 1,

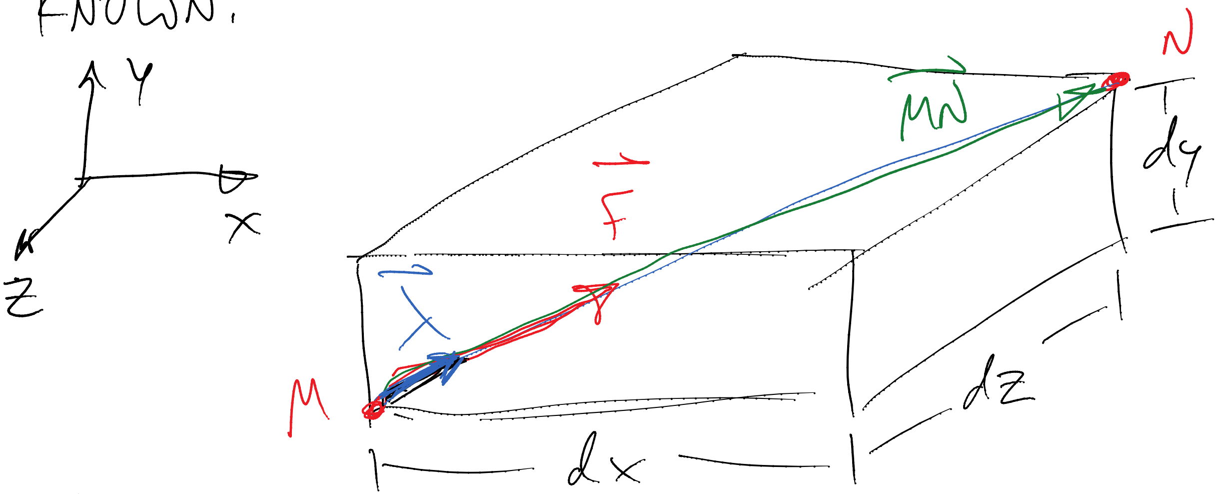
$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

THE DIRECTION COSINES CAN BE FOUND IF THE RECTANGULAR COMPONENTS OF A VECTOR ARE KNOWN:

$$\cos\theta_x = \frac{F_x}{F}, \quad \cos\theta_y = \frac{F_y}{F}, \quad \cos\theta_z = \frac{F_z}{F}$$

ALTERNATELY, THE COMPONENTS CAN BE FOUND IF THE LINE OF ACTION OF THE FORCE IS KNOWN.

KNOWN.



$$\overrightarrow{MN} = d_x\,\hat{i} + d_y\,\hat{j} + d_z\,\hat{k}$$

$$\vec{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}\left(d_x\,\hat{i} + d_y\,\hat{j} + d_z\,\hat{k}\right)$$

WHERE $d = \sqrt{d_x^2 + d_y^2 + d_z^2}$

$$\vec{F} = F\vec{\lambda} = \frac{F}{d}\left(d_x\,\hat{i} + d_y\,\hat{j} + d_z\,\hat{k}\right)$$

<u>OR</u> $F_x = \dfrac{F d_x}{d}$, $F_y = \dfrac{F d_y}{d}$, $F_z = \dfrac{F d_z}{d}$

$\vec{R} = \sum \vec{F}$ RESULTANT FORCE VECTOR

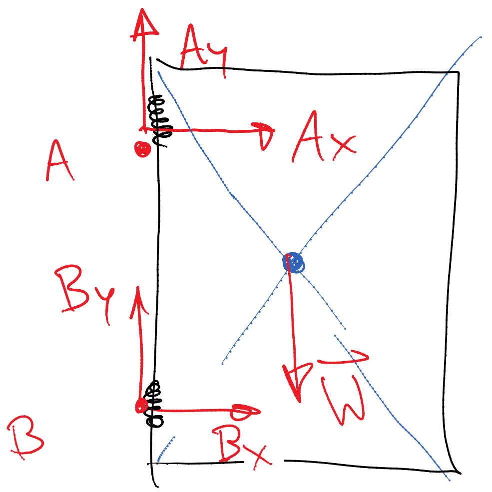
$$R_x\,\hat{i} + R_y\,\hat{j} + R_z\,\hat{k} = \left(\sum F_x\right)\hat{i} + \left(\sum F_y\right)\hat{j} + \left(\sum F_z\right)\hat{k}$$

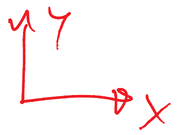
<u>OR</u> $R_x = \sum F_x$, $R_y = \sum F_y$, $R_z = \sum F_z$

NEWTON'S LAW STATES THAT A PARTICLE IS IN EQUILIBRIUM WHEN THE RESULTANT FORCE IS ZERO.

$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$

THREE EQUATIONS, AT MOST THREE UNKNOWNS.





$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_A = 0$

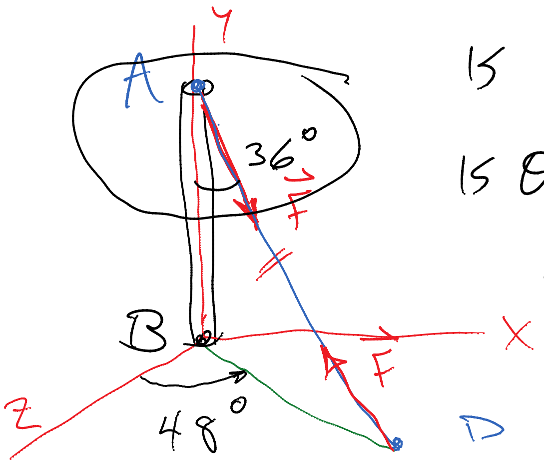
INDETERMINATE
STATICS
PROBLEM

EXAMPLE PROB.

TENSION IN AD IS
85 LB

FIND COMPONENTS OF
AD AT POINT A

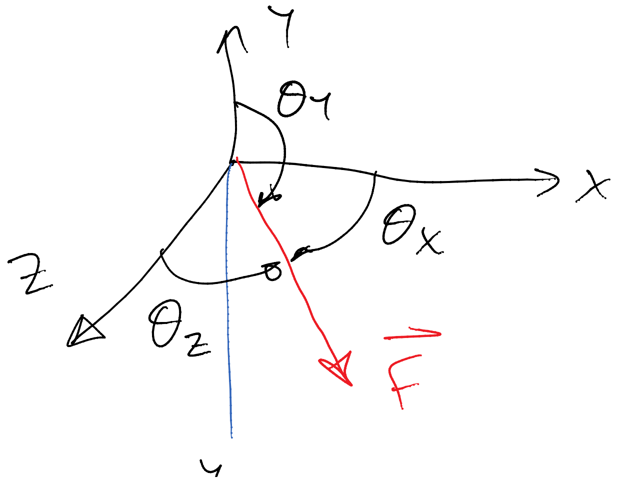
AND THE DIRECTION ANGLES: $\theta_x, \theta_y, \theta_z$



IS $\theta_z = 48°$? NO

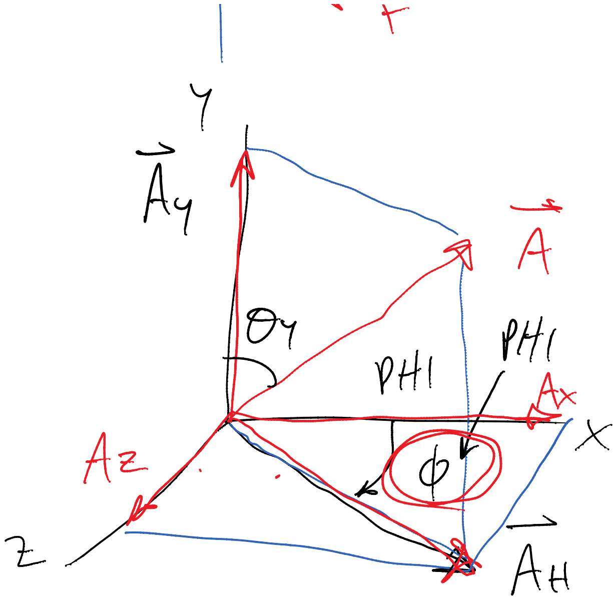
IS $\theta_x = 90° - 48°$
NO

IS $\theta_y = 36°$?
NO



DIRECTION ANGLES ARE
MEASURED FROM THE
POSITIVE COORDINATE
AXES TO THE VECTOR

$0° \le \theta_x \le 180°$



$$0^\circ \le \theta_x \le 180^\circ$$

$$A_y = \boxed{A\cos\theta_y}\bigstar, \quad A_H = A\cdot\sin\theta_y$$

NOW RESOLVE A_H INTO A_z AND A_x

$$A_x = A_H\cdot\cos\phi = \boxed{A\sin\theta_y\cdot\cos\phi}\bigstar$$

$$A_z = A_H\sin\phi = \boxed{A\sin\theta_y\cdot\sin\phi}\bigstar$$

SOLVE FOR COMPONENTS IN EX. PROBLEM:

$$\boxed{F_x =} (85^{LB})\cdot\sin(180-36^\circ)\cdot\cos\overset{\phi}{(90-48^\circ)} = \boxed{37.1^{LB}}$$

$$\boxed{F_y =} (85^{LB})\cos(180-36^\circ) = \boxed{-68.8^{LB}}$$

$$\boxed{F_z =} (85^{LB})\sin(180-36^\circ)\cdot\sin(90-48^\circ) = \boxed{33.4^{LB}}$$

THE DIRECTION ANGLES ARE:

$$\frac{F_x}{F} = \cos\theta_x, \quad \boxed{\theta_x} = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{37.1}{85}\right) = \boxed{64.1^\circ}$$

$$\boxed{\theta_y} = \cos^{-1}\left(\frac{F_y}{F}\right) = \cos^{-1}\left(\frac{-68.8}{85}\right) = \boxed{144^\circ} = 180-36^\circ$$

$$\boxed{\theta_z} = \cos^{-1}\left(\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{33.4}{85}\right) = \boxed{66.9^\circ}$$

SANITY CHECK

Prob. 2.30

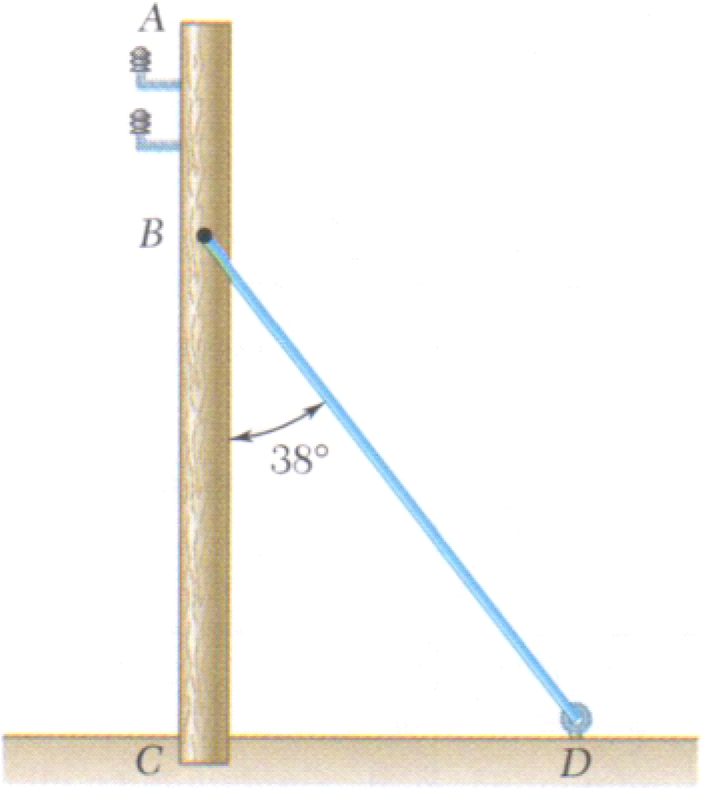


Fig. ***P2.29*** **and** ***P2.30***

2.30 The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** has a 180-N component along line *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component in a direction perpendicular to *AC*.

Prob. 2.45

2.45 Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

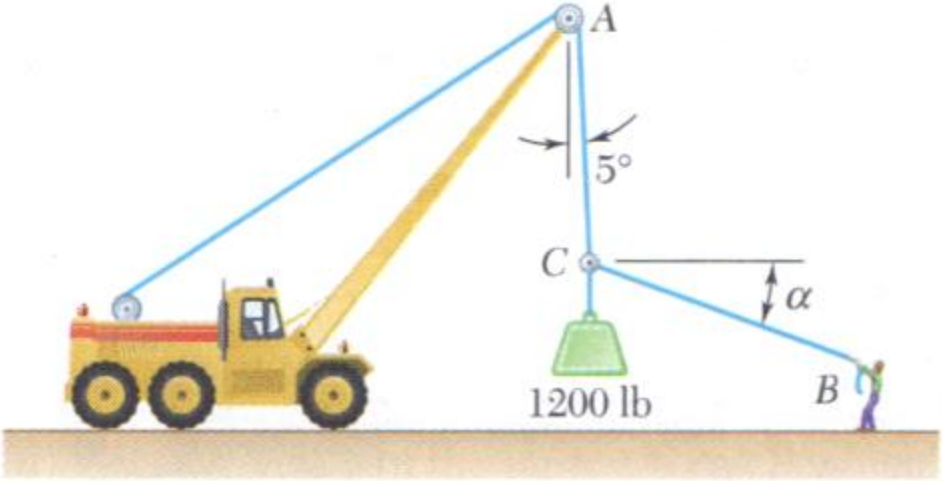
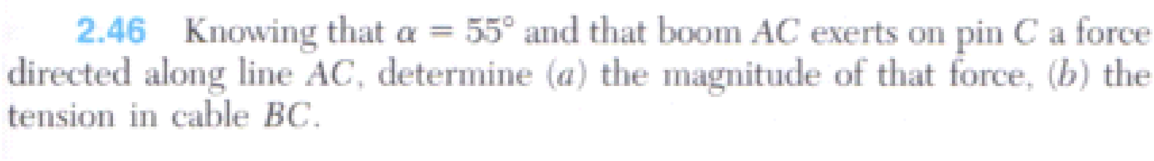


Fig. P2.45

Prob. 2.46



2.46 Knowing that $\alpha = 55°$ and that boom *AC* exerts on pin *C* a force directed along line *AC*, determine (*a*) the magnitude of that force, (*b*) the tension in cable *BC*.

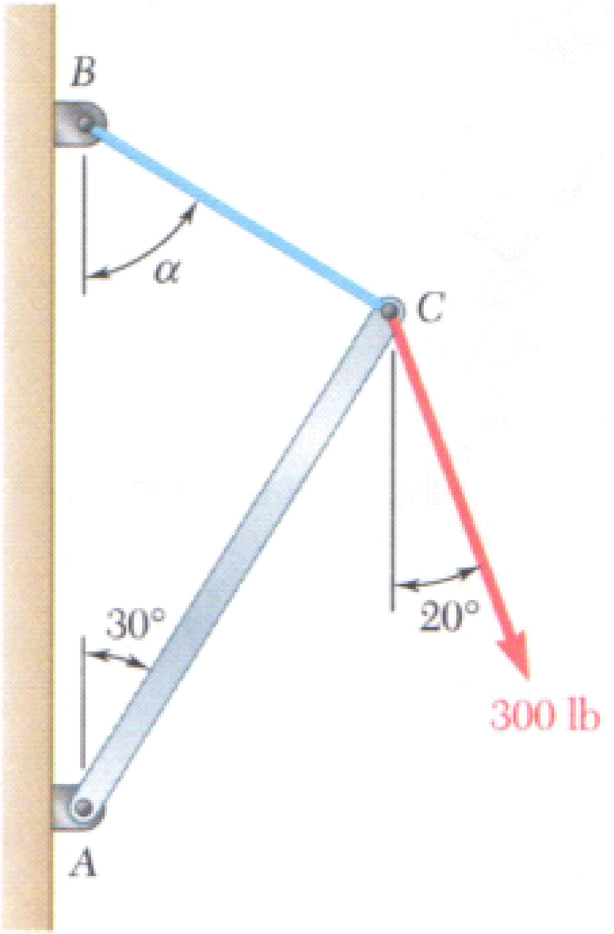


Fig. P2.46

Prob. 2.57

2.57 Two cables tied together at *C* are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (*a*) the magnitude of the largest force **P** which may be applied at *C*, (*b*) the corresponding value of α.

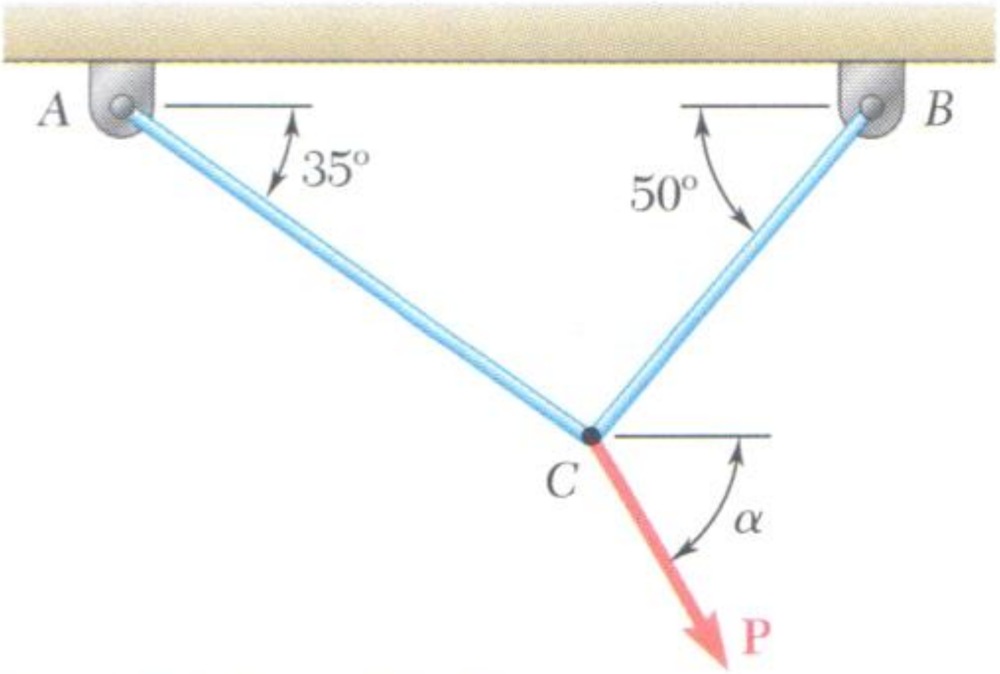


Fig. P2.57 and P2.58

Prob. 2.58

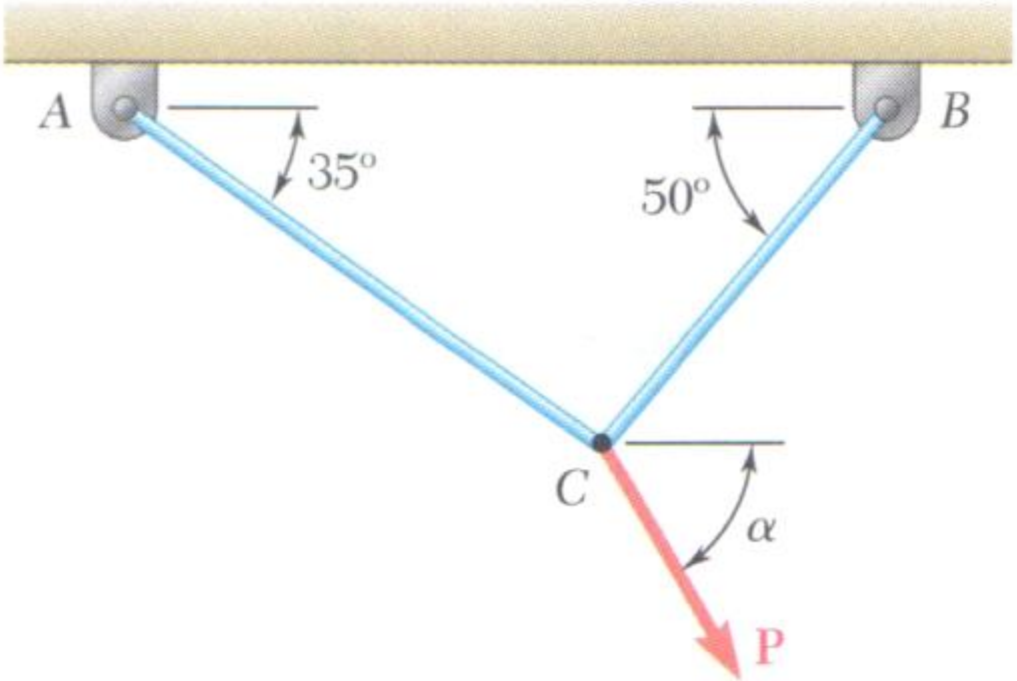


Fig. P2.57 and P2.58

2.58 Two cables tied together at *C* are loaded as shown. Knowing that the maximum allowable tension is 1200 N in cable *AC* and 600 N in cable *BC*, determine (*a*) the magnitude of the largest force **P** which may be applied at *C*, (*b*) the corresponding value of *α*.

Prob. 2.65

2.65 A 160-kg load is supported by the rope-and-pulley arrangement shown. Knowing that $\beta = 20°$, determine the magnitude and direction of the force **P** which should be exerted on the free end of the rope to maintain equilibrium. (*Hint.* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chap. 4.)

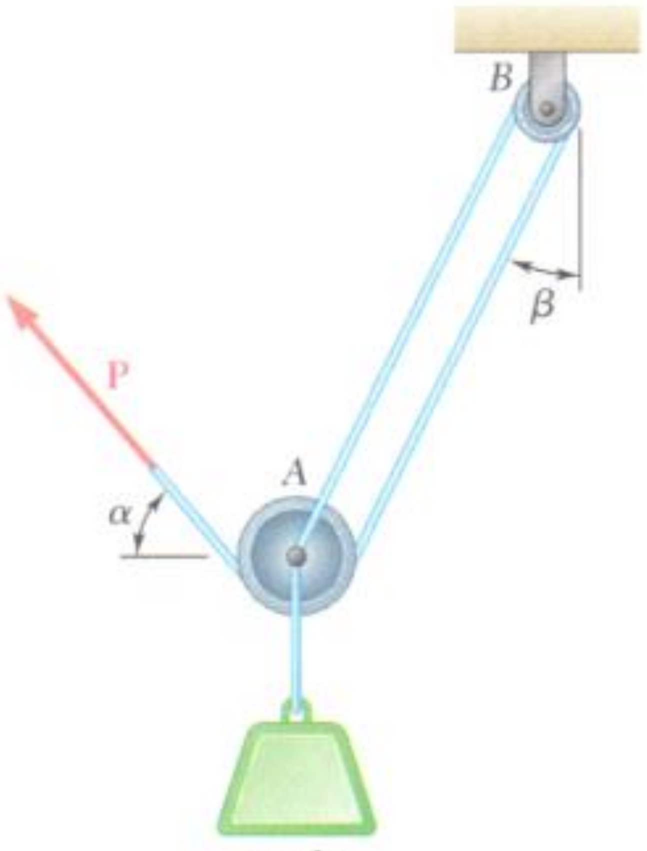


Fig. P2.65 and P2.66

Prob. 2.66

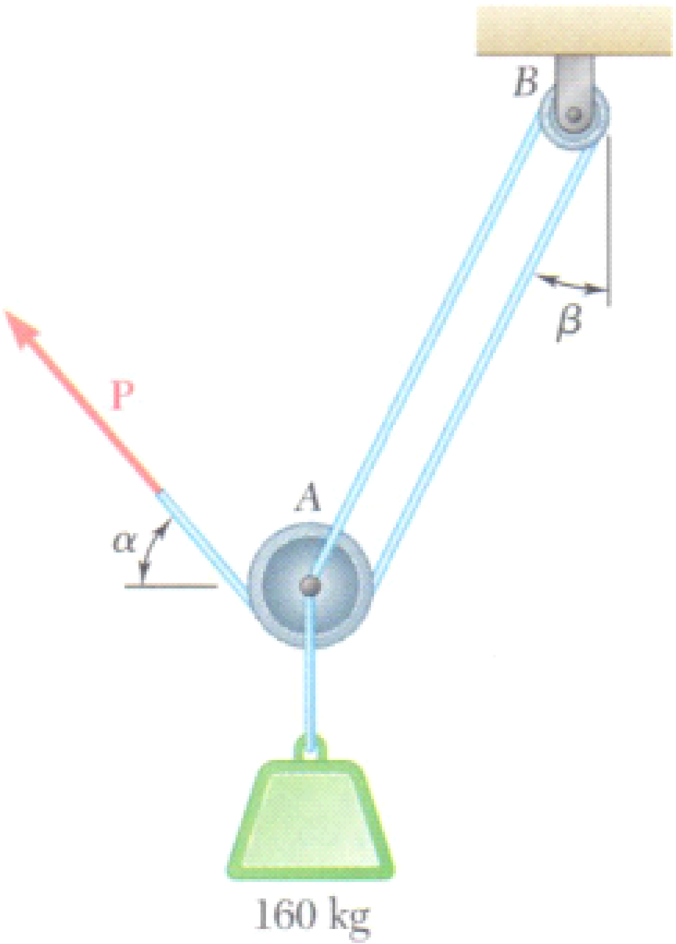


Fig. P2.65 and P2.66

2.66 A 160-kg load is supported by the rope-and-pulley arrangement shown. Knowing that $\alpha = 40°$, determine (*a*) the angle β, (*b*) the magnitude of the force **P** which should be exerted on the free end of the rope to maintain equilibrium. (See the hint for Prob. 2.65.)

Prob. 2.71

2.71 Determine (*a*) the x, y, and z components of the 600-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

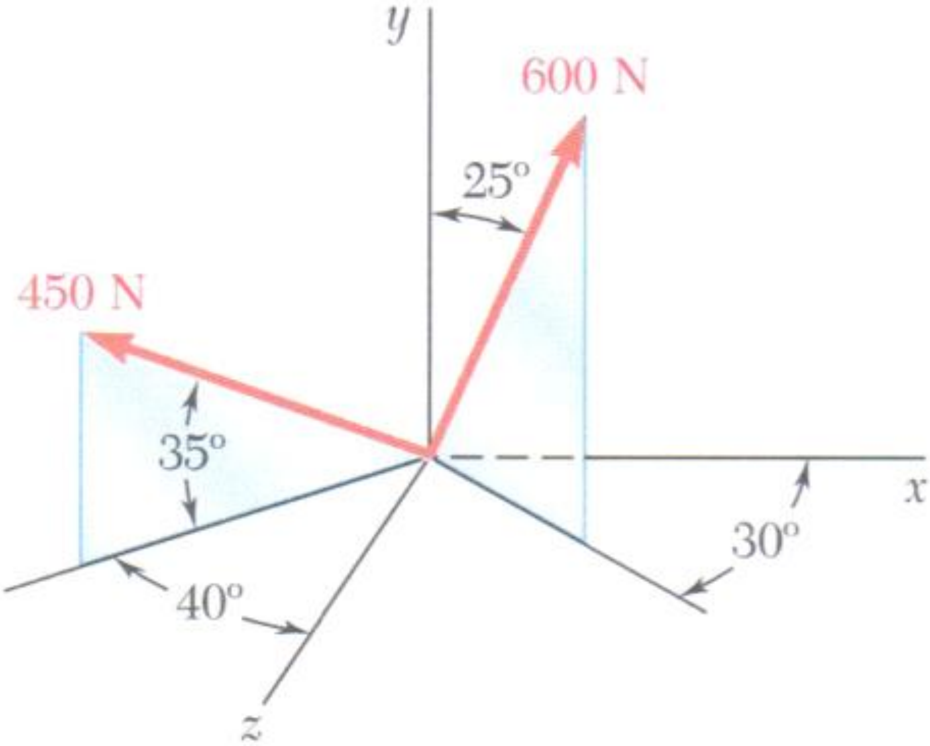


Fig. P2.71 and P2.72

Prob. 2.72

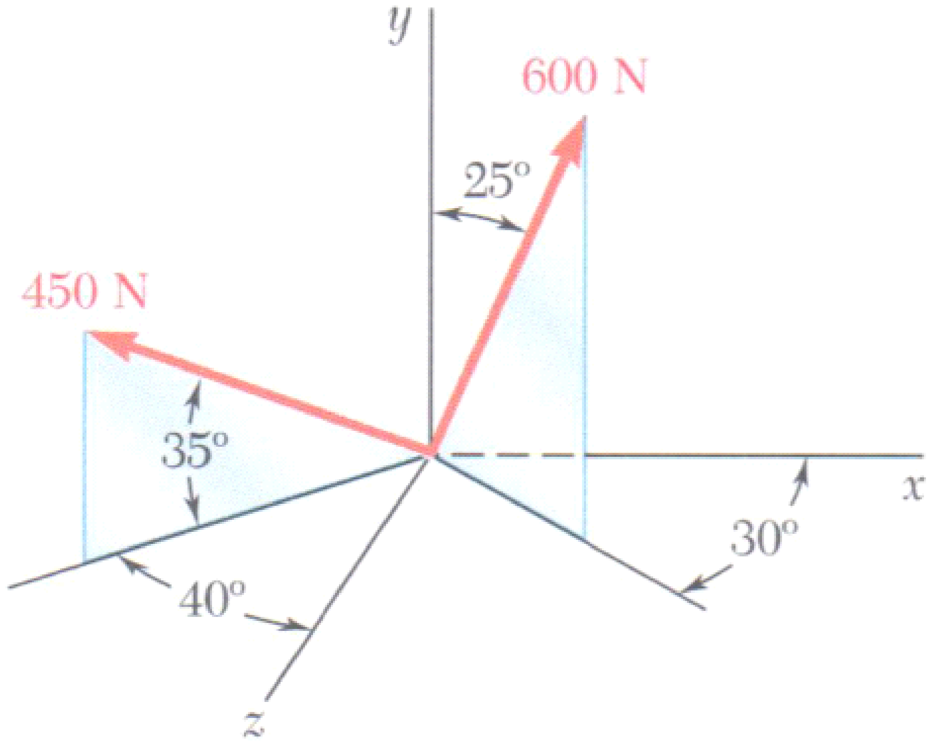


Fig. P2.71 and P2.72

2.72 Determine (*a*) the *x*, *y*, and *z* components of the 450-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

Prob. 2.73

2.73 The end of the coaxial cable AE is attached to the pole AB, which is strengthened by the guy wires AC and AD. Knowing that the tension in wire AC is 120 lb, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

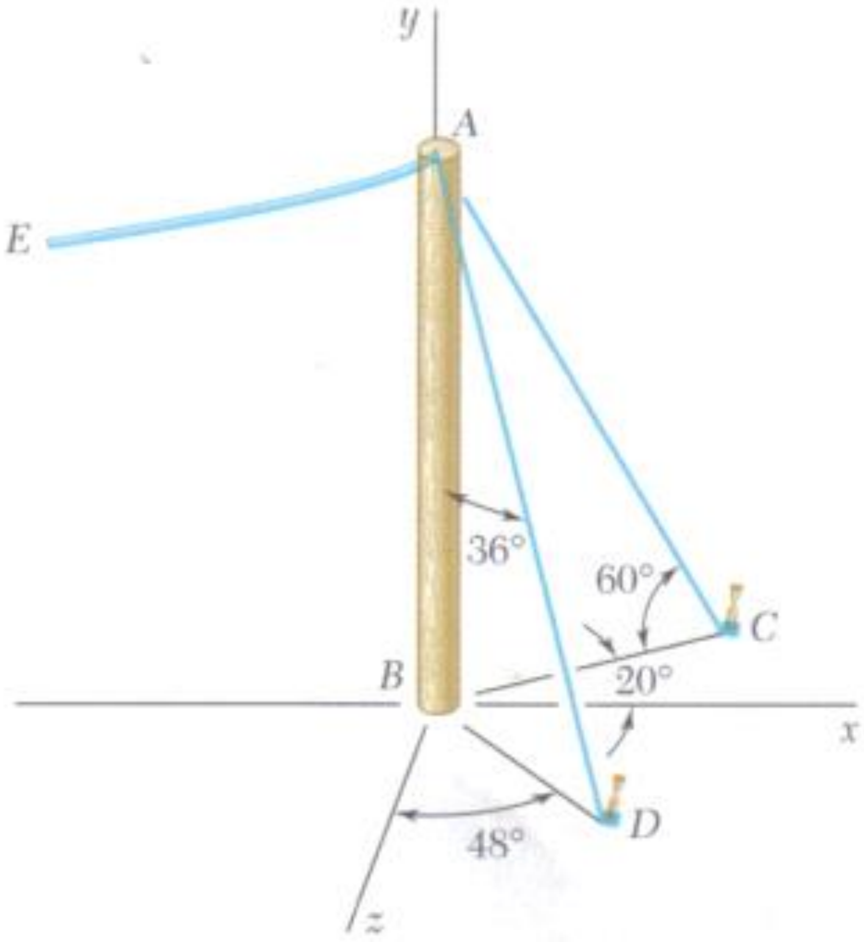


Fig. P2.73 and P2.74

Prob. 2.74

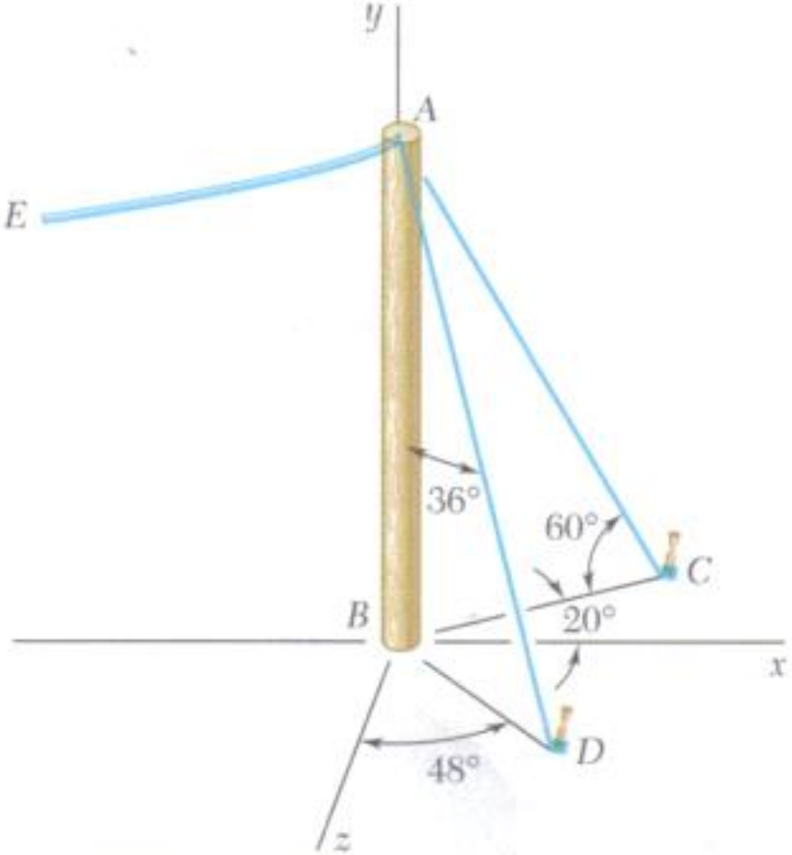


Fig. P2.73 and P2.74

2.74 The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in wire *AD* is 85 lb, determine (*a*) the components of the force exerted by the wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

Prob. 2.87

2.87 A transmission tower is held by three guy wires anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 525 lb, determine the components of the force exerted by the wire on the bolt at *B*.

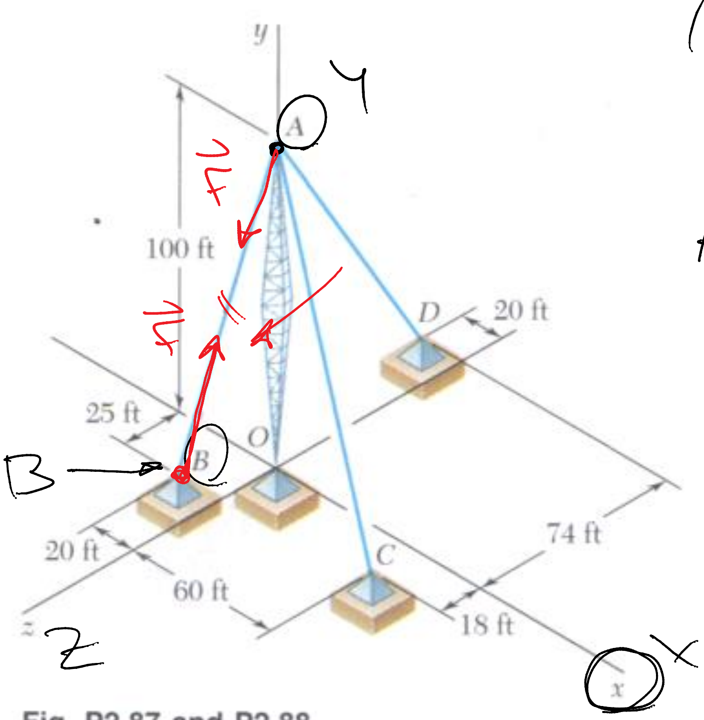


Fig. P2.87 and P2.88

$T_{AB} = 525^{LB}$ FIND $\vec{T}_{AB}$

LOCATE POINTS A AND B:

$A(0, 100, 0)^{ft}$, $B(-20, 0, 25)^{ft}$

$d_x = X_A - X_B = (0) - (-20) = 20^{ft}$

$d_y = Y_A - Y_B = (100) - (0) = 100^{ft}$

$d_z = Z_A - Z_B = (0) - (25) = -25^{ft}$

$d = \sqrt{20^2 + 100^2 + 25^2} = 105^{ft}$

$F_x = F \cdot \frac{d_x}{d} = (525^{LB})\left(\frac{20\cancel{ft}}{105\cancel{ft}}\right) = 100^{LB}$

$F_y = F \cdot \frac{d_y}{d} = (525^{LB})\left(\frac{100}{105}\right) = 500^{LB}$

$F_z = F \frac{d_z}{d} = (525)\left(\frac{-25}{105}\right) = -125^{LB}$

$$\boxed{\vec{T}_{AB} = (100)\hat{i} + (500)\hat{j} + (-125)\hat{k}\ ^{LB}}$$

Prob. 2.88

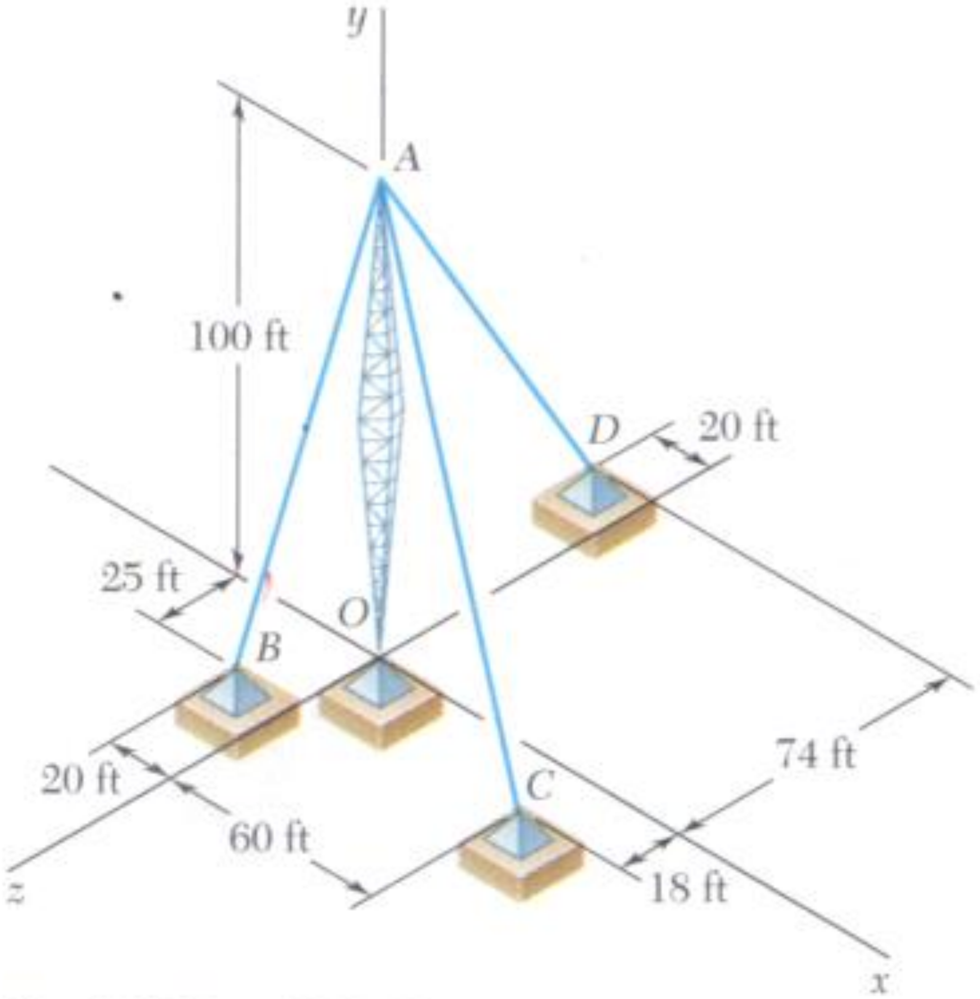


Fig. P2.87 and P2.88

2.88 A transmission tower is held by three guy wires anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AD* is 315 lb, determine the components of the force exerted by the wire on the bolt at *D*.

Prob. 2.93

2.93 Knowing that the tension is 425 lb in cable *AB* and 510 lb in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

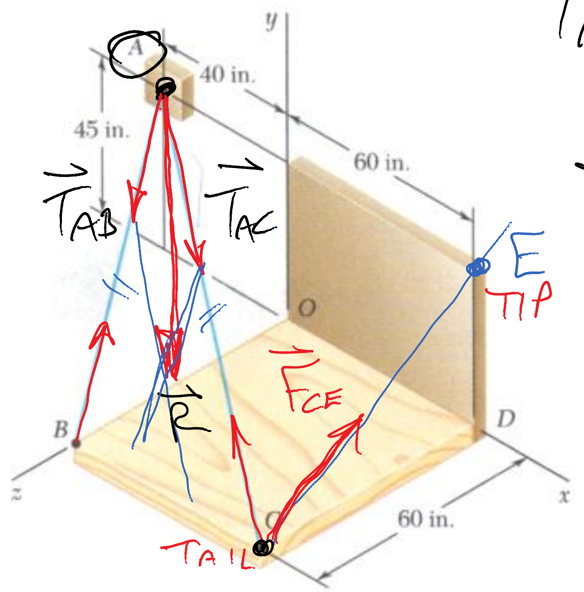


Fig. P2.93 and P2.94

$T_{AB} = 425^{LB}$, $T_{AC} = 510^{LB}$, FIND $\vec{R}$

DIRECTION ANGLES

$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC}$

LOCATE POINTS:

$dx = X_E - X_C$

$dy = Y_E - Y_C$

$dz = Z_E - Z_C$

$A(-40, 45, 0)^{IN}$, $B(0, 0, 60)^{IN}$

$C(60, 0, 60)^{IN}$

$\vec{T}_{AB}$: $dx = X_B - X_A = (0) - (-40) = 40^{IN}$

$dy = Y_B - Y_A = (0) - (45) = -45^{IN}$

$dz = Z_B - Z_A = (60) - (0) = 60^{IN}$

$d = \sqrt{40^2 + 45^2 + 60^2} = 85^{IN}$

$F_x = F \cdot \frac{dx}{d} = (425^{LB})\left(\frac{40^{IN}}{85^{IN}}\right) = 200^{LB}$

$F_y = F \cdot \frac{dy}{d} = (425)\left(\frac{-45}{85}\right) = -225^{LB}$

$F_z = F \cdot \frac{dz}{d} = (425)\left(\frac{60}{85}\right) = 300^{LB}$

$\boxed{\vec{T}_{AB} = (200)\hat{i} + (-225)\hat{j} + (300)\hat{k}^{LB}}$ ★

$\vec{T}_{AC}$: $dx = X_C - X_A = (60) - (-40) = 100^{IN}$

$$d_y = Y_C - Y_A = (0) - (45) = -45^{\text{IN}}$$

$$d_z = Z_C - Z_A = (60) - (0) = 60^{\text{IN}}$$

$$d = \sqrt{100^2 + 45^2 + 60^2} = 125^{\text{IN}}$$

SIG. DIGITS:

1.0

1.00

0.001 1E-3

0.0010

$$F_x = F \cdot \frac{d_x}{d} = (510^{\text{LB}})\left(\frac{100^{\text{IN}}}{125^{\text{IN}}}\right) = 408^{\text{LB}}$$

$$F_y = F \cdot \frac{d_y}{d} = (510)\left(\frac{-45}{125}\right) = -183.6^{\text{LB}}$$

$$F_z = F \cdot \frac{d_z}{d} = (510)\left(\frac{60}{125}\right) = 244.8^{\text{LB}}$$

$$\boxed{\vec{T}_{AC} = (408)\hat{i} + (-183.6)\hat{j} + (244.8)\hat{k}^{\text{ LB}}}$$

$$\vec{R} = (200 + 408)\hat{i} + (-225 - 183.6)\hat{j} + (300 + 244.8)\hat{k}^{\text{ LB}}$$

$$\boxed{\vec{R} = (608)\hat{i} + (-408.6)\hat{j} + (544.8)\hat{k}^{\text{ LB}}}$$

$$\boxed{|\vec{R}|} = \sqrt{608^2 + 408.6^2 + 544.8^2} = \boxed{913^{\text{LB}}}$$

$$\boxed{\theta_x} = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{608}{913}\right) = \boxed{48.2^\circ}$$

$$\boxed{\theta_y} = \cos^{-1}\left(\frac{R_y}{R}\right) = \cos^{-1}\left(\frac{-408}{913}\right) = \boxed{117^\circ}$$

$$\boxed{\theta_z} = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{544.8}{913}\right) = \boxed{53.4^\circ}$$

Prob. 2.94

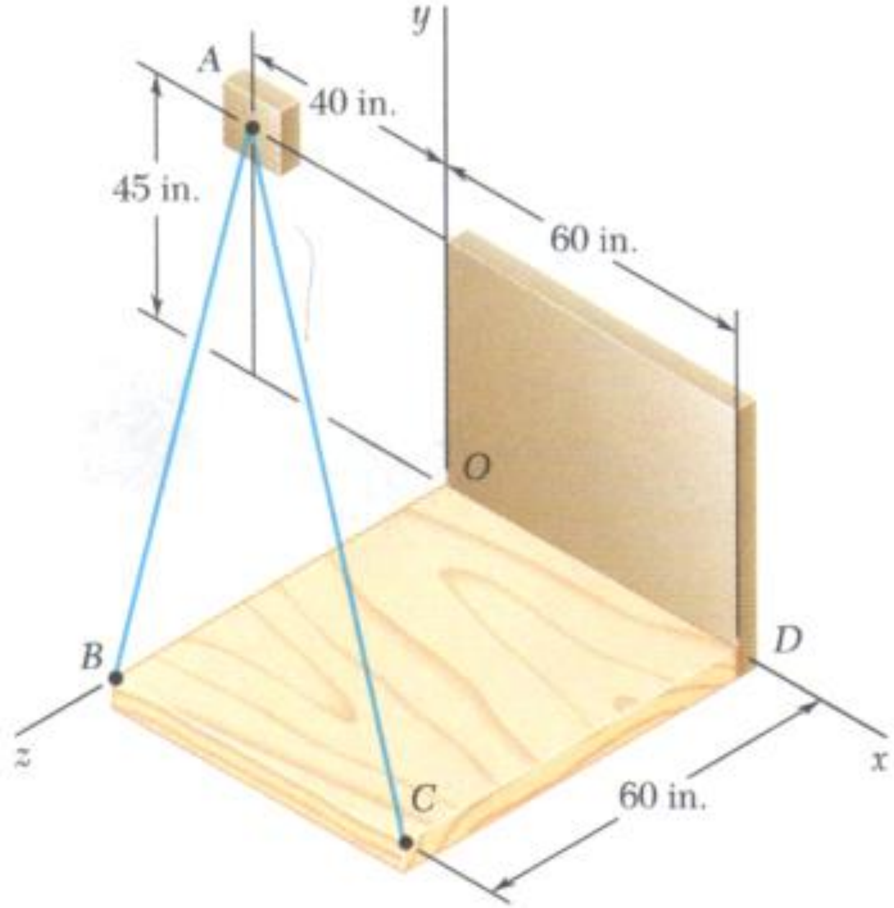
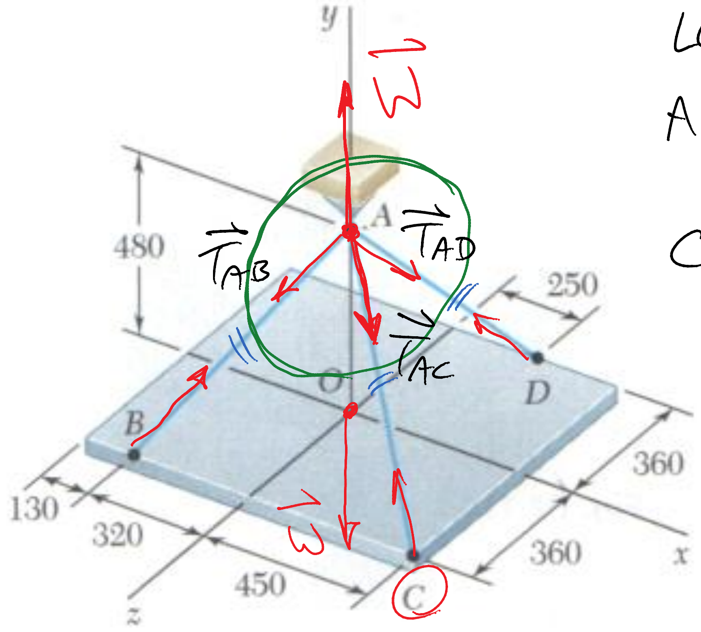


Fig. P2.93 and P2.94

2.94 Knowing that the tension is 510 lb in cable AB and 425 lb in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

Prob. 2.109

2.109 A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.



Dimensions in mm

Fig. P2.109 and P2.110

LOCATE POINTS:

$A(0,480,0)^{mm}$, $B(-320,0,360)^{mm}$

$C(450,0,360)^{mm}$, $D(250,0,-360)^{mm}$

$\vec{T}_{AB}$: $dx = X_B - X_A$

$dx = (-320) - (0) = -320^{mm}$

$dy = Y_B - Y_A$

$dy = (0) - (480) = -480^{mm}$

$dz = Z_B - Z_A = (360) - (0) = 360^{mm}$

$$d = \sqrt{320^2 + 480^2 + 360^2} = 680^{mm}$$

$$F_x = F \cdot \frac{dx}{d} = T_{AB}\left(\frac{-320}{680}\right) = -0.470\, T_{AB}$$

$$F_y = F \cdot \frac{dy}{d} = T_{AB}\left(\frac{-480}{680}\right) = -0.706\, T_{AB}$$

$$F_z = F \cdot \frac{dz}{d} = T_{AB}\left(\frac{360}{680}\right) = 0.529\, T_{AB}$$

$$\boxed{\vec{T}_{AB} = (-0.470\, T_{AB})\hat{i} + (-0.706\, T_{AB})\hat{j} + (0.529\, T_{AB})\hat{k}\ N}$$

$\vec{T}_{AC}$: $dx = X_C - X_A = (450) - (0) = 450^{mm}$

$dy = Y_C - Y_A = (0) - (480) = -480^{mm}$

$dz = 360^{mm}$, $d = 750^{mm}$

$$F_x = F \cdot \frac{d_x}{d} = (60^N)\left(\frac{450}{750}\right) = 36.0^N$$

$$F_y = F \cdot \frac{d_y}{d} = (60)\left(\frac{-480}{750}\right) = -38.4^N$$

$$F_z = 28.8^N$$

$$\vec{T}_{AC} = (36)\hat{i} + (-38.4)\hat{j} + (28.8)\hat{k}^N$$

⋮

$$\vec{T}_{AD} = (0.385\, T_{AD})\hat{i} + (-0.738\, T_{AD})\hat{j} + (-0.554\, T_{AD})\hat{k}^N$$

$\Sigma \vec{F} = m\vec{a}$

$$\vec{W} = (W)\hat{j}^N$$

EQUILIBRIUM EQUATIONS: $\Sigma \vec{F} = 0$

$\Sigma F_x = 0 : (-0.470\, T_{AB}) + (36) + (0.385\, T_{AD}) = 0$ EQN. 1

$T_{AB} = 0.819\, T_{AD} + 76.6$ EQN. 2

$\Sigma F_y = 0 : (-0.706\, T_{AB}) + (-38.4) + (-0.738\, T_{AD}) + (W) = 0$ EQN. 3

$\Sigma F_z = 0 : (0.529\, T_{AB}) + (28.8) + (-0.554\, T_{AD}) = 0$

$$0.529(0.819\, T_{AD} + 76.6) + 28.8 - 0.554\, T_{AD} = 0$$

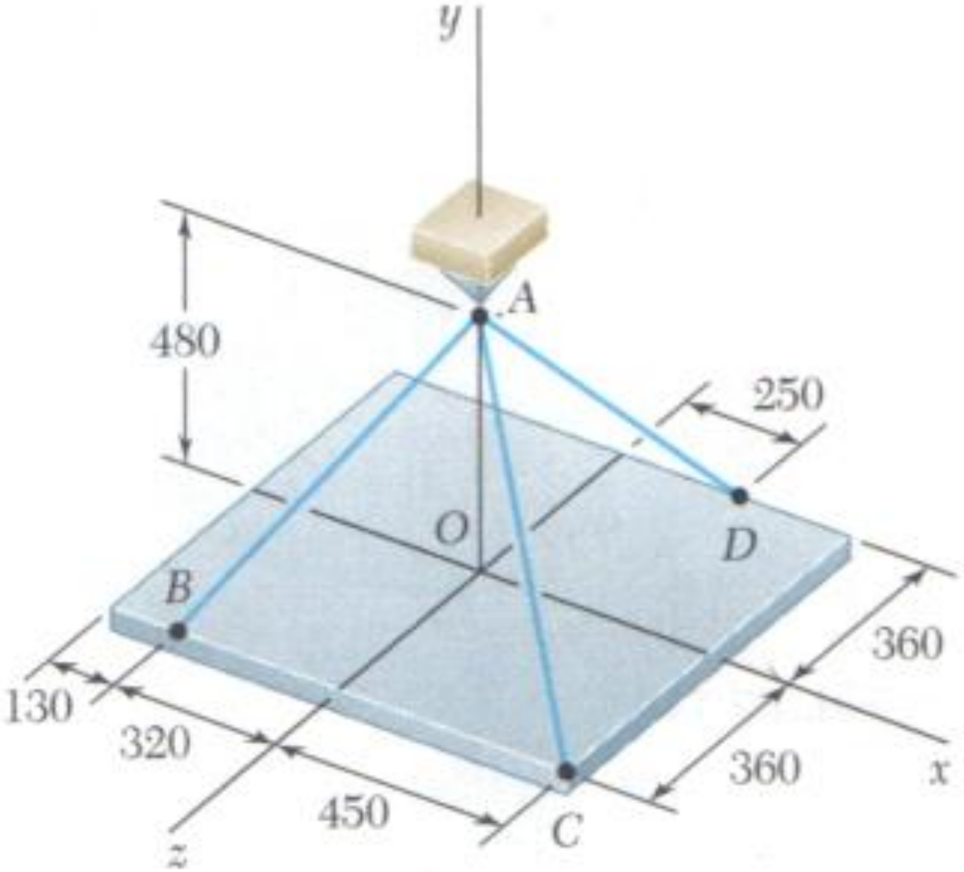
$$T_{AD} = 566^N$$

$$T_{AB} = 0.819(566) + 76.6 = 540^N$$

$$T_{AB} = 0.819(566) + 76.6 = 540^{N}$$

$$\boxed{W} = 0.706(540) + 38.4 + 0.738(566) = 838^{N}$$

Prob. 2.110



Dimensions in mm

Fig. P2.109 and P2.110

2.110 A rectangular plate is supported by three cables as shown. Knowing that the tension in cable *AD* is 520 N, determine the weight of the plate.