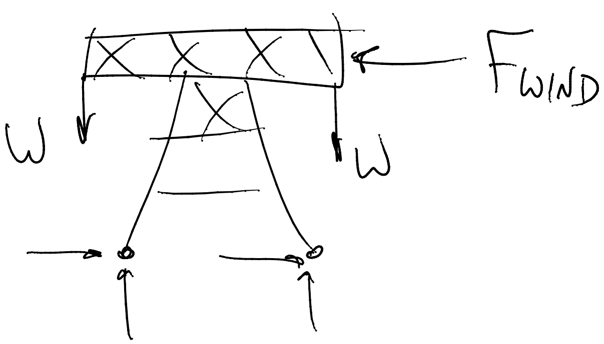
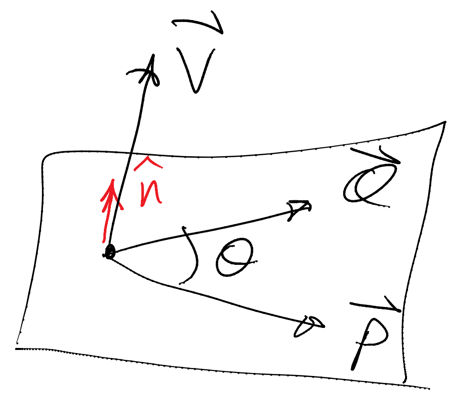
SO FAR, WE HAVE EXAMINED THE STATICS OF A PARTICLE, i.e., $\Sigma \vec{F} = 0$ AT A SINGLE POINT. IN MANY CASES, WE MUST ANALYZE THE BEHAVIOR OF A RIGID BODY, WHERE FORCES MAY NOT ALL BE FOCUSED ON A SINGLE POINT.



TO DO THIS ANALYSIS, WE MUST BE PROFICIENT AT CALCULATING MOMENTS (TORQUES) ON THE BODY. THIS WILL REQUIRE SOME MORE MATHEMATICAL TOOLS:

VECTOR CROSS PRODUCT

VECTOR DOT PRODUCT



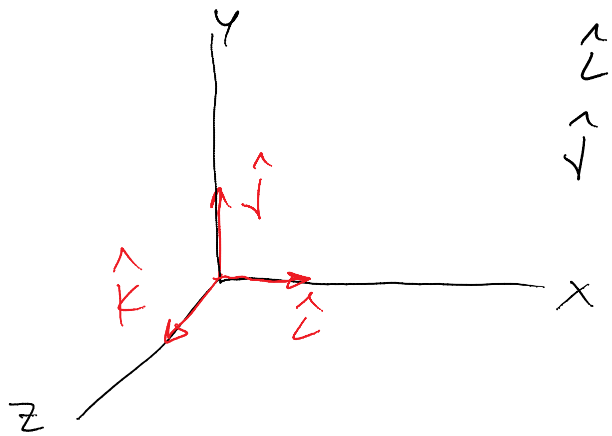
$$\vec{V} = \vec{P} \times \vec{Q} = \hat{n}\,|\vec{P}| \cdot |\vec{Q}| \cdot \sin\theta$$

$\hat{n}$ = PERPENDICULAR TO PLANE DEFINED BY $\vec{P}$ AND $\vec{Q}$

IS $\vec{V} \neq \vec{Q} \times \vec{P}$ $\qquad \vec{Q} \times \vec{P} = -\vec{V}$

DUE TO $\sin\theta$, IF $\vec{P}$ AND $\vec{Q}$ ARE PARALLEL,

$\vec{P} \times \vec{Q} = 0$.



$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{i} \times \hat{k} = -\hat{j}$

$$\vec{V} = \vec{P} \times \vec{Q} = (P_x\hat{i} + P_y\hat{j} + P_z\hat{k}) \times (Q_x\hat{i} + Q_y\hat{j} + Q_z\hat{k})$$

$$= \boxed{P_x Q_x(\hat{i} \times \hat{i})} + P_x Q_y(\hat{i} \times \hat{j}) + P_x Q_z(\hat{i} \times \hat{k})$$

$+$ 6 MORE TERMS

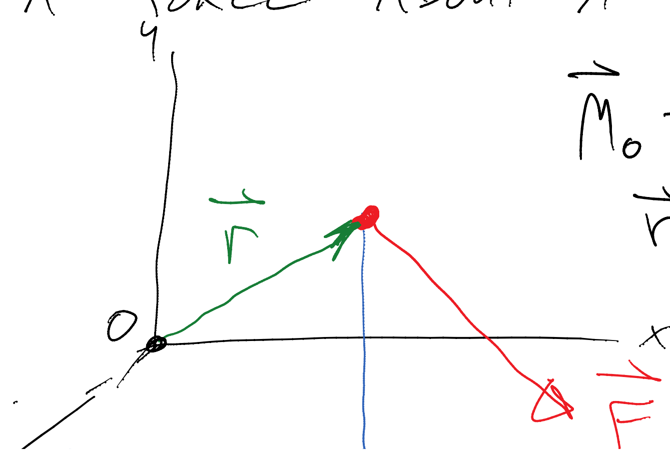
$$\vec{V} = \boxed{(P_y Q_z - P_z Q_y)}\,\hat{i} + (P_z Q_x - P_x Q_z)\,\hat{j} + (P_x Q_y - P_y Q_x)\,\hat{k}$$

OR

$$\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

USE THE CROSS PRODUCT TO DETERMINE THE MOMENT OF A FORCE ABOUT A POINT.

COMPUTE MOMENT ABOUT THE ORIGIN

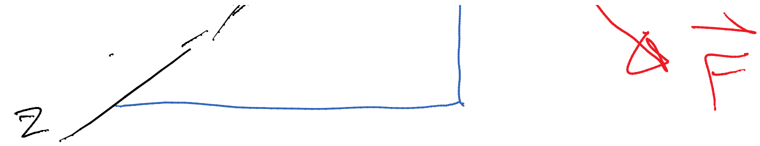


$\vec{M}_o = \vec{r} \times \vec{F}$

$\vec{r}$ = POSITION VECTOR

$\vec{F}$ = FORCE VECTOR

ORIGIN



VECTOR

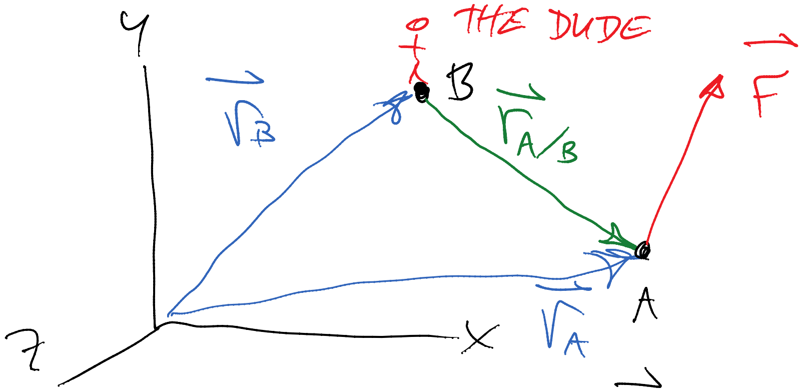
$$\vec{r} = (r_x)\,\hat{i} + (r_y)\,\hat{j} + (r_z)\,\hat{k}$$

$$\vec{F} = (F_x)\,\hat{i} + (F_y)\,\hat{j} + (F_z)\,\hat{k}$$

$$\boxed{\vec{M}_0 = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}}$$

MOMENT ABOUT THE ORIGIN

TO CALCULATE THE MOMENT OF A FORCE ABOUT ANY ARBITRARY POINT,



CALCULATE MOMENT THAT $\vec{F}$ MAKES ABOUT POINT B:

$$\boxed{\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}}$$

$\vec{r}_A$ = POSITION VECTOR FROM ORIGIN TO A

$\vec{r}_{A/B}$ = POSITION VECTOR POINT A W.R.T. POINT B

$\vec{r}_B$ = POSITION VECTOR FROM ORIGIN TO B

$$A = (\cancel{B})\left(\frac{A}{\cancel{B}}\right)$$

$$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$$

$$\boxed{\vec{M}_B = \vec{r}_{A/B} \times \vec{F}}$$

IN SOME CASES, WE WILL WANT TO KNOW THE MOMENT OF A FORCE ABOUT AN AXIS. THIS MOMENT WILL TEND TO CAUSE THE RIGID BODY TO ROTATE ABOUT THIS AXIS. TO DO THIS, WE WILL USE THE VECTOR DOT PRODUCT.

$$\vec{P} \cdot \vec{Q} = |\vec{P}| \cdot |\vec{Q}| \cdot \cos\theta$$

DUE TO $\cos\theta$, IF THE TWO VECTORS ARE PERPENDICULAR, $\vec{P} \cdot \vec{Q} = 0$

FOR THE UNIT VECTORS, $\hat{i} \cdot \hat{i} = 1$, $\hat{i} \cdot \hat{k} = 0$

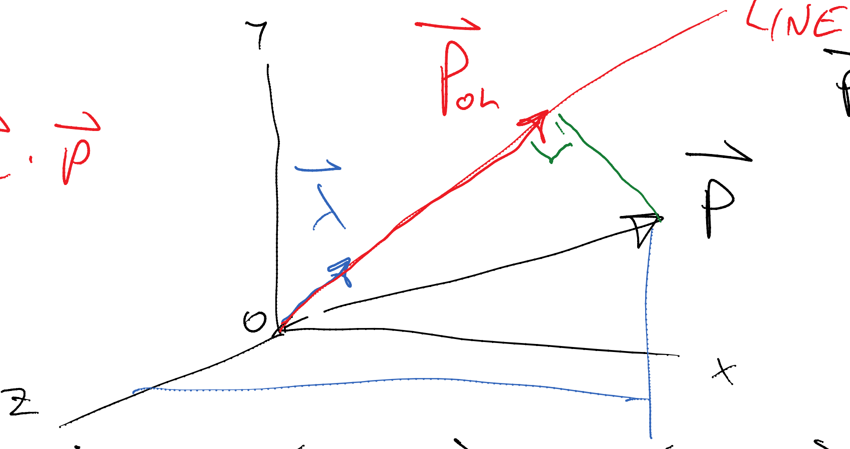
$$\vec{P} \cdot \vec{Q} = (P_x \hat{i} + P_y \hat{j} + P_z \hat{k}) \cdot (Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k})$$

$$\vec{P} \cdot \vec{Q} = (P_x Q_x)(\hat{i} \cdot \hat{i}) + (P_x Q_y)(\hat{i} \cdot \hat{j}) + \cdots \text{ 7 TERMS}$$

$$\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z = \text{SCALAR}$$

THE DOT PRODUCT CAN BE USED TO FIND THE PROJECTION OF A VECTOR ONTO A GIVEN AXIS.

$\vec{P} \cdot \vec{Q} \overset{\checkmark}{=} \vec{Q} \cdot \vec{P}$



$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$

$$\vec{\lambda} = (\cos\theta_x)\hat{i} + (\cos\theta_y)\hat{j} + (\cos\theta_z)\hat{k}$$

$$P_{OL} = \vec{P} \cdot \vec{\lambda} = P_x \cos\theta_x + P_y \cos\theta_y + P_z \cos\theta_z$$

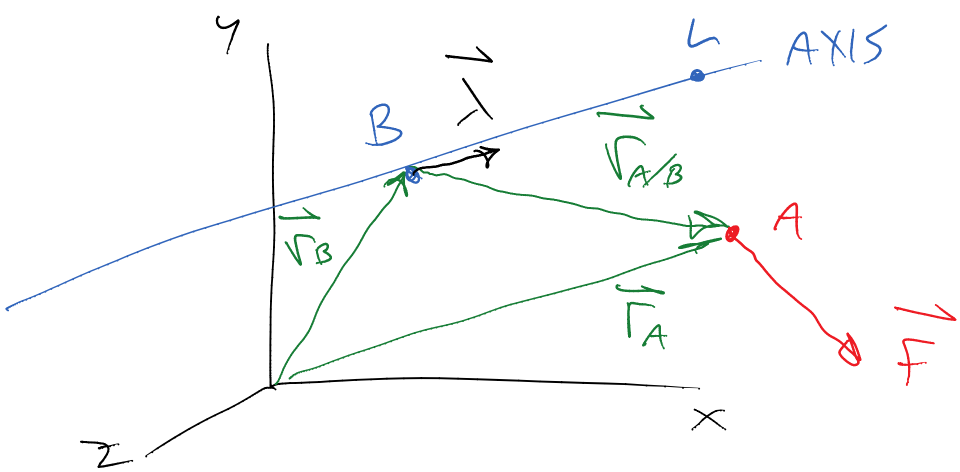
USING THE DOT PRODUCT, THE MOMENT OF A FORCE ABOUT A GIVEN AXIS IS:

$$M_{OL} = \vec{M}_O \cdot \vec{\lambda}$$

THE VECTOR COMPONENT OF $\vec{M}_O$ ALONG OL IS:

$$\vec{M}_{OL} = M_{OL} \cdot \vec{\lambda}$$

IF THE AXIS DOES NOT PASS THROUGH THE ORIGIN, CHOOSE AN ARBITRARY POINT B ON THE AXIS.



THE MOMENT ABOUT B DUE TO $\vec{F}$ IS

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

THE PROJECTION OF THIS MOMENT ONTO THE AXIS IS:

$$M_{BL} = \vec{\lambda}_{BL} \cdot \vec{M}_B$$

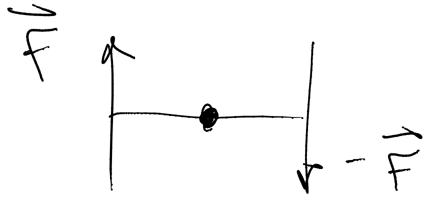
THE VECTOR COMPONENT OF $\vec{M}_B$ ALONG BL IS:

$$\vec{M}_{BL} = M_{BL} \cdot \vec{\lambda}$$ "SOAK TIME"

MOMENT OF A COUPLE

MOMENT OF A COUPLE

A SPECIAL ARRANGEMENT OF FORCES IS CALLED A COUPLE.

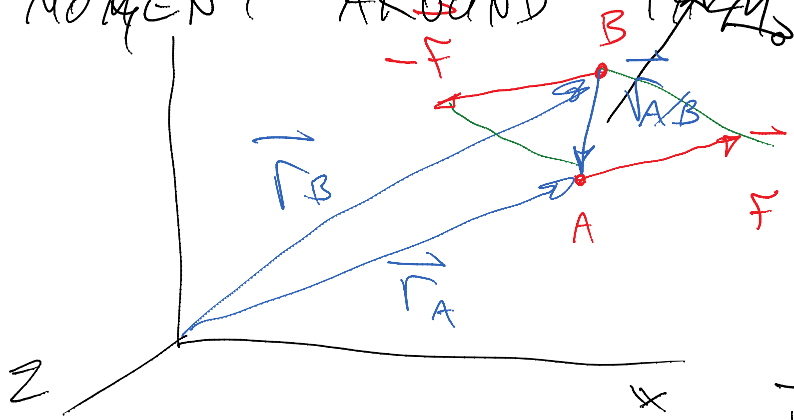


TWO EQUAL AND OPPOSITE PARALLEL FORCES

A COUPLE DOES NOT TEND TO DISPLACE A RIGID BODY, ONLY ROTATE IT.

IN THE FUTURE, WE WILL NEED TO MANIPULATE COUPLES TO SIMPLIFY OUR ANALYSIS OF RIGID BODIES.

THE MOMENT CREATED BY A COUPLE CAN BE COMPUTE BY FINDING THE RESULTANT MOMENT AROUND THE ORIGIN:



$$\vec{M}_o = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F})$$

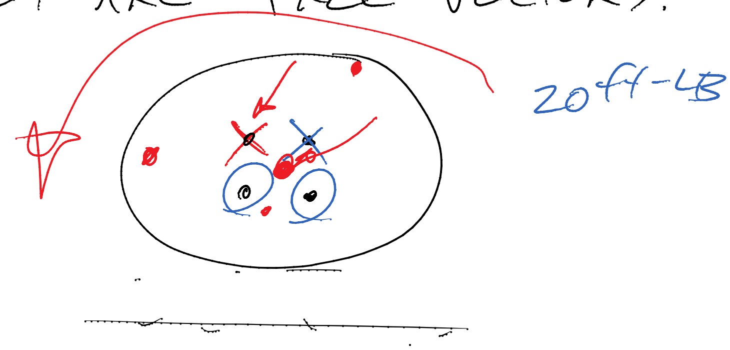
$$\vec{M}_o = (\vec{r}_A - \vec{r}_B) \times (\vec{F})$$

$$\vec{M}_o = \vec{r}_{A/B} \times \vec{F}$$

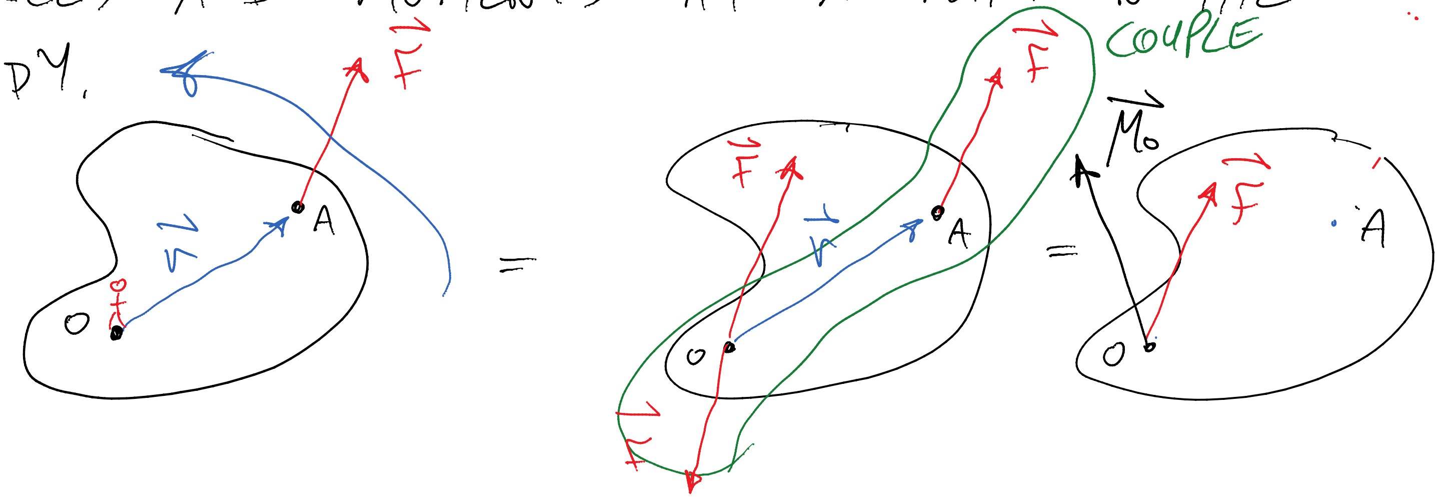
$\vec{r}_{A/B}$ = POSITION VECTOR OF A W.R.T. B

IF MORE THAN ONE COUPLE IS PRESENT ON A

BODY, THE RESULTING MOMENT CAN BE ADDED VECTORIALLY SINCE THEY ARE FREE VECTORS.



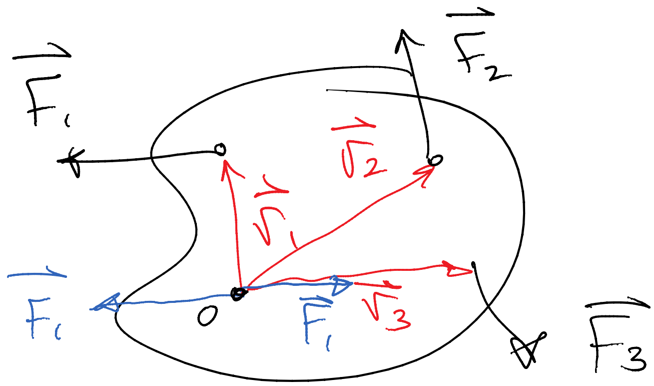
TO ANALYZE RIGID BODIES, WE NEED TO BE ABLE TO RESOLVE FORCES ON THE BODY TO FORCES AND MOMENTS AT A POINT ON THE BODY.



THE MOMENT DUE TO THE COUPLE IS:

$$\vec{M}_0 = \vec{r} \times \vec{F}$$

IF THERE IS MORE THAN ONE FORCE, EACH



CAN BE RESOLVED INTO A MOMENT AND A FORCE AT A POINT.

THE RESULTANT FORCE IS

$$\vec{R} = \Sigma \vec{F}$$

THE RESULTANT MOMENT IS

$$\vec{M}_O = \Sigma \vec{M} = \Sigma \left(\vec{r}_i \times \vec{F}_i \right)$$

Prob. 3.12

3.12 It is known that a force with a moment of 960 N · m about D is required to straighten the fence post CD. If $d = 2.80$ m, determine the tension that must be developed in the cable of winch puller AB to create the required moment about point D.

$M_D = 960\ \text{N·m}$

$d = 2.80\ \text{m}$

FIND $|\vec{F}|$

$A(-2.8, 0)$ m

$C(0.2, 0.875)$ m

$D(0, 0)$ m

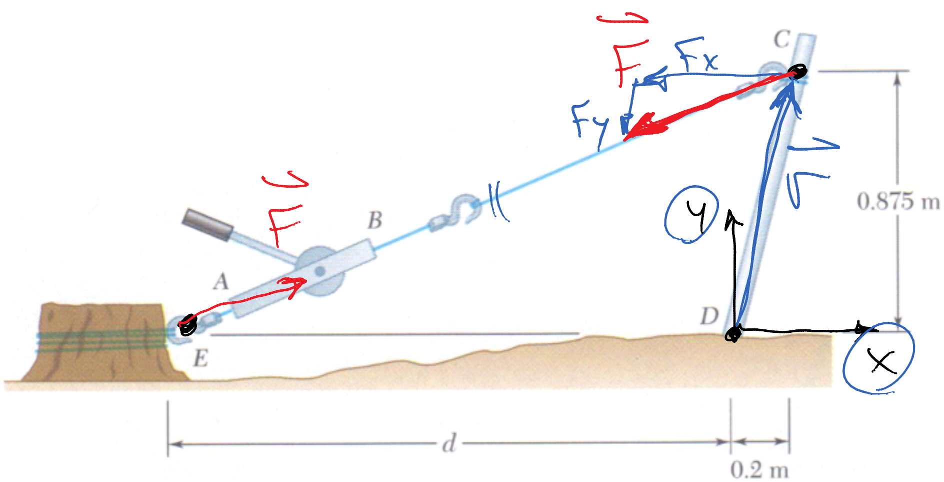


Fig. P3.11, P3.12 and *P3.13*

$\vec{F}$: $d_x = X_A - X_C = (-2.8) - (0.2) = -3.0$ m

$d_y = Y_A - Y_C = (0) - (0.875) = -0.875$ m

$d = \sqrt{(3)^2 + (0.875)^2} = 3.125$ m

$F_x = F\dfrac{d_x}{d} = F\left(\dfrac{-3.0}{3.125}\right) = -0.96F$

$F_y = F\dfrac{d_y}{d} = F\left(\dfrac{-0.875}{3.125}\right) = -0.28F$

$$\boxed{\vec{F} = (-0.96F)\,\hat{i} + (-0.28F)\,\hat{j}\ \text{N}}$$

$\vec{r} = (d_x)\,\hat{i} + (d_y)\,\hat{j} =$

$\vec{r} = (X_C - X_D)\,\hat{i} + (Y_C - Y_D)\,\hat{j}$

$\vec{r} = (0.2 - 0)\,\hat{i} + (0.875 - 0)\,\hat{j}$

$\vec{r} = (0.2)\,\hat{i} + (0.875)\,\hat{j}$ m

$$\vec{r} = (0.2)\,\hat{i} + (0.875)\,\hat{j} \ \text{m}$$

$$\boxed{\vec{M}_D = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.2 & 0.875 & 0 \\ -0.96F & -0.28F & 0 \end{vmatrix}}$$

$$\vec{M}_D = \left[(0.875)(0) - (0)(-0.28F)\right]\hat{i}$$

$$- \left[(0.2)(0) - (0)(-0.96F)\right]\hat{j}$$

$$+ \left[(0.2)(-0.28F) - (0.875)(-0.96F)\right]\hat{k} \ \text{N}\cdot\text{m}$$

$$\vec{M}_D = (0.784F)\,\hat{k} \ \text{N}\cdot\text{m} = (960)\,\hat{k} \ \text{N}\cdot\text{m}$$

$$0.784F = 960$$

$$\boxed{F = 1224 \ \text{N}}$$

Prob. 3.13

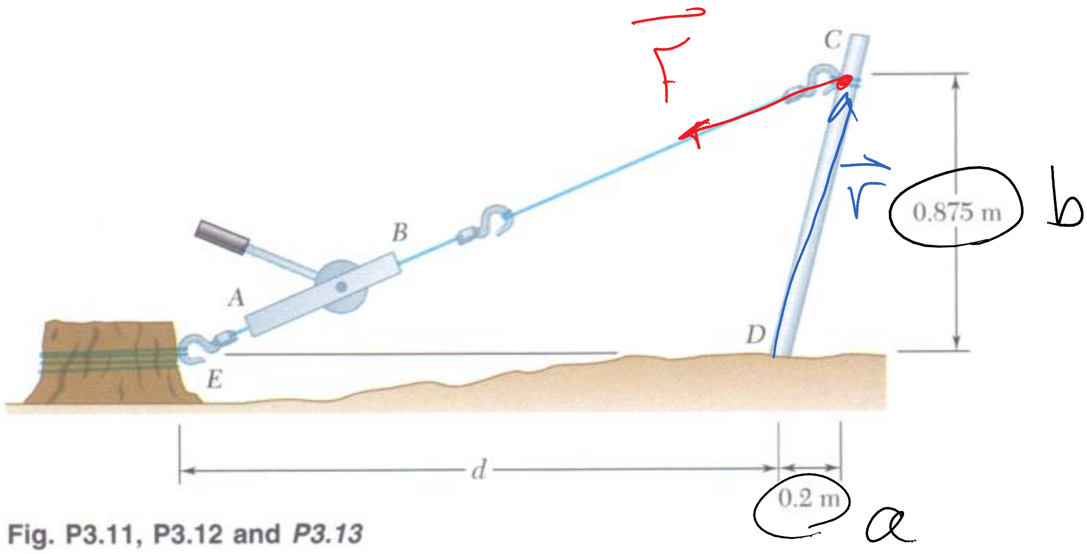


Fig. P3.11, P3.12 and *P3.13*

3.13 It is known that a force with a moment of 960 N · m about *D* is required to straighten the fence post *CD*. If the capacity of winch puller *AB* is 2400 N, determine the minimum value of distance *d* to create the specified moment about point *D*.

$$A(-d, 0), \quad C(a, b)$$

$$\vec{T}_{ac} : dx = X_A - X_c = -d - a = -(d+a) = -c$$

$$dy = Y_A - Y_c = 0 - b = -b$$

$$d = \sqrt{c^2 + b^2}$$

$$F_x = F \frac{dx}{d} = T_{AC} \cdot \frac{-c}{\sqrt{c^2+b^2}}$$

$$F_y = F \frac{dy}{d} = T_{AC} \cdot \frac{-b}{\sqrt{c^2+b^2}}$$

$$\vec{T}_{AC} = (F_x)\hat{i} + (F_y)\hat{j}$$

$$\vec{r} = (dx)\hat{i} + (dy)\hat{j} = (a)\hat{i} + (b)\hat{j}$$

Prob. 3.21

3.21 Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Knowing that the tensions in cables AB and BC are 555 N and 660 N, respectively, determine the moment about O of the resultant force exerted on the tree by the cables at B.

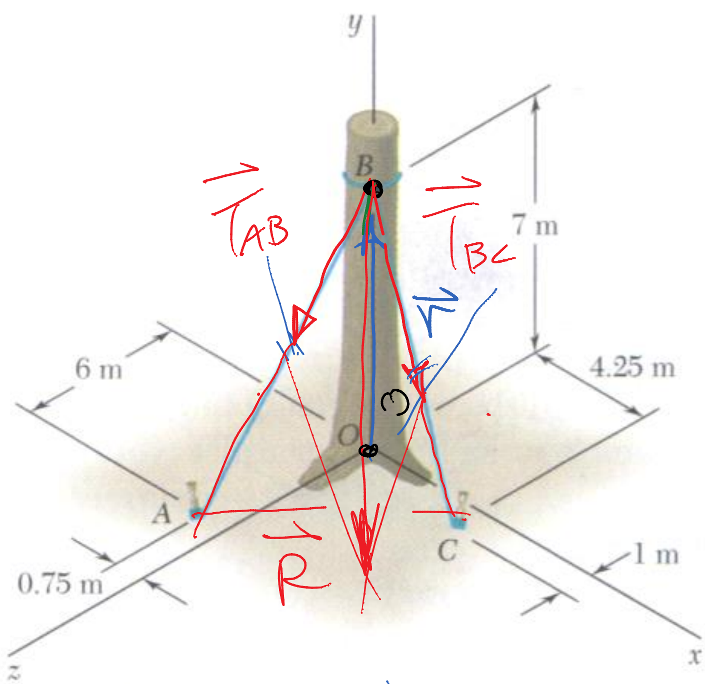


Fig. P3.21

$T_{AB} = 555^N$, $T_{BC} = 660^N$

FIND $\vec{M}_o$

$A(-0.75, 0, 6)^m$

$B(0, 7, 0)^m$

$C(4.25, 0, 1)^m$

$$\vec{T}_{AB} = d_x = X_A - X_B = (-0.75) - (0) = -0.75^m$$

$$d_y = Y_A - Y_B = (0) - (7) = -7^m$$

$$d_z = Z_A - Z_B = (6) - (0) = 6^m$$

$$d = \sqrt{0.75^2 + 7^2 + 6^2} = 9.25^m$$

$$F_x = F\frac{d_x}{d} = (555^N)\left(\frac{-0.75^m}{9.25^m}\right) = -45^N$$

$$F_y = F\frac{d_y}{d} = (555)\left(\frac{-7}{9.25}\right) = -420^N$$

$$F_z = F\frac{d_z}{d} = (555)\left(\frac{6}{9.25}\right) = 360^N$$

$F_z = F \frac{dz}{d} = (555)(9.25) \quad 360$

$$\boxed{\vec{T}_{AB} = (-45)\hat{i} + (-420)\hat{j} + (360)\hat{k} \ N}$$

$\vec{T}_{BC}$: $d_x = X_C - X_B = (4.25) - (0) = 4.25^m$

$d_y = Y_C - Y_B = (0) - (7) = -7^m$

$d_z = Z_C - Z_B = (1) - (0) = 1^m$

$d = \sqrt{4.25^2 + 7^2 + 1^2} = 8.25^m$

$F_x = F \frac{dx}{d} = (660^N)\left(\frac{4.25}{8.25}\right) = 340^N$

$F_y = F \frac{dy}{d} = (660)\left(\frac{-7}{8.25}\right) = -560^N$

$F_z = F \frac{dz}{d} = (660)\left(\frac{1}{8.25}\right) = 80^N$

$$\boxed{\vec{T}_{BC} = (340)\hat{i} + (-560)\hat{j} + (80)\hat{k} \ N}$$

$\vec{R} = \vec{T}_{AB} + \vec{T}_{BC}$

$\vec{R} = (-45 + 340)\hat{i} + (-420 - 560)\hat{j} + (360 + 80)\hat{k} \ N$

$$\boxed{\vec{R} = (295)\hat{i} + (-980)\hat{j} + (440)\hat{k} \ N}$$

$\vec{r} = (0)\hat{i} + (7)\hat{j} + (0)\hat{k} \ m$

$$dx = x_B - x_o = (0) - (0) = 0^m$$

$$dy = y_B - y_o = (7) - (0) = 7^m$$

$$dz = z_B - z_o = (0) - (0) = 0^m$$

$$\vec{r} = (dx)\hat{i} + (dy)\hat{j} + (dz)\hat{k}$$

$$\boxed{\vec{r} = (7)\hat{j}\ m}$$

$$\vec{M}_o = \vec{r} \times \vec{R} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 7 & 0 \\ 295 & -980 & 440 \end{vmatrix}$$

$$\vec{M}_o = [(7)(440) - (0)(-980)]\hat{i}$$

$$- [(0)(440) - (0)(295)]\hat{j}$$

$$+ [(0)(-980) - (7)(295)]\hat{k}\ N\cdot m$$

$$\boxed{\vec{M}_o = (3080)\hat{i} + (-2065)\hat{k}\ N\cdot m}$$

Prob. 3.22

3.22 A farmer uses a rope and pulley to lift a bale of hay of mass 26 kg. Determine the moment about *A* of the resultant force exerted on the pulley by the rope if the center of the pulley *C* lies 0.3 m below point *B* and 7.1 m above the ground.

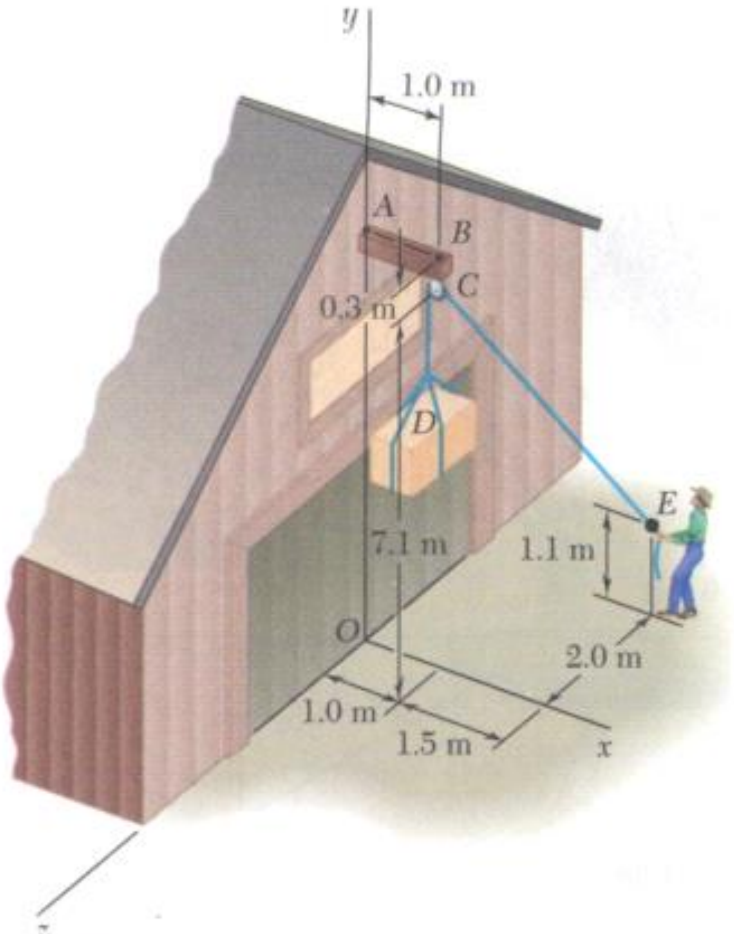


Fig. P3.22

Prob. 3.47

3.47 The 0.61 × 1.00-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 66 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *D*.

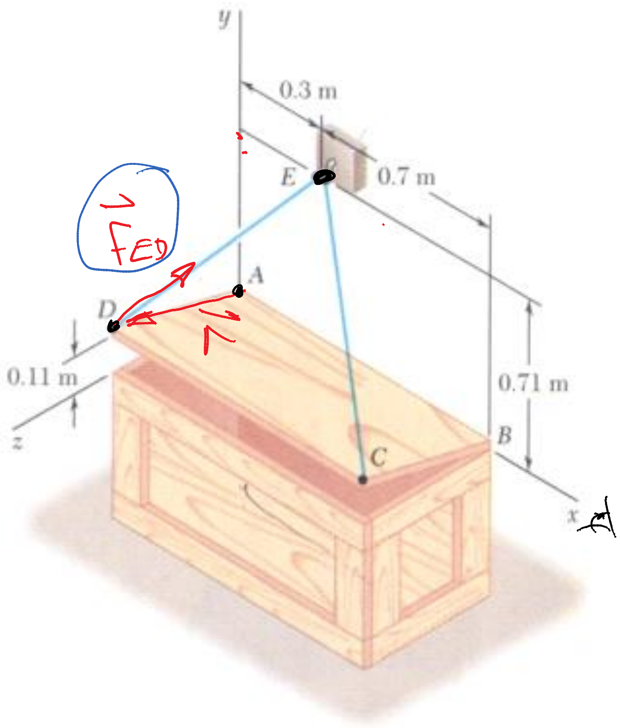
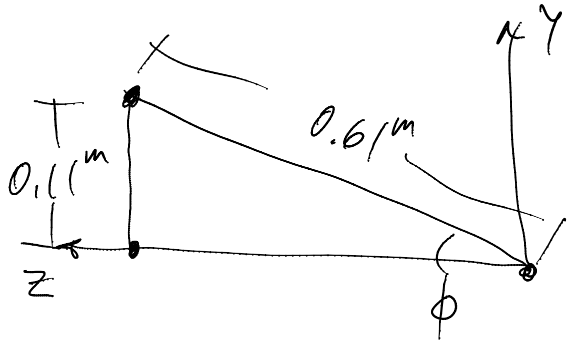


Fig. P3.47 and P3.48

$F_{ED} = 66^N$, FIND $\vec{M}_o$

LOCATE POINTS:



$$\phi = \sin^{-1}\left(\frac{0.11}{0.61}\right) = 10.38°$$

$Y_D = 0.11^m$, $X_D = 0^m$

$Z_D = 0.61 \cdot \cos 10.38° = 0.600^m$

$D(0, 0.11, 0.6)^m$, $E(0.3, 0.71, 0)^m$

$\vec{F}_{ED}$: $d_x = X_E - X_D = 0.3^m$, $d_y = 0.6^m$, $d_z = -0.6^m$

$d = 0.9^m$

$$F_x = F_{ED}\frac{d_x}{d} = (66^N)\left(\frac{0.3}{0.9}\right) = 22^N, \quad F_y = 44^N, \quad F_z = -44^N$$

$$\vec{F}_{ED} = (22)\hat{i} + (44)\hat{j} + (-44)\hat{k}\ ^N$$

$$\vec{r} = (0)\hat{i} + (0.11)\hat{j} + (0.6)\hat{k}\ ^m$$

$| \quad \hat{i} \quad \hat{j} \quad \hat{k} \quad |$

$$\vec{M}_o = \vec{r} \times \vec{T}_{DE} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.11 & 0.6 \\ 22 & 44 & -44 \end{vmatrix}$$

$$\vec{M}_o = \left[(0.11)(-44) - (0.6)(44)\right]\hat{i}$$

$$- \left[(0)(-44) - (0.6)(22)\right]\hat{j}$$

$$+ \left[(0)(44) - (0.11)(22)\right]\hat{k}$$

$$\boxed{\vec{M}_o = (-31.2)\,\hat{i} + (13.2)\,\hat{j} + (-2.42)\,\hat{k} \ \text{N}\cdot\text{m}}$$

Prob. 3.48

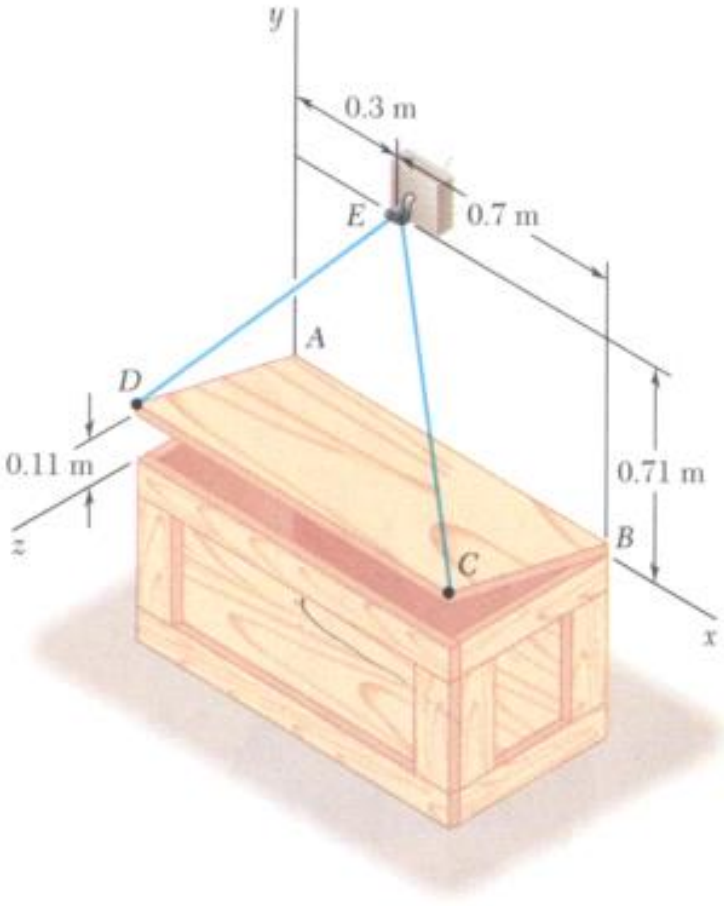


Fig. P3.47 and P3.48

3.48 The 0.61 × 1.00-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 66 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *C*.

Prob. 3.57

3.57 A sign erected on uneven ground is guyed by cables EF and EG. If the force exerted by cable EF at E is 46 lb, determine the moment of that force about the line joining points A and D.

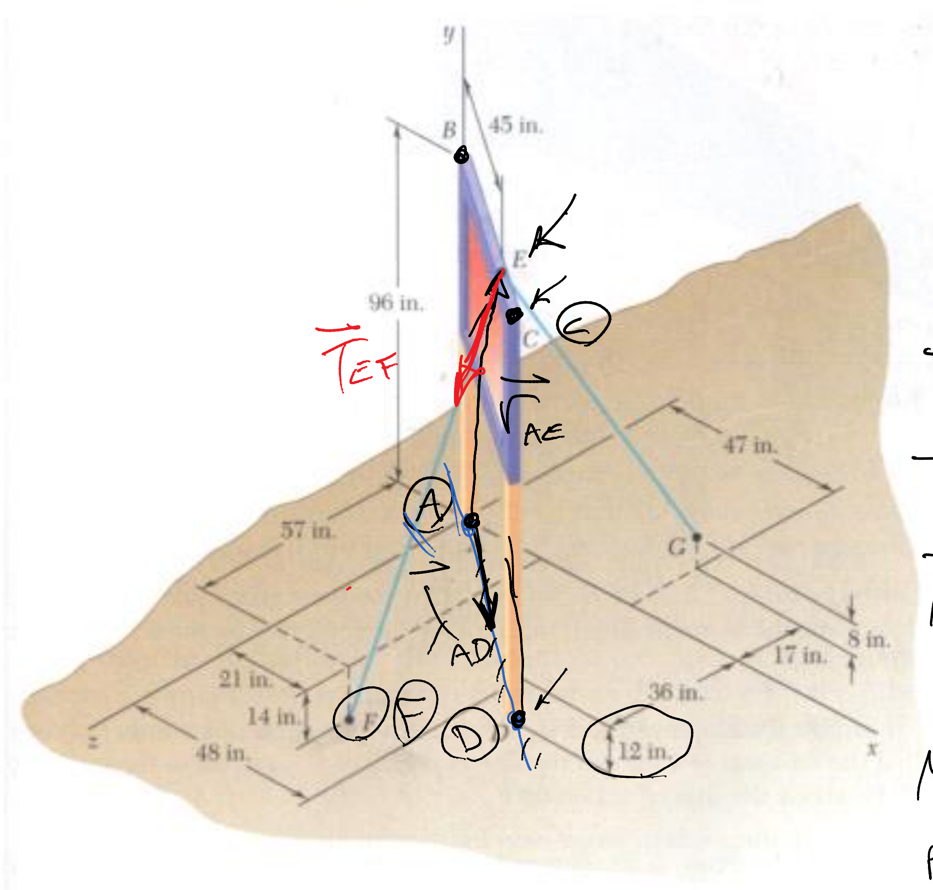


Fig. P3.57 and P3.58

FIND

$\vec{T}_{EF} = (F_x)\hat{i} + (F_y)\hat{j} + (F_z)\hat{k}$

$\vec{r}_{AE} = (r_x)\hat{i} + (r_y)\hat{j} + (r_z)\hat{k}$

$\vec{\lambda}_{AD} = (\lambda_x)\hat{i} + (\lambda_y)\hat{j} + (\lambda_z)\hat{k}$

$\vec{M}_A = \vec{r}_{AE} \times \vec{T}_{EF}$ ★

MOMENT ABOUT A

$M_{AD} = \vec{\lambda}_{AD} \cdot \vec{M}_A$ ★

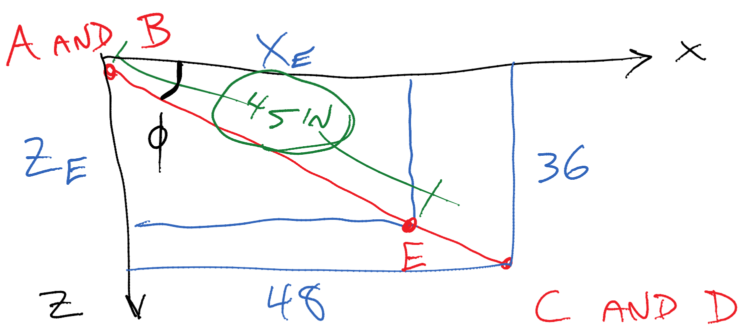
PROJECTION OF $\vec{M}_A$ ONTO AD

$\vec{M}_{AD} = M_{AD} \cdot \vec{\lambda}_{AD}$

VECTOR COMPONENT OF $\vec{M}_A$ ALONG AD

LOCATE POINTS:

$D(48, -12, 36)$



POINT: $Y_E = 96^{IN}$

$\phi = \tan^{-1}\left(\frac{36}{48}\right) = 36.9°$

$X_E = 45 \cdot \cos 36.9° = 36^{IN}$

$Z_E = 45 \cdot \sin 36.9° = 27^{IN}$

$E(36, 96, 27)^{IN}$; $F(21, -14, 57)^{IN}$

$\vec{T}_{EF}$: $d_x = X_F - X_E = 21 - 36 = -15^{IN}$

$$d_y = Y_F - Y_E = -14 - 96 = -110^{IN}$$

$$d_z = 30^{IN}$$

$$d = \sqrt{15^2 + 110^2 + 30^2} = 115^{IN}$$

$$F_x = F \cdot \frac{d_x}{d} = (46^{LB})\left(\frac{-15}{115}\right) = -6^{LB}$$

$$F_y = F \cdot \frac{d_y}{d} = (46^{LB})\left(\frac{-110}{115}\right) = -44^{LB}$$

$$F_z = F \cdot \frac{d_z}{d} = (46)\left(\frac{30}{115}\right) = 12^{LB}$$

$$\boxed{\vec{T}_{EF} = (-6)\hat{i} + (-44)\hat{j} + (12)\hat{k} \ ^{LB}}$$

$$\boxed{\vec{r}_{AE} = (36)\hat{i} + (96)\hat{j} + (27)\hat{k} \ ^{IN}}$$

$\vec{\lambda}_{AD}$: $\underline{d_x = X_D - X_A = 48^{IN}}$, $d_y = -12^{IN}$, $d_z = 36^{IN}$

$$d = \sqrt{48^2 + 12^2 + 36^2} = 61.2^{IN}$$

$$\vec{\lambda}_{AD} = \left(\frac{d_x}{d}\right)\hat{i} + \left(\frac{d_y}{d}\right)\hat{j} + \left(\frac{d_z}{d}\right)\hat{k}$$

$$\vec{\lambda}_{AD} = \left(\frac{48}{61.2}\right)\hat{i} + \left(\frac{-12}{61.2}\right)\hat{j} + \left(\frac{36}{61.2}\right)\hat{k}$$

$$\boxed{\vec{\lambda}_{AD} = (0.784)\hat{i} + (-0.196)\hat{j} + (0.588)\hat{k}}$$

$$\vec{M}_A = \vec{r}_{AE} \times \vec{T}_{EF} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 36 & 96 & 27 \\ & & 12 \end{vmatrix}$$

$$\begin{vmatrix} & & \\ -6 & -44 & 12 \end{vmatrix}$$

$$\boxed{\vec{M}_A = (2340)\hat{i} + (-594)\hat{j} + (-1008)\hat{k} \text{ IN-LB}}$$

$$M_{AD} = \vec{\lambda}_{AD} \cdot \vec{M}_A$$

$$M_{AD} = (0.784)(2340) + (-0.196)(-594) + (0.588)(-1008)$$

$$\boxed{M_{AD} = 1358 \text{ IN}\cdot\text{LB}}$$

PROJECTION

$$\vec{M}_{AD} = M_{AD} \cdot \vec{\lambda}_{AD}$$

$$\vec{M}_{AD} = (1358 \text{ IN-LB})\left[(0.784)\hat{i} + (-0.196)\hat{j} + (0.588)\hat{k}\right]$$

$$\boxed{\vec{M}_{AD} = (1065)\hat{i} + (-266)\hat{j} + (798)\hat{k} \text{ IN-LB}}$$

Prob. 3.58

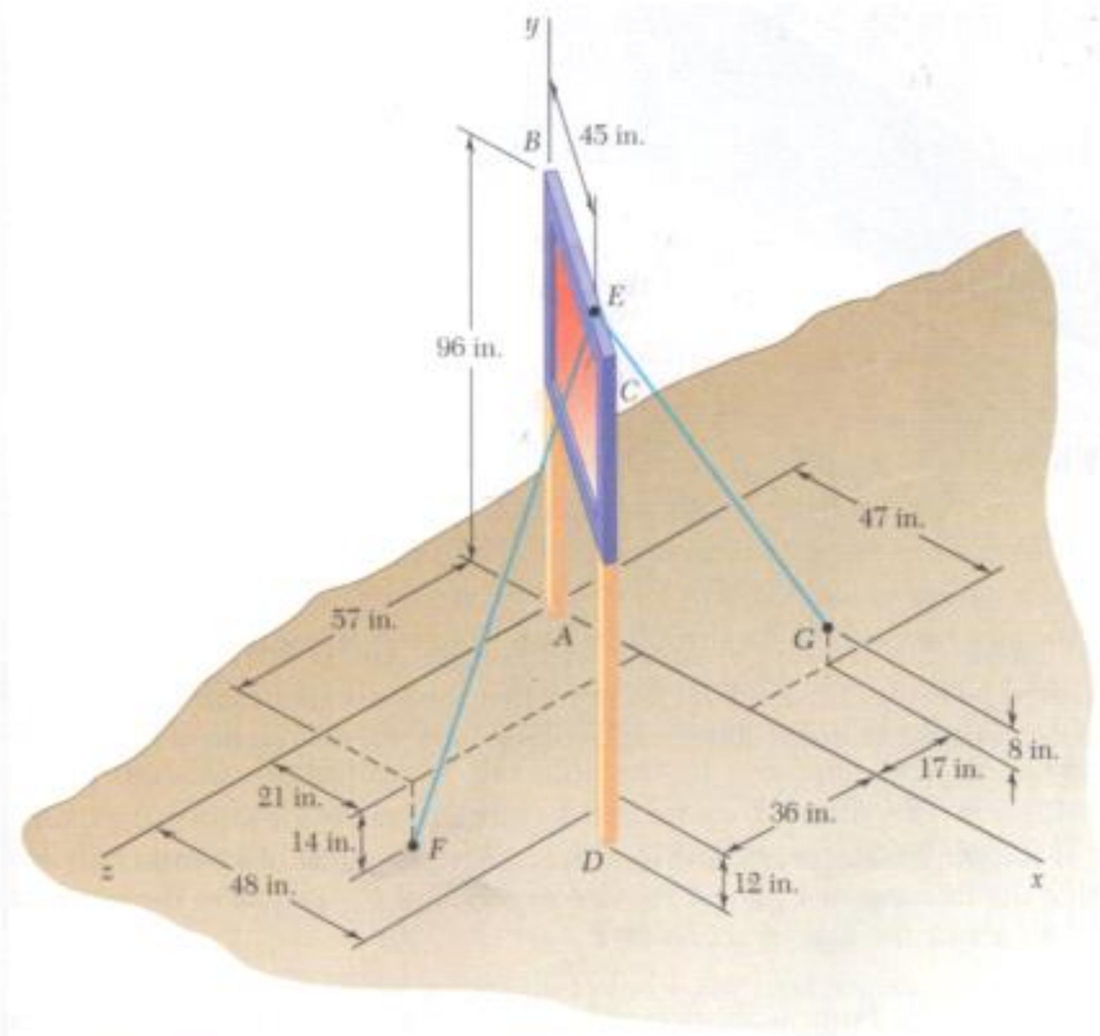


Fig. P3.57 and P3.58

3.58 A sign erected on uneven ground is guyed by cables *EF* and *EG*. If the force exerted by cable *EG* at *E* is 54 lb, determine the moment of that force about the line joining points *A* and *D*.

Prob. 3.76

3.76 If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

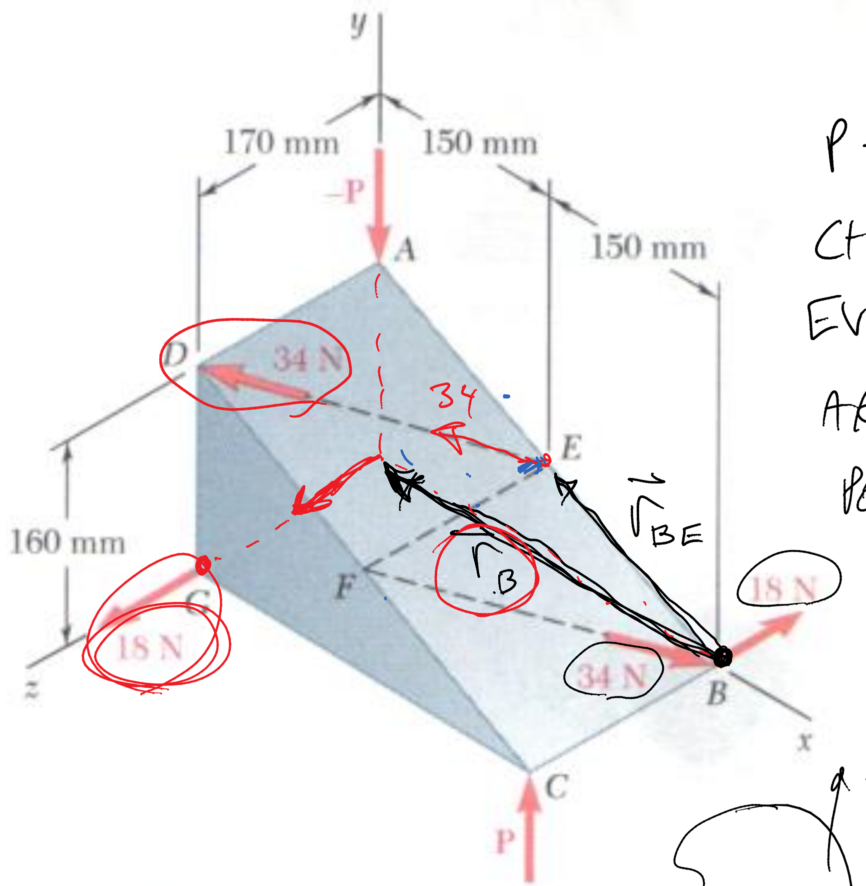


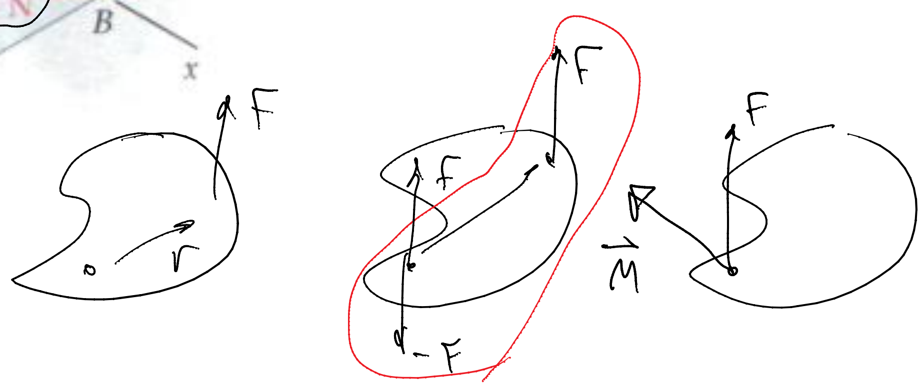
Fig. P3.76

$P = 0$, FIND $\vec{M}$

CHOICE OF POINT TO EVALUATE $\vec{M}$ IS ARBITRARY. CHOOSE POINT B TO SIMPLIFY.

$\vec{r}_B$: $dx = x_G - x_B$

$dx = 0 - 300 = -300$



$$\vec{M}_B = \vec{r}_B \times \vec{F}_G + \vec{r}_{BE} \times \vec{F}_E$$

LOCATE POINTS: $B(300, 0, 0)^{mm}$, $E(150, 80, 0)^{mm}$,

$D(0, 160, 170)^{mm}$

$$\boxed{\vec{r}_B = (-300)\,\hat{i}\ mm}$$

$\vec{r}_{BE}$: $dx = x_E - x_B = 150 - 300 = -150^{mm}$

$d_y = 80^{mm}, \quad d_z = 0$

$$\boxed{\vec{r}_{BE} = (-150)\hat{i} + (80)\hat{j}}$$

$$\boxed{\vec{F}_G = (18)\hat{k}^{\,N}}$$

$\vec{F}_E$: $d_x = X_D - X_E = 0 - 150 = -150^{mm}$

$d_y = 80^{mm}, \quad d_z = 170^{mm}, \quad d = 240.4^{mm}$

$$F_x = F \cdot \frac{d_x}{d} = (34^N)\left(\frac{-150}{240.4}\right) = -21.2^N$$

$$F_y = F \cdot \frac{d_y}{d} = (34)\left(\frac{80}{240.4}\right) = 11.3^N, \quad F_z = 24.0^N$$

$$\boxed{\vec{F}_E = (-21.2)\hat{i} + (11.3)\hat{j} + (24.0)\hat{k}^{\,N}}$$

$$\vec{r}_B \times \vec{F}_G = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -300 & 0 & 0 \\ 0 & 0 & 18 \end{vmatrix}$$

$$\boxed{= (5400)\hat{j} \ N \cdot mm}$$

$$\vec{r}_E \times \vec{F}_E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -150 & 80 & 0 \\ -21.2 & 11.3 & 24 \end{vmatrix}$$

$$\boxed{= (1920)\hat{i} + (3600)\hat{j} + (0)\hat{k} \ N \cdot mm}$$

1

$$\boxed{= (1920)\hat{i} + (3600)\hat{j} + (0)\hat{k} \quad N\cdot mm}$$

$$\vec{M}_B = (1920)\hat{i} + (5400 + 3600)\hat{j}$$

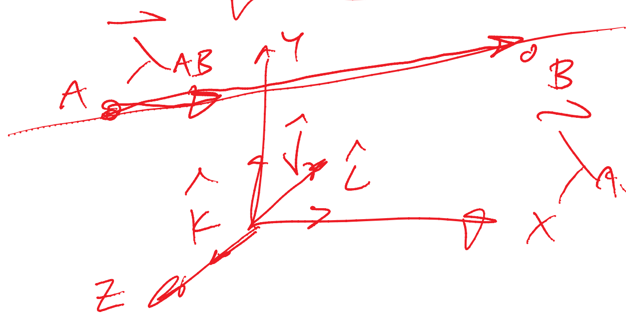
$$\boxed{\vec{M}_B = (1920)\hat{i} + (9000)\hat{j} \quad N\cdot mm}$$

$$|\vec{M}_B| = \sqrt{1920^2 + 9000^2} = 9202 \; N\cdot mm$$

$$\theta_x = \cos^{-1}\left(\frac{1920}{9202}\right) = \boxed{77.9^\circ}, \quad \theta_y \boxed{= 12.0^\circ}, \quad \theta_z = \cos^{-1}\left(\frac{0}{9202}\right)$$

$$\boxed{\theta_z = 90^\circ}$$

$\vec{\lambda}$: UNIT VECTOR



$$\vec{\lambda}_{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

Prob. 3.77

3.77 If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

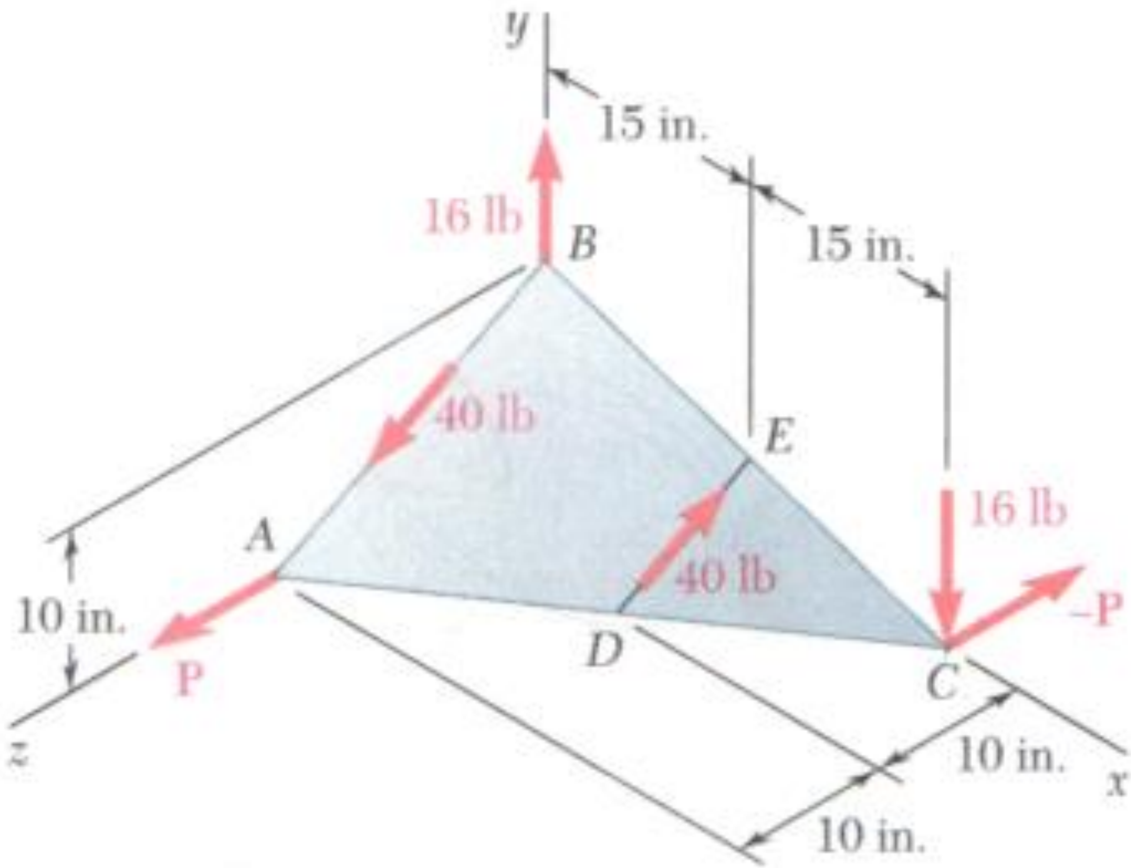


Fig. P3.77

Prob. 3.96

3.96 To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at B a 175-N force directed along line AB. Replace that force with an equivalent force-couple system at C.

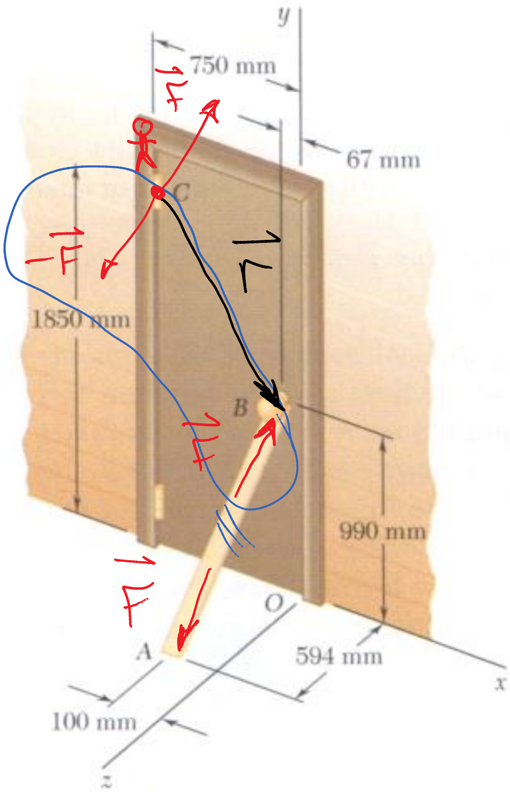


Fig. P3.96

$\vec{M}_A = \vec{r} \times \vec{F}$

FIND POINTS: $A(-100, 0, 594)^{mm}$

$B(-67, 990, 0)^{mm}$, $C(-750, 1850, 0)^{mm}$

$\vec{F}$: $|\vec{F}| = 175^N$

$d_x = X_B - X_A = (-67) - (-100) = 33^{mm}$

$d_y = Y_B - Y_A = 990 - 0 = 990^{mm}$

$d_z = Z_B - Z_A = 0 - 594 = -594^{mm}$

$d = \sqrt{33^2 + 990^2 + 594^2} = 1155^{mm}$

$F_x = F \cdot \frac{d_x}{d} = (175^N)\left(\frac{33}{1155}\right) = 5.00^N$

$F_y = F \cdot \frac{d_y}{d} = (175)\left(\frac{990}{1155}\right) = 150^N$

$F_z = F \frac{d_z}{d} = (175)\left(\frac{-594}{1155}\right) = -90^N$

$\boxed{\vec{F} = (5)\hat{i} + (150)\hat{j} + (-90)\hat{k}^{\,N}}$

$\vec{r} = (d_x)\hat{i} + (d_y)\hat{j} + (d_z)\hat{k}$

$d_x = X_B - X_C = (-67) - (-750) = 683^{mm}$

$d_y = Y_B - Y_C = 990 - 1850 = -860^{mm}$

$$d_z = z_B - z_C = 0 - 0 = 0$$

$$\boxed{\vec{r} = (683)\hat{i} + (-860)\hat{j} + (0)\hat{k} \text{ mm}}$$

$$\vec{M}_c = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 683 & -860 & 0 \\ 5 & 150 & -90 \end{vmatrix}$$

$$\vec{M}_c = [(-860)(-90) - (0)(150)]\hat{i}$$

$$- [(683)(-90) - (0)(5)]\hat{j}$$

$$+ [(683)(150) - (-860)(5)]\hat{k} \text{ N}\cdot\text{mm}$$

$$\vec{M}_c = \left[(77400)\hat{i} + (61470)\hat{j} + (106,750)\hat{k} \text{ N}\cdot\text{mm}\right]\left(\frac{\text{m}}{1000\text{ mm}}\right)$$

$$\boxed{\vec{M}_c = (77.4)\hat{i} + (61.5)\hat{j} + (106.7)\hat{k} \text{ N}\cdot\text{m}}$$

Prob. 3.97

3.97 A 110-N force acting in a vertical plane parallel to the yz plane is applied to the 220-mm-long horizontal handle AB of a socket wrench. Replace the force with an equivalent force-couple system at the origin O of the coordinate system.

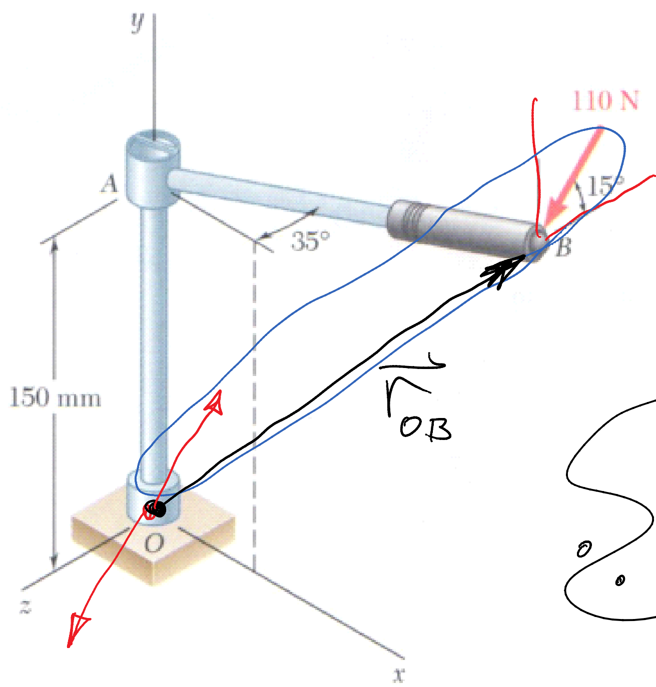
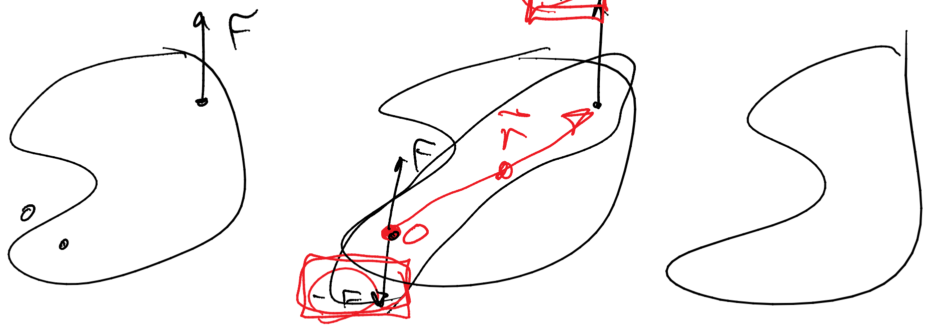


Fig. P3.97

$$\vec{F} = (-110 \cdot \sin 15^\circ)\hat{j} + (110 \cdot \cos 15^\circ)\hat{k}$$

$$\vec{M}_o = \vec{r}_{OB} \times \vec{F}$$



Prob. 3.119

3.119 As plastic bushings are inserted into a 60-mm-diameter cylindrical sheet metal enclosure, the insertion tools exert the forces shown on the enclosure. Each of the forces is parallel to one of the coordinate axes. Replace these forces with an equivalent force-couple system at C.

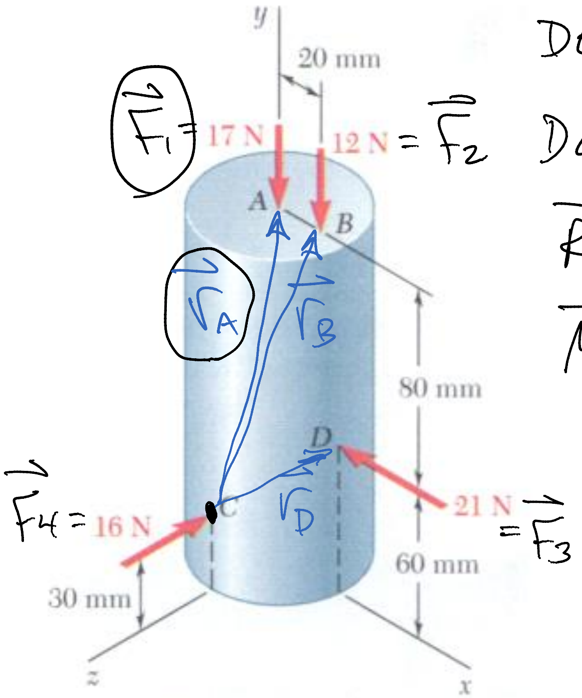


Fig. P3.119

DEFINE FORCES $(\hat{i}, \hat{j}, \hat{k})$

DEFINE POSITION VECTORS $(\hat{i}, \hat{j}, \hat{k})$

$$\vec{R} = \sum \vec{F}$$

$$\vec{M}_C = \sum (\vec{r}_i \times \vec{F}_i)$$

$$\vec{F}_1 = (-17)\hat{j} \text{ N} \qquad \vec{F}_4 = (-16)\hat{k} \text{ N}$$

$$\vec{F}_2 = (-12)\hat{j} \text{ N}$$

$$\vec{F}_3 = (-21)\hat{i}$$

$$\boxed{\vec{R} = (-21)\hat{i} + (-29)\hat{j} + (-16)\hat{k} \text{ N}}$$

$\vec{r}_{CA}$:

$$dx = X_A - X_C = 0 - 0 = 0$$

$$dy = Y_A - Y_C = 140 - 30 = 110 \text{ mm}$$

$$dz = Z_A - Z_C = 0 - 30 = -30 \text{ mm}$$

$$\boxed{\vec{r}_{CA} = (0)\hat{i} + (110)\hat{j} + (-30)\hat{k} \text{ mm}}$$

$$\boxed{\vec{r}_{CB} = (20)\hat{i} + (110)\hat{j} + (-30)\hat{k} \text{ mm}}$$

$$\boxed{\vec{r}_{CD} = (30)\hat{i} + (30)\hat{j} + (-30)\hat{k} \text{ mm}}$$

$$\left| \quad \hat{i} \quad \hat{j} \quad \hat{k} \quad \right|$$

$$\vec{M}_1 = \vec{r}_{CA} \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 110 & -30 \\ 0 & -17 & 0 \end{vmatrix}$$

$$\vec{M}_1 = (-510)\,\hat{i} \quad N \cdot mm$$

$$\vec{M}_2 = (-360)\,\hat{i} + (-240)\,\hat{k} \quad N \cdot mm$$

$$\vec{M}_3 = (0)\,\hat{i} + (630)\,\hat{j} + (630)\,\hat{k} \quad N \cdot mm$$

$$\boxed{\vec{M}_C = (-870)\,\hat{i} + (630)\,\hat{j} + (390)\,\hat{k} \quad N \cdot mm}$$

Prob. 3.120

3.120 Two 150-mm-diameter pulleys are mounted on line shaft AD. The belts at B and C lie in vertical planes parallel to the yz plane. Replace the belt forces shown with an equivalent force-couple system at A.

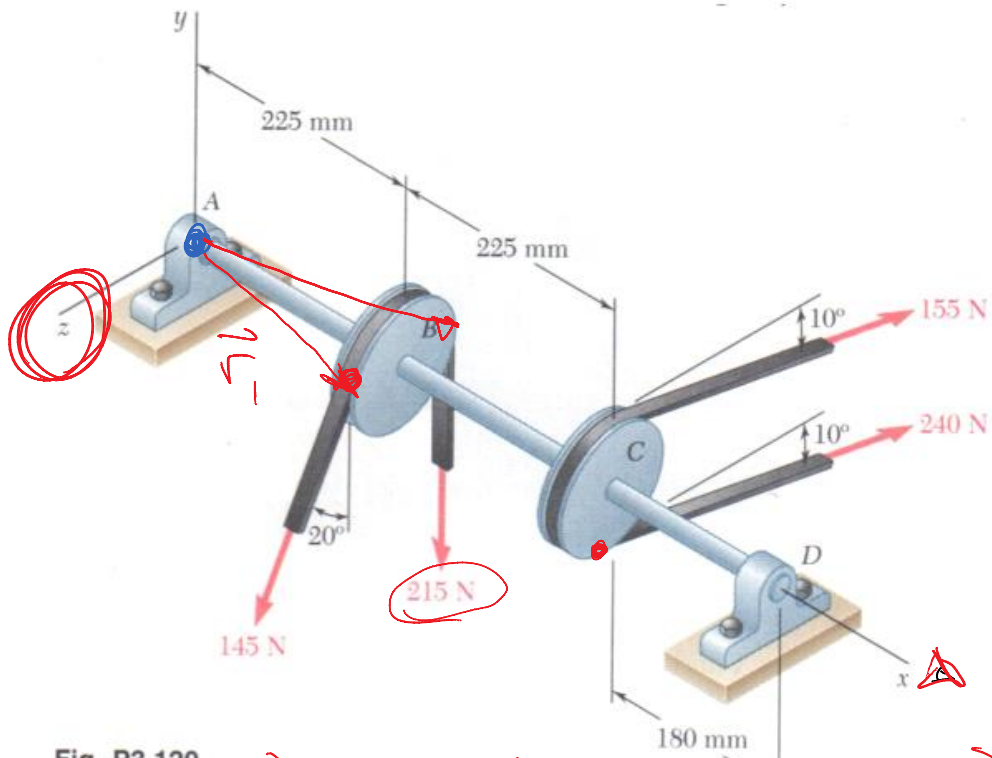
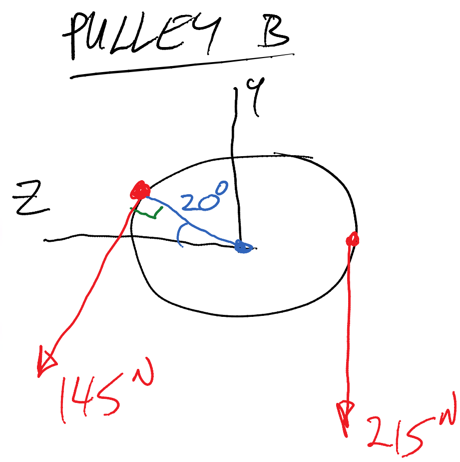


Fig. P3.120



$$\vec{r}_1 = (225)\hat{i} + (75 \cdot \sin 20^\circ)\hat{j} + (75 \cdot \cos 20^\circ)\hat{k} \quad \text{mm}$$

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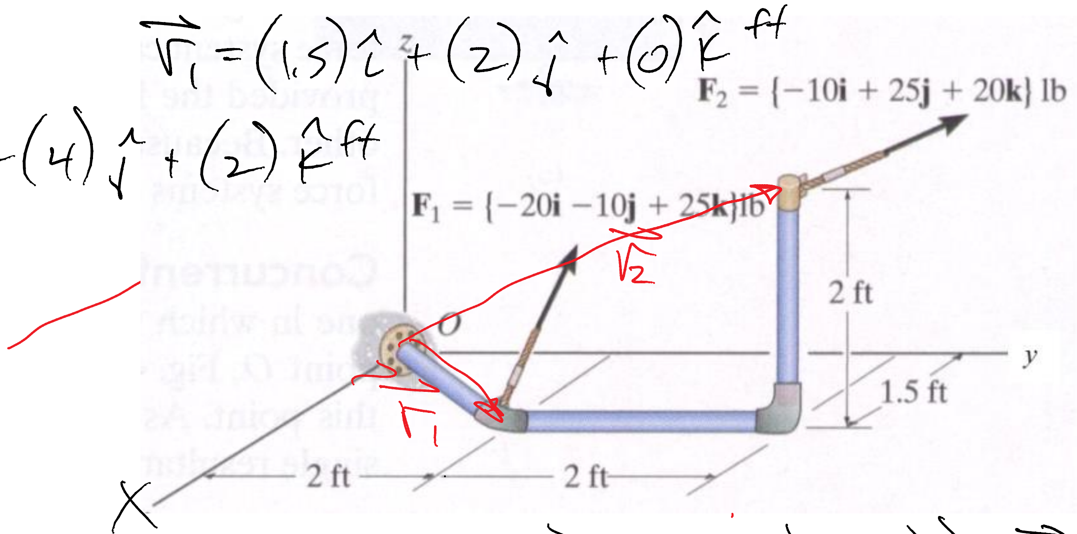
$\vec{r}_1 = (1.5)\hat{i} + (2)\hat{j} + (0)\hat{k}$ ft

$\vec{r}_2 = (1.5)\hat{i} + (4)\hat{j} + (2)\hat{k}$ ft



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$\vec{M}_o = \sum (\vec{r}_i \times \vec{F}_i) \qquad \vec{R} = \sum \vec{F}_i$

$$\underset{r}{\vec{M}_1 = \vec{r}_1} \times \underset{r}{\vec{F}_1} = \begin{array}{c} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{vmatrix} \end{array} \begin{array}{l} \\ \rightarrow r \\ \rightarrow F \end{array}$$

$$\vec{M}_2 = \vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5 & 4 & 2 \\ -10 & 25 & 20 \end{vmatrix}$$

$$\vec{P} \times \vec{Q} = (\qquad) \times (\qquad)$$

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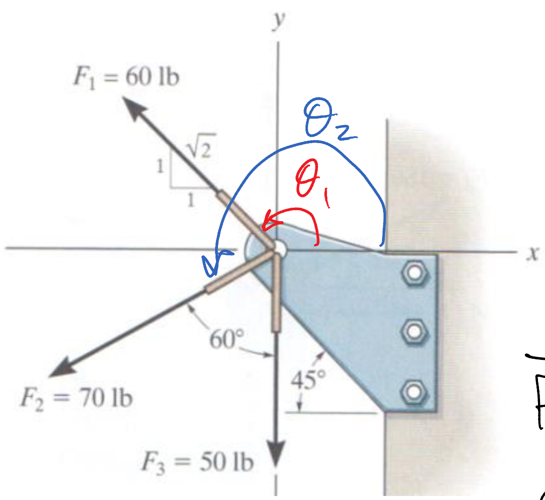
1. (25 points) Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



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$\vec{F}_1 = (F_1 \cdot \cos\theta_1)\hat{i} + (F_1 \cdot \sin\theta_1)\hat{j}$

$\theta_1 = 90 + 45° = 135°$

$\vec{F}_1 = (60 \cdot \cos 135°)\hat{i} + (60 \cdot \sin 135°)\hat{j}$ LB

$\vec{F}_2 = (F_2 \cdot \cos\theta_2)\hat{i} + (F_2 \cdot \sin\theta_2)\hat{j}$

$\theta_2 = 180 + 30 = 210°$

$\vec{F}_2 = (70 \cdot \cos 210°)\hat{i} + (70 \cdot \sin 210°)\hat{j}$ LB

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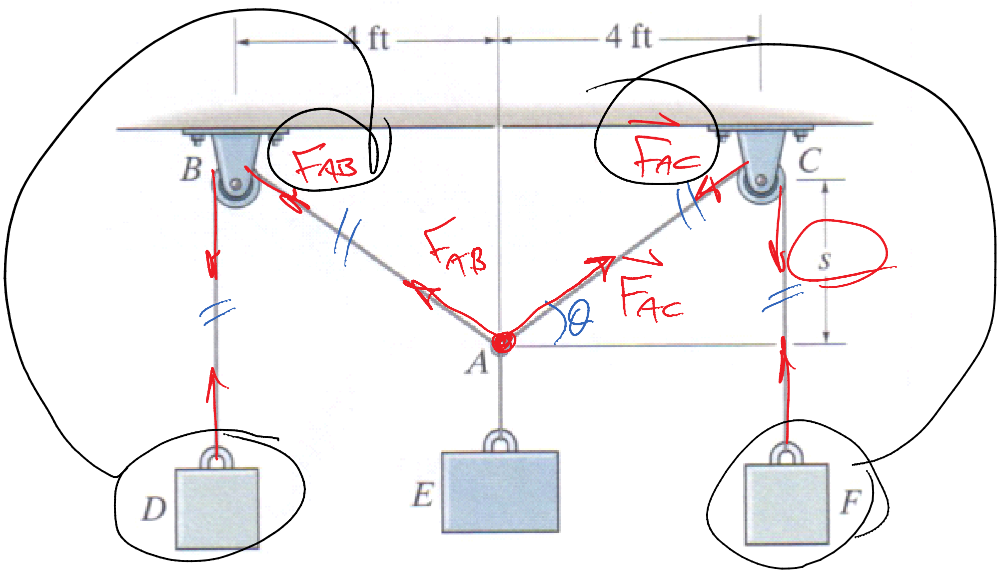
2. (15 points) Blocks D and F weigh 5 lb each and block E weighs 8 lb. Determine the sag s for equilibrium. Neglect the size of the pulleys.

$$|\vec{F}_{AB}| = |\vec{F}_{AC}| = 5^{LB}$$

$$\theta = TAN^{-1}\left(\frac{5}{4}\right)$$



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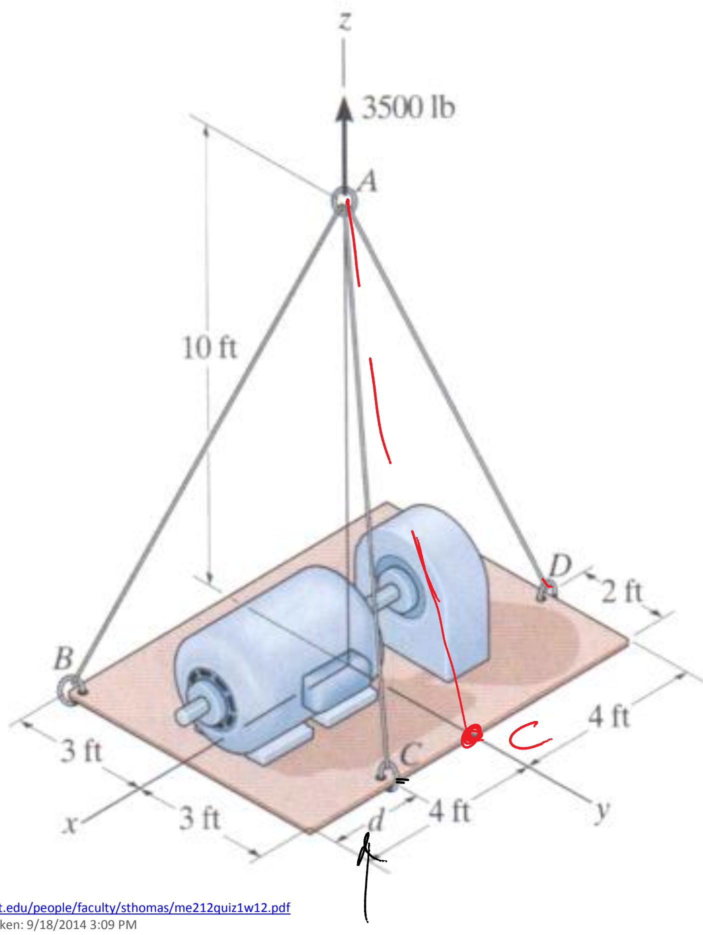
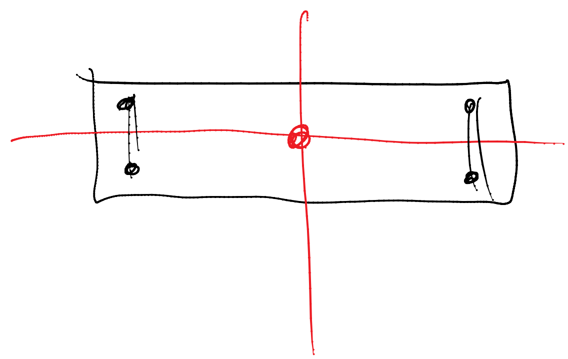
3. (60 points) Determine the force in each cable needed to support the 3500-lb platform. Set $d = 4$ ft.





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