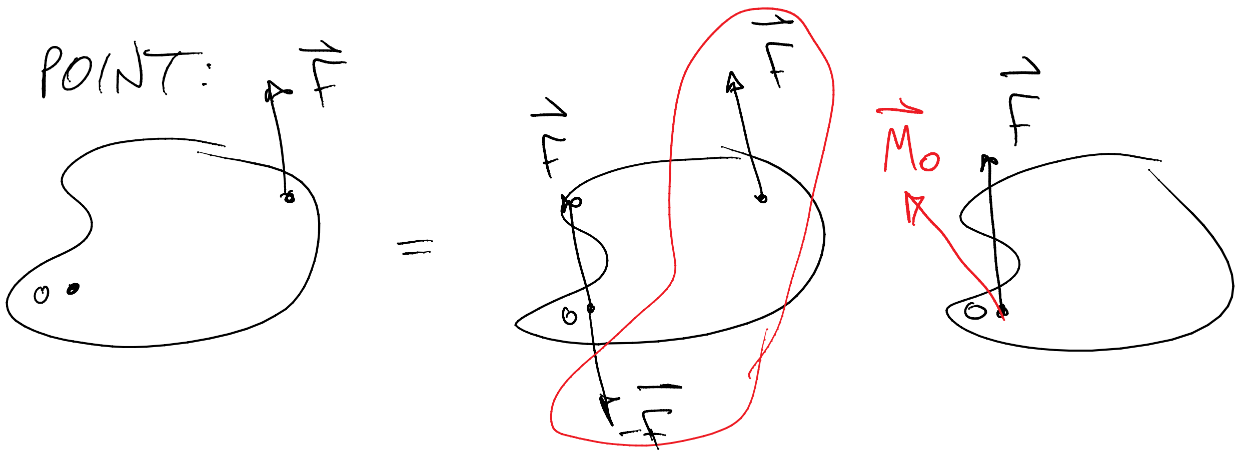
Chapter 4

EQUILIBRIUM OF RIGID BODIES

SO FAR, WE HAVE LEARNED HOW TO REPLACE A FORCE (OR FORCES) ON A RIGID BODY WITH A FORCE-COUPLE SYSTEM AT A POINT:



FOR THE BODY TO BE IN EQUILIBRIUM,

$$\Sigma \vec{F} = 0, \quad \Sigma \vec{M}_O = 0$$

RESOLVING THE VECTOR EQUATIONS INTO COMPONENTS:

$$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$$

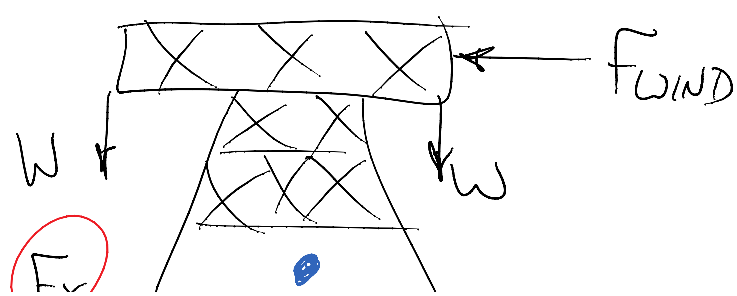
$$\Sigma M_x = 0, \quad \Sigma M_y = 0, \quad \Sigma M_z = 0$$

} SIX EQUATIONS, AT MOST SIX UNKNOWNS

FOR OUR 2-D TRUSS

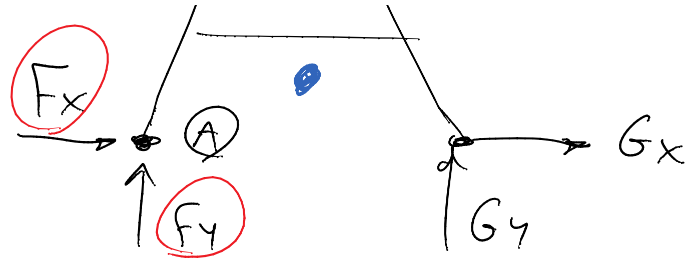
$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$



$$\Sigma F_y = 0$$

$$\Sigma M_A = 0$$



FREE-BODY DIAGRAMS

ANALYSIS REQUIRES THAT WE "FREE" THE BODY FROM ITS RESTRAINTS. THIS DIAGRAM SHOWS ALL FORCES, MOMENTS AND DIMENSIONS.

Prob. 4.1

4.1 Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

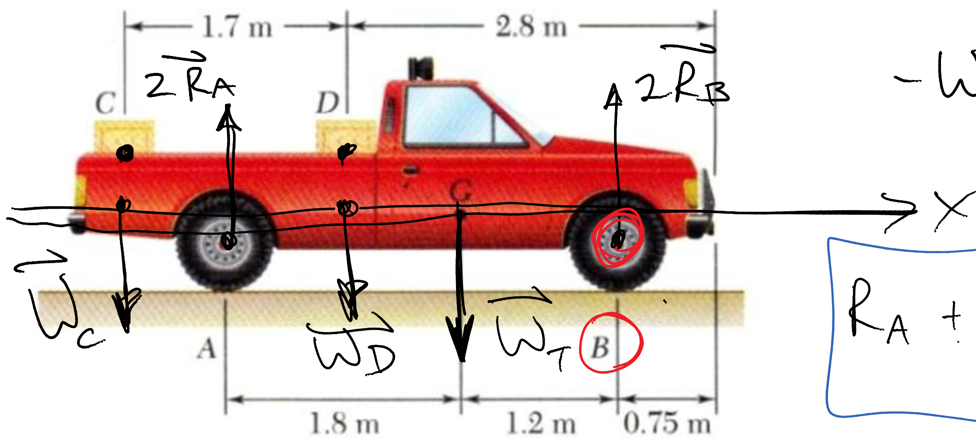


Fig. P4.1

$$\Sigma F_y = 0$$

$$-W_C + 2R_A - W_D - W_T + 2R_B = 0$$

$$R_A + R_B = \frac{1}{2}(W_C + W_D + W_T) \quad \text{EQN. (1)}$$

$$\Sigma M_B = 0 \; +\circlearrowleft :$$

$$(1.2^m)(W_T) + (2.05^m)(W_D) - (3.0^m)(2R_A) + (3.75^m)(W_C) = 0$$

$$R_A = 0.2W_T + 0.342W_D + 0.625W_C \quad \text{EQN. (2)}$$

$$W_C = W_D = mg = (350^{kg})(9.81 \tfrac{m}{s^2}) = 3430^N$$

$$W_T = (1400^{kg})(9.81 \tfrac{m}{s^2}) = 1.37 \times 10^{4\,N}$$

$$\boxed{R_A} = (0.2)(1.37\times10^4) + 0.342(3430) + 0.625(3430) = \boxed{6060^N}$$

$$R_A = (6060^N)\left(\frac{LB}{4.45N}\right) = 1361^{LB}$$

EQN. (1):

$$\boxed{R_B} = \frac{1}{2}(3430 + 3430 + 1.37\times10^4) - 6060 = \boxed{4220^N}$$

Prob. 4.3

4.3 A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

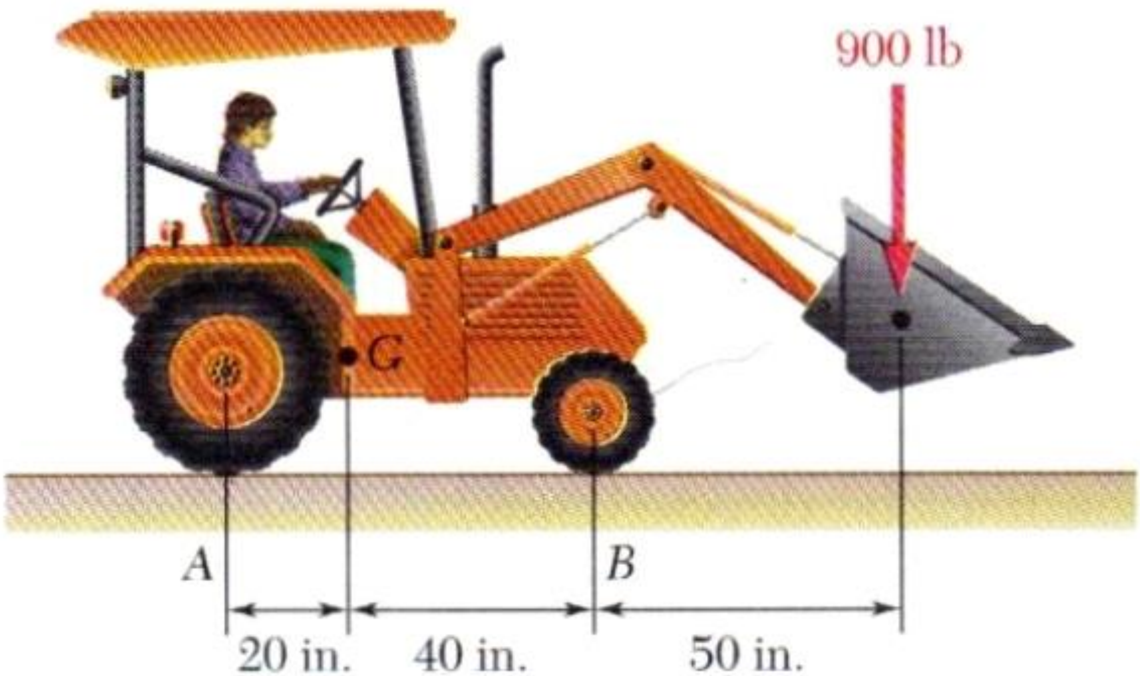


Fig. P4.3

Prob. 4.4

4.4 For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

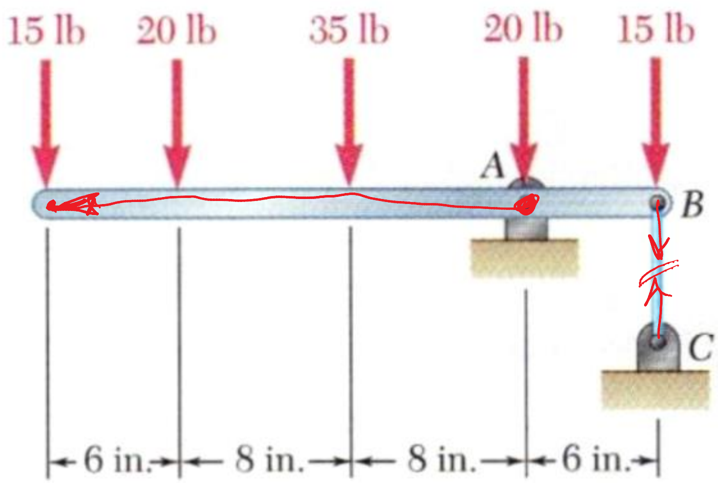
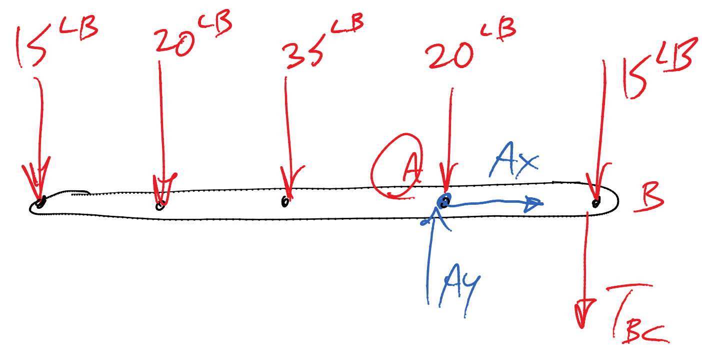
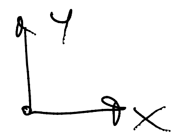


Fig. P4.4



$$\sum F_x = 0$$



$$A_x = 0$$

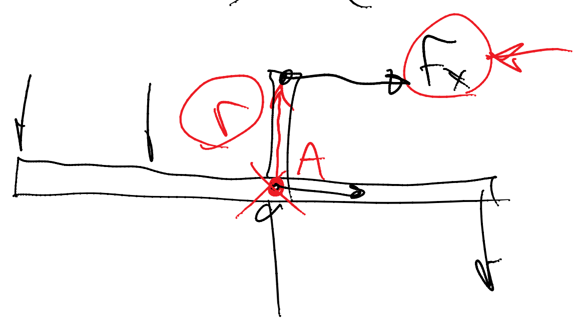
$$\sum F_y = 0 : -15 - 20 - 35 - 20 - 15 + A_y - T_{BC} = 0$$

$$\boxed{A_y = 105 + T_{BC}}$$

$$\sum M_A = 0 \;\; +\circlearrowleft : \;\hat{k}\text{-DIRECTION}$$

$$+(22^{IN})(15^{LB}) + (16^{IN})(20^{LB}) + (8^{IN})(35^{LB}) - (6^{IN})(15^{LB})$$

$$-(6^{IN})T_{BC} = 0$$



$$\boxed{T_{BC} = 140^{LB}}$$

$$\boxed{A_y = 105 + 140 = 245^{LB}}$$

Prob. 4.7

4.7 A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (*a*) the vertical force **P** which should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

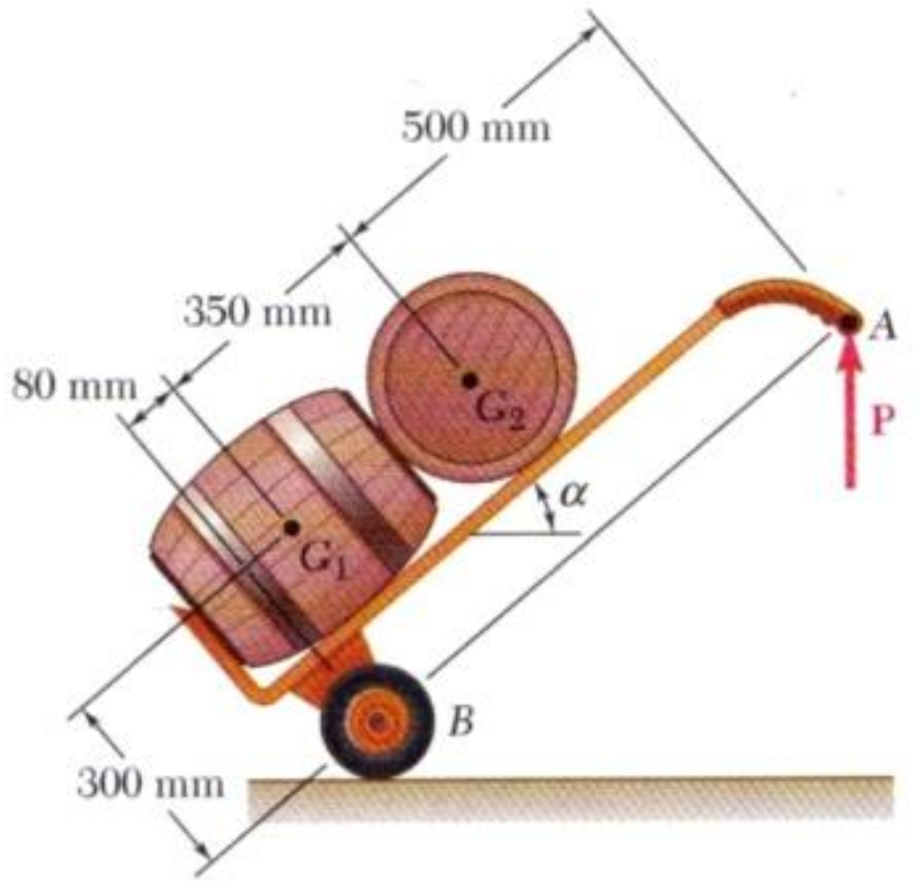


Fig. P4.7

Prob. 4.12

4.12 For the beam and loading shown, determine the range of the distance *a* for which the reaction at *B* does not exceed 100 lb downward or 200 lb upward.

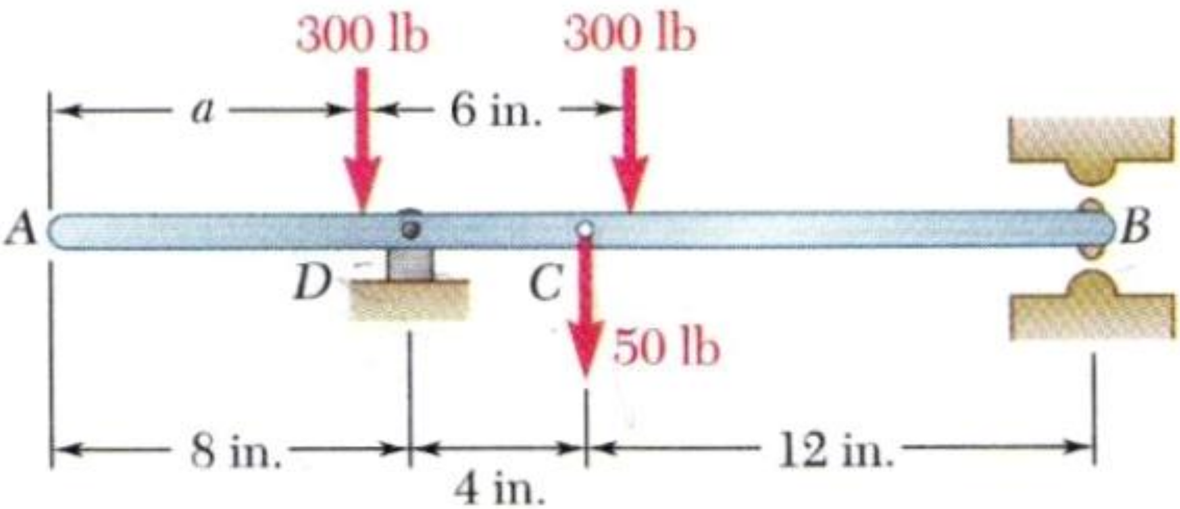


Fig. P4.12

Prob. 4.15

4.15 The bracket BCD is hinged at C and attached to a control cable at B. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at C.

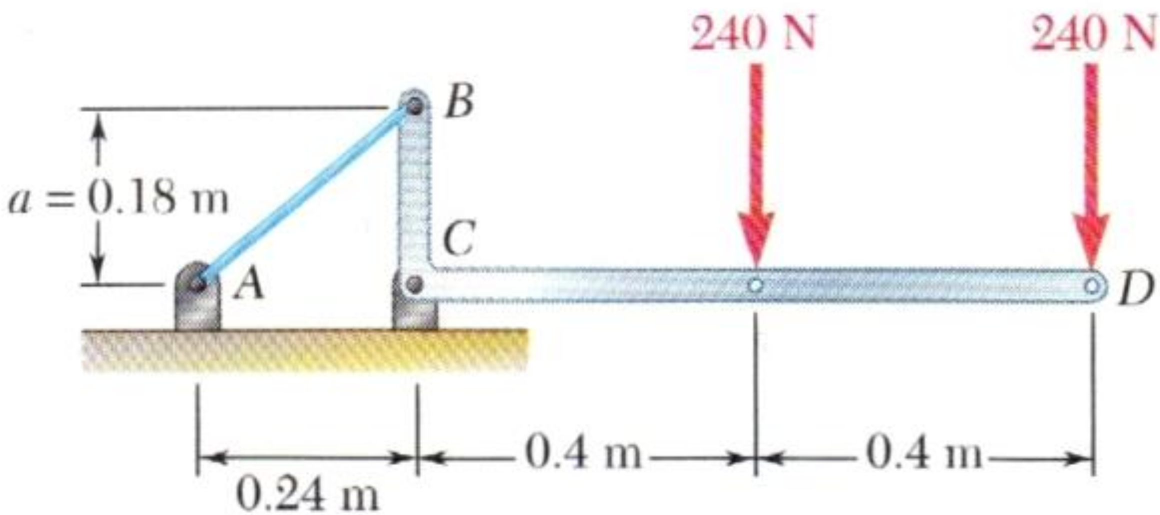


Fig. P4.15

Prob. 4.16

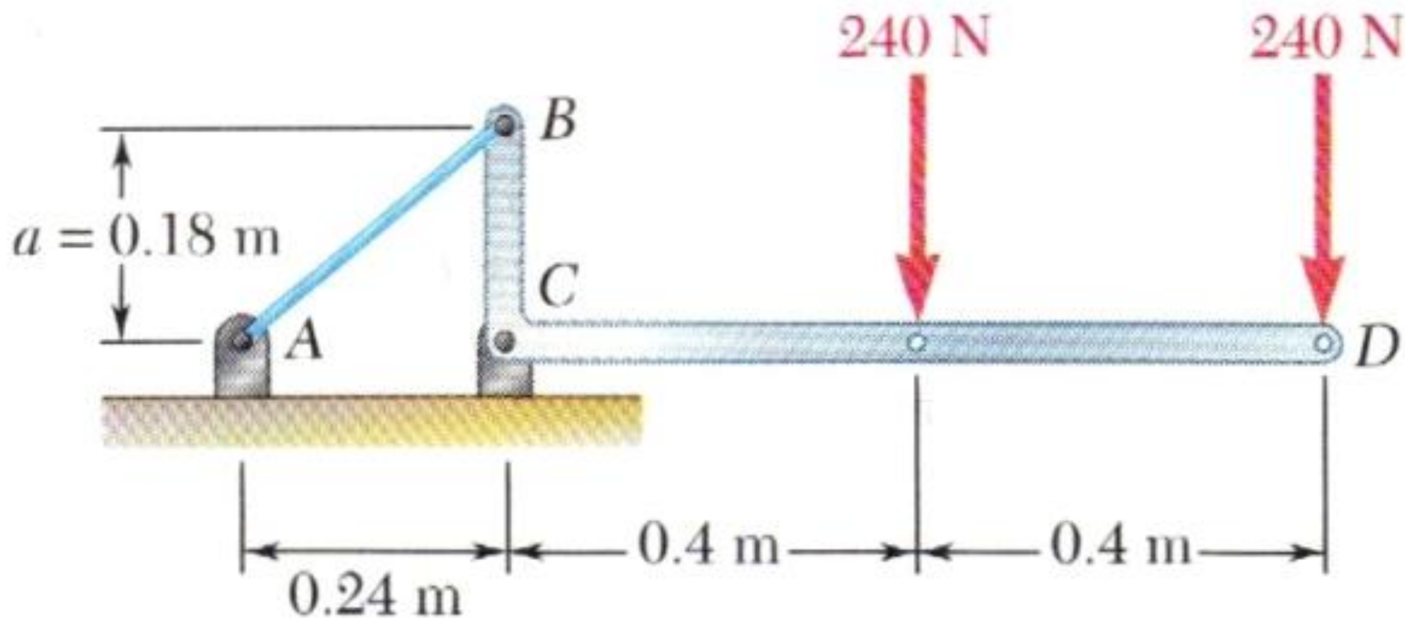


Fig. P4.15

4.16 Solve Prob. 4.15, assuming that $a = 0.32$ m.

Prob. 4.17

4.17 The required tension in cable AB is 200 lb. Determine (*a*) the vertical force **P** which must be applied to the pedal, (*b*) the corresponding reaction at C.

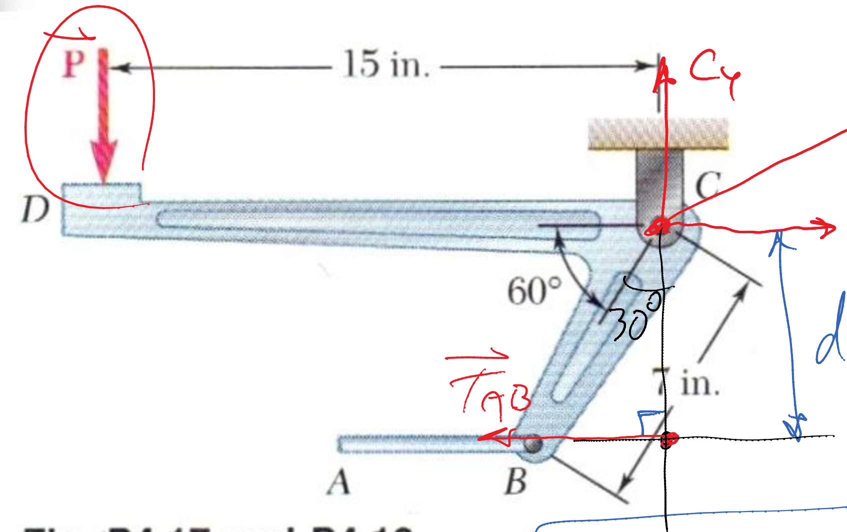


Fig. P4.17 and P4.18

PINNED CONNECTION

$$\sum M_c = 0 \circlearrowleft + :$$

$$\cos 30° = \frac{d}{7.0}$$

$$d = 7 \cdot \cos 30° = 6.06^{IN}$$

$$+(15^{IN})P - (6.06^{IN})(200^{LB}) = 0$$

$$P = 80.8^{LB}$$

$$\sum F_x = 0: \quad C_x - 200^{LB} = 0$$

$$C_x = 200^{LB}$$

$$\sum F_y = 0: \quad C_y - 80.8 = 0$$

$$C_y = 80.8^{LB}$$

$$\vec{C} = (200)\hat{i} + (80.8)\hat{j}^{\,LB}$$

$$\theta = \tan^{-1}\left(\frac{80.8}{200}\right) = 22.0°$$

Prob. 4.20

4.20 The lever *BCD* is hinged at *C* and attached to a control rod at *B*. Determine the maximum force **P** which can be safely applied at *D* if the maximum allowable value of the reaction at *C* is 1000 N.

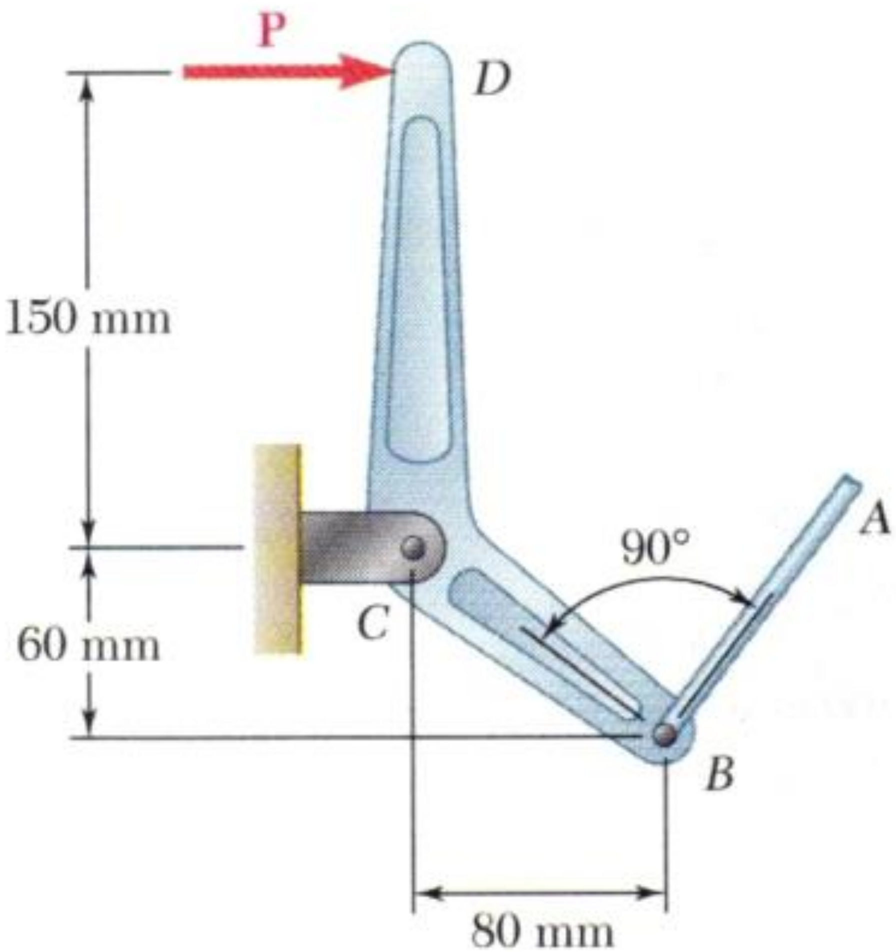


Fig. ***P4.19*** **and P4.20**

Prob. 4.29

4.29 Neglecting friction, determine the tension in cable *ABD* and the reaction at support *C*.

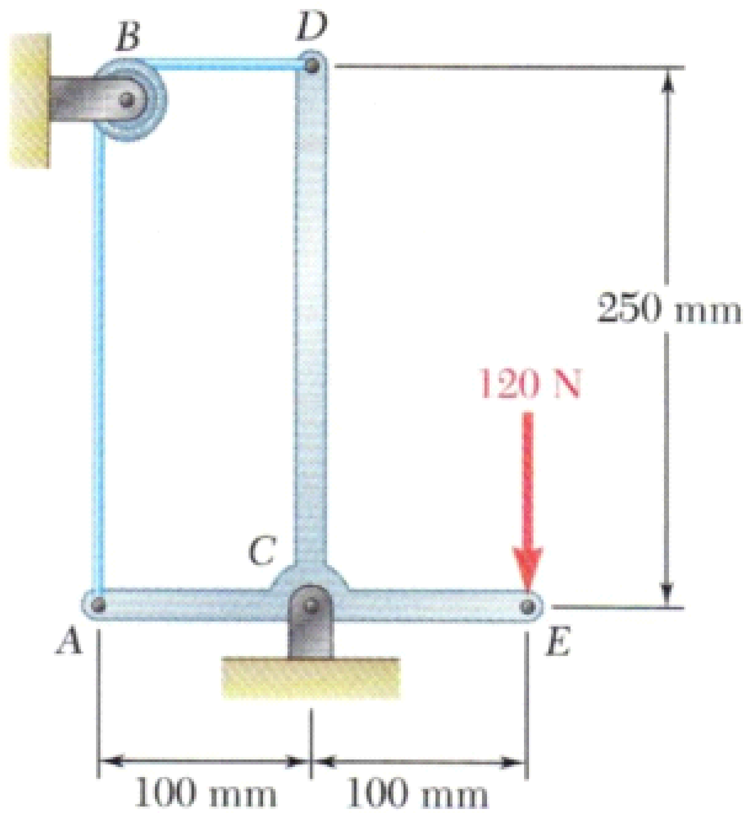


Fig. P4.29

Prob. 4.30

4.30 Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ADB*, (*b*) the reaction at *C*.

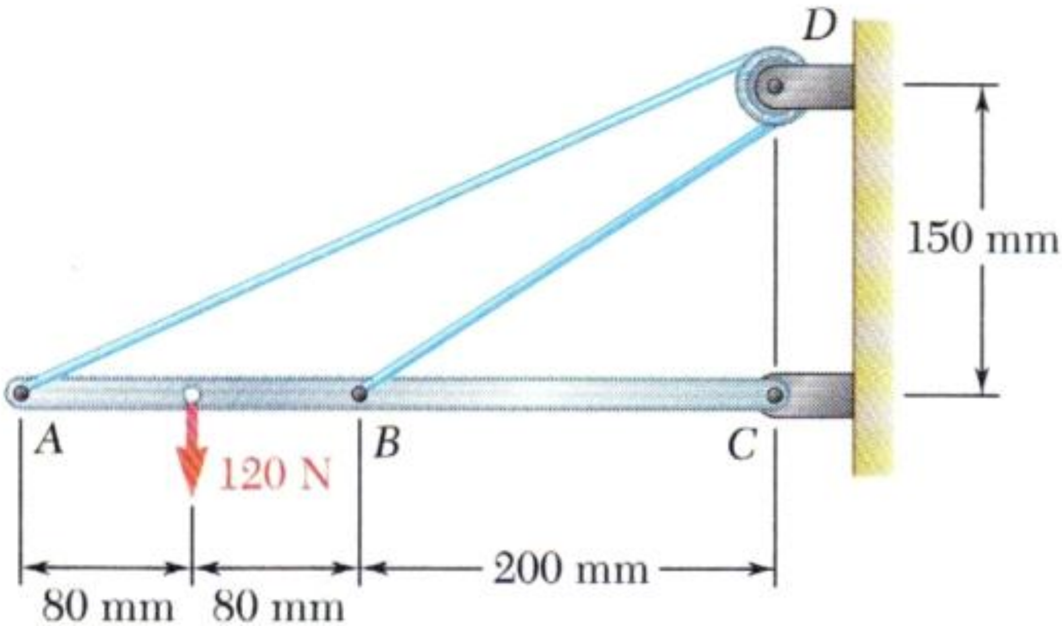


Fig. P4.30

Prob. 4.52

4.52 Rod *AB* is acted upon by a couple **M** and two forces, each of magnitude *P*. (*a*) Derive an equation in θ, P, M, and l which must be satisfied when the rod is in equilibrium. (*b*) Determine the value of θ corresponding to equilibrium when $M = 150\ \text{N}\cdot\text{m}$, $P = 200$ N, and $l = 600$ mm.

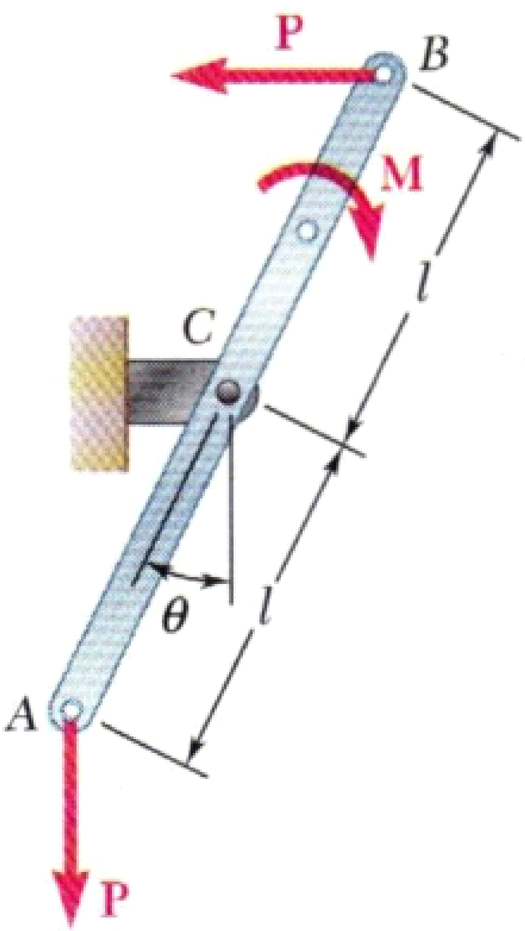


Fig. P4.52

Prob. 4.53

4.53 A tension Q is maintained in the cord shown as it passes over pulleys of diameter d. (*a*) Neglecting the weight of the rod and pulleys, express the magnitude of the force **P** corresponding to equilibrium in terms of Q, a, d, and θ. (*b*) Knowing that $Q = 10$ lb, $a = 5$ in., $d = 0.8$ in., and $\theta = 30°$, determine the magnitude P.

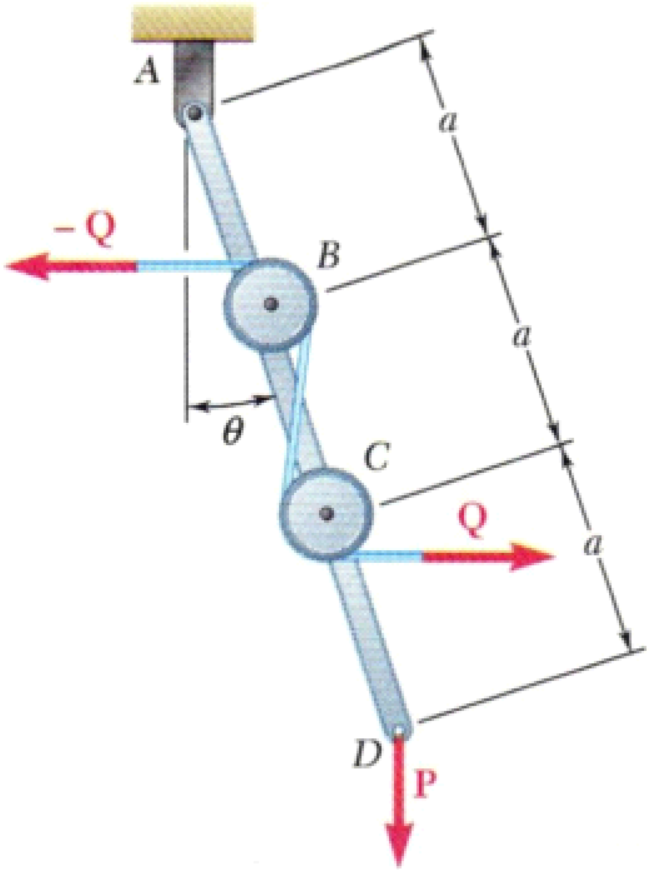


Fig. P4.53

Prob. 4.77

4.77 and 4.78 Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D. The tension may be assumed to be the same in portions AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B.

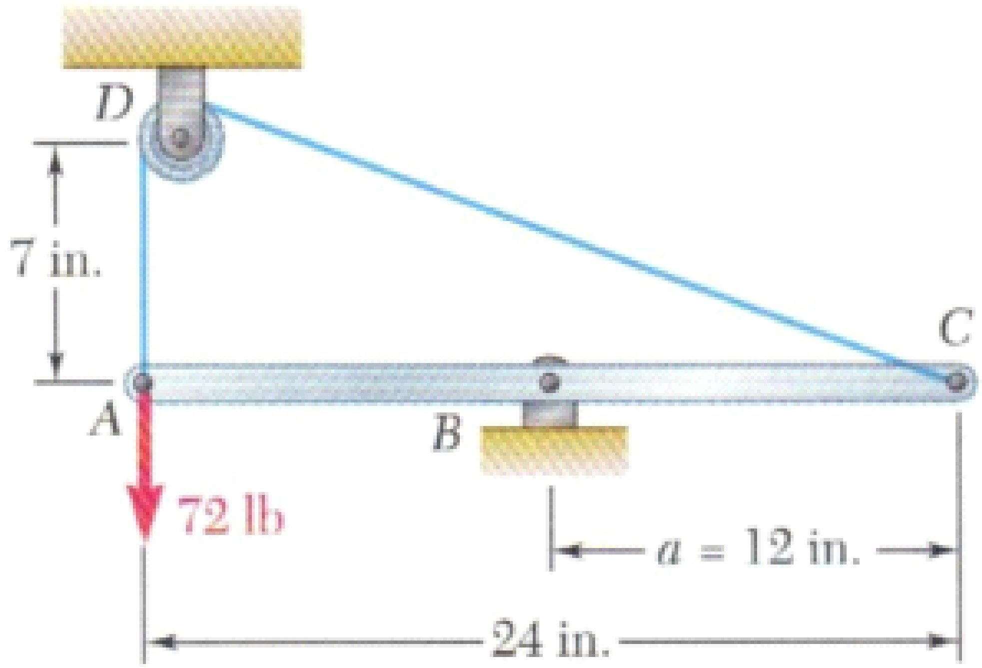


Fig. P4.77

Prob. 4.78

4.77 and 4.78 Member ABC is supported by a pin and bracket at B and by an inextensible cord attached at A and C and passing over a frictionless pulley at D. The tension may be assumed to be the same in portions AD and CD of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at B.

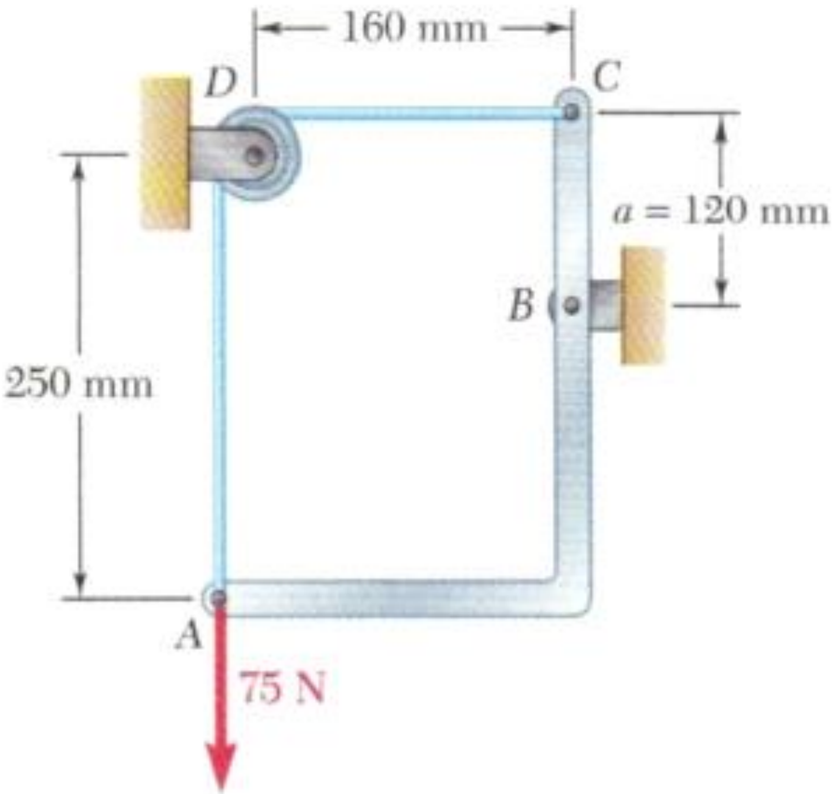


Fig. *P4.78*

Prob. 4.91

4.91 Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 30 mm and the radius of spool C is 40 mm. Knowing that $T_B = 80$ N and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

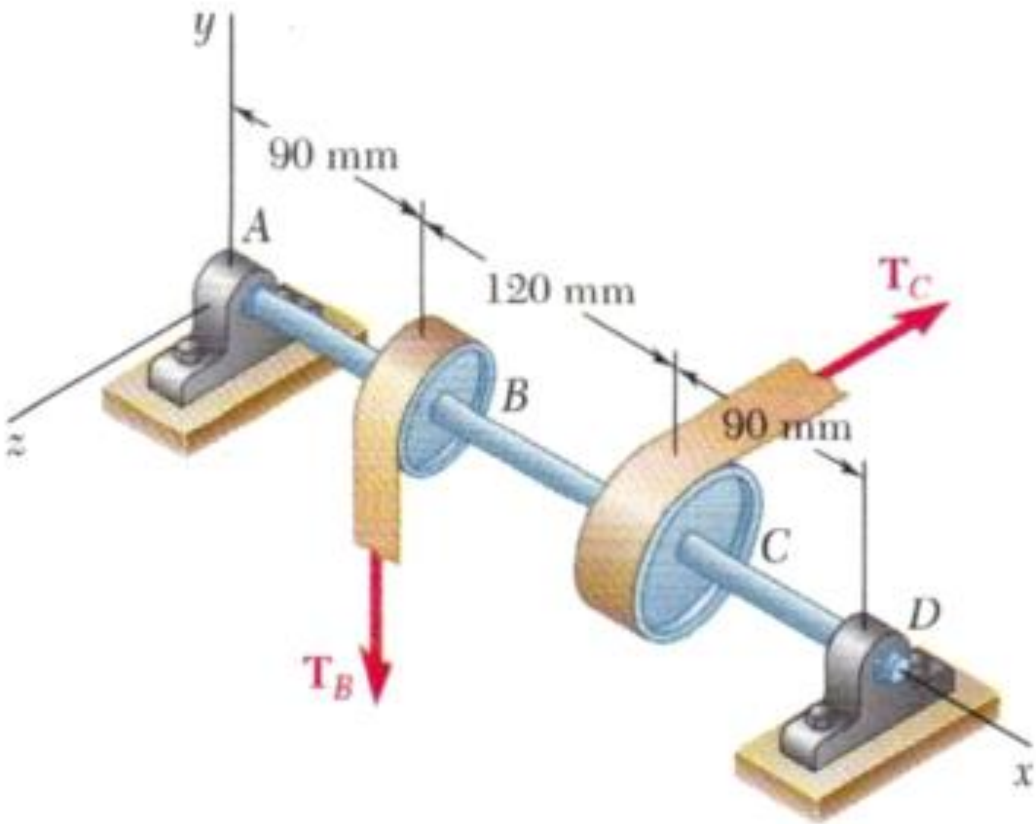


Fig. P4.91

Prob. 4.92

4.92 Solve Prob. 4.91, assuming that the spool *C* is replaced by a spool of radius 50 mm.

4.91 Two tape spools are attached to an axle supported by bearings at *A* and *D*. The radius of spool *B* is 30 mm and the radius of spool *C* is 40 mm. Knowing that $T_B = 80$ N and that the system rotates at a constant rate, determine the reactions at *A* and *D*. Assume that the bearing at *A* does not exert any axial thrust and neglect the weights of the spools and axle.

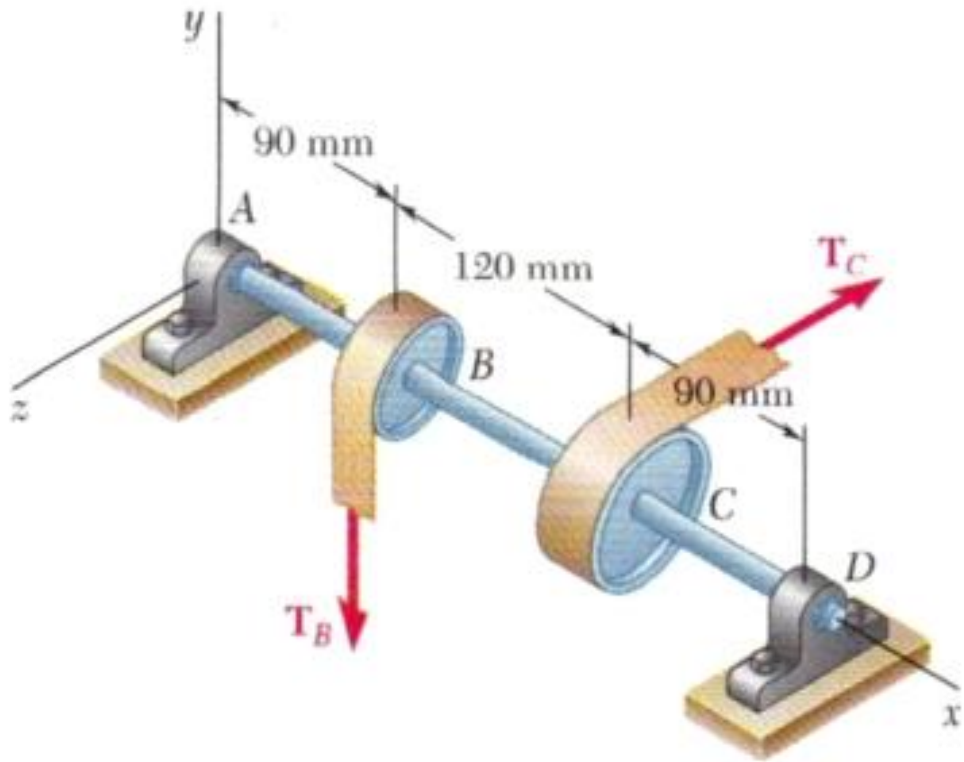


Fig. P4.91

Prob. 4.93

4.93 Two transmission belts pass over sheaves welded to an axle supported by bearings at B and D. The sheave at A has a radius of 2.5 in., and the sheave at C has a radius of 2 in. Knowing that the system rotates at a constant rate, determine (*a*) the tension T, (*b*) the reactions at B and D. Assume that the bearing at D does not exert any axial thrust and neglect the weights of the sheaves and axle.

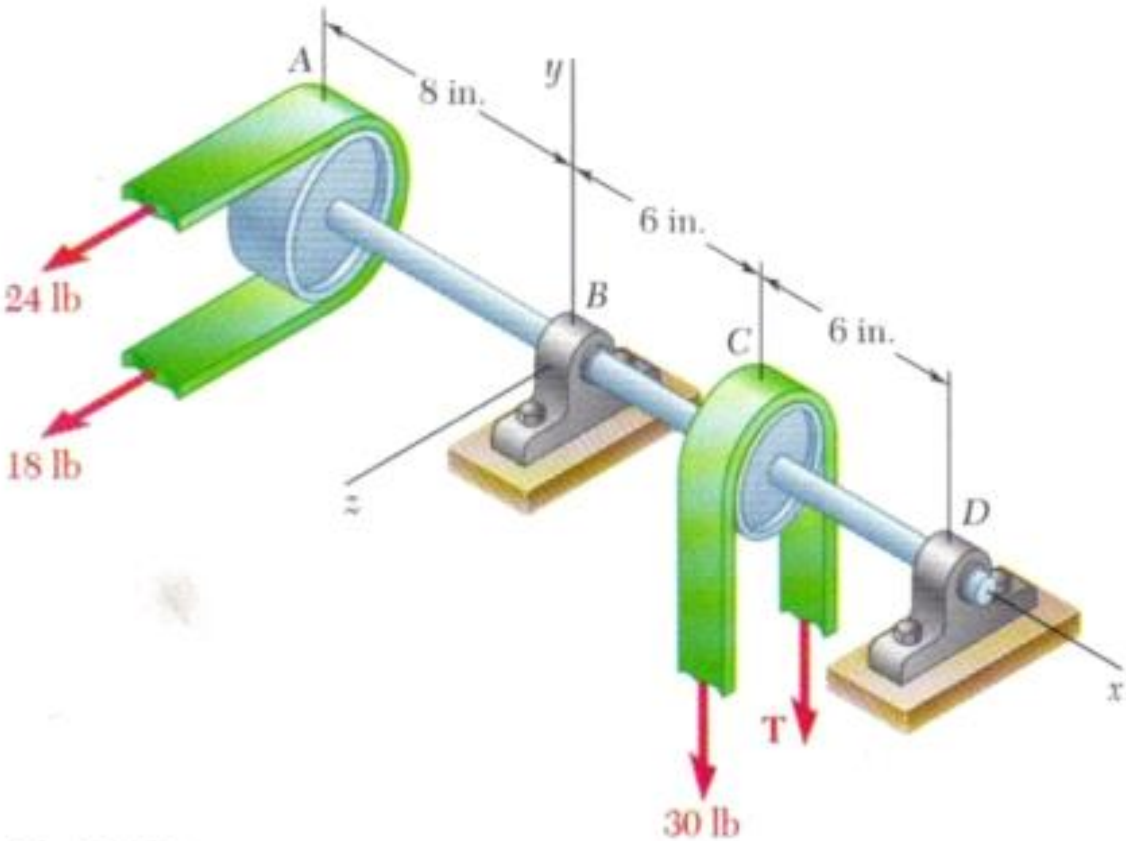


Fig. P4.93

Prob. 4.115

4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at A and B.

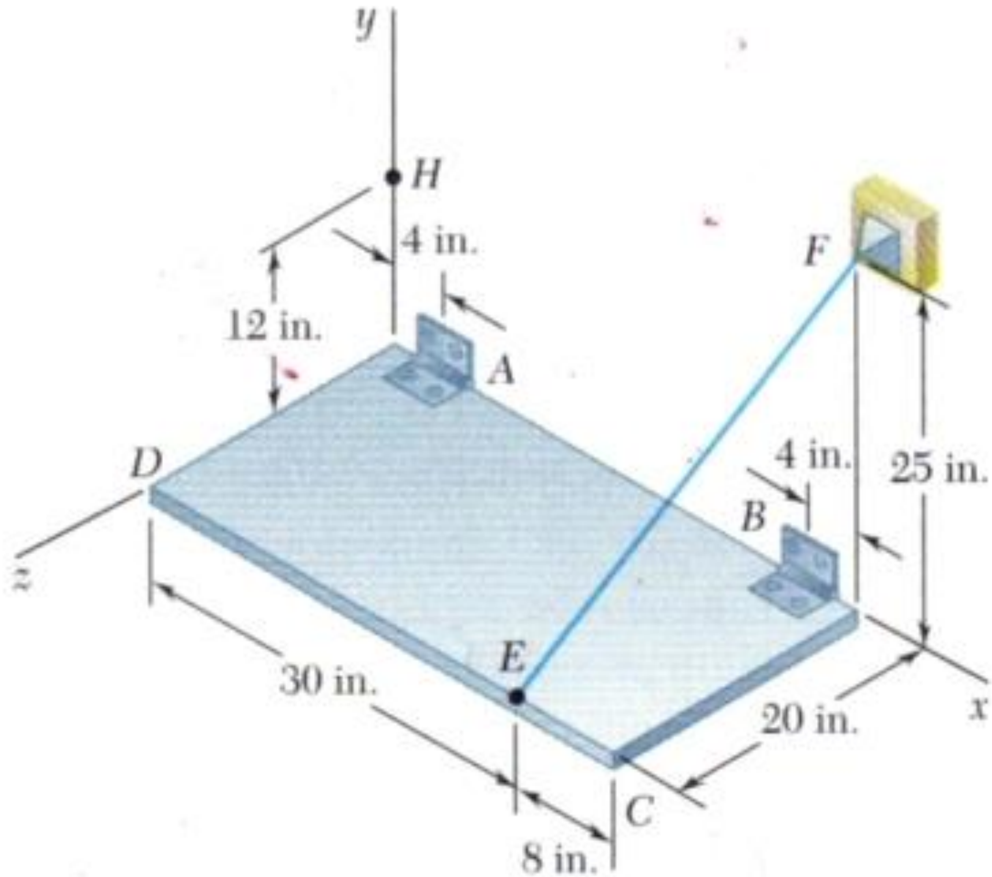


Fig. P4.115

Prob. 4.116

4.116 Solve Prob. 4.115, assuming that cable *EF* is replaced by a cable attached at points *E* and *H*.

4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at *A* and *B* and by cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

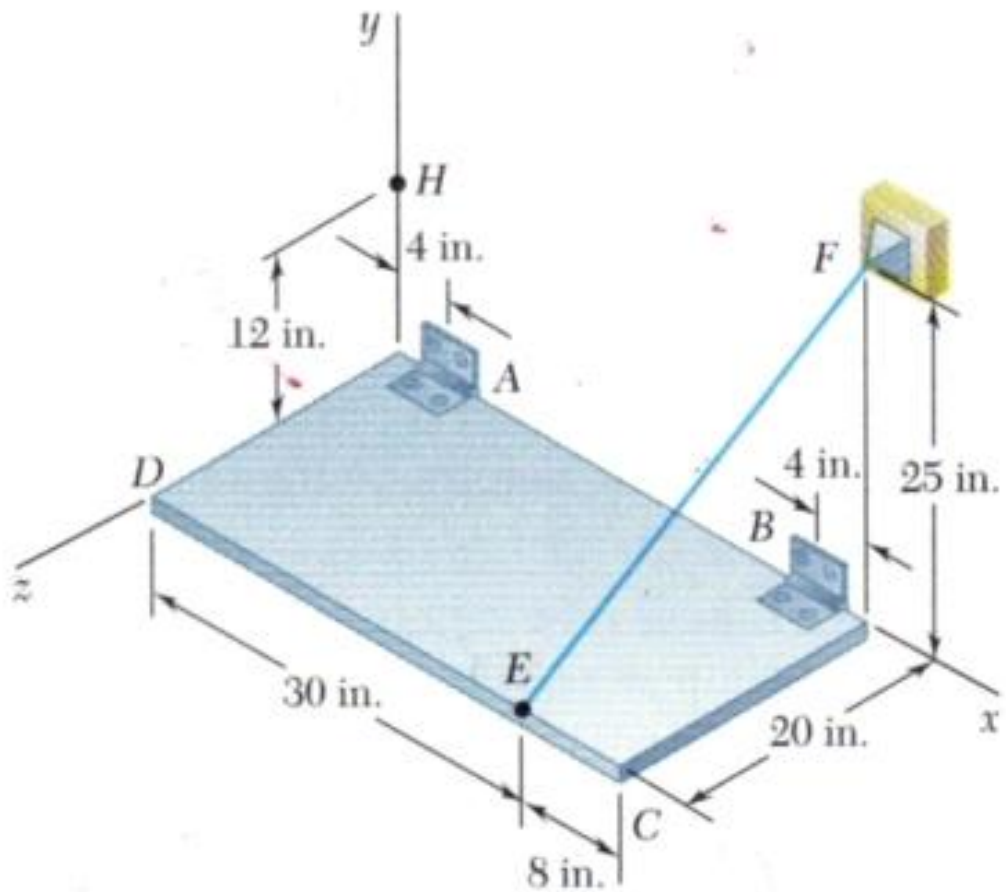


Fig. P4.115

Prob. 4.135

4.135 The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 60-lb load is applied at *C* as shown, determine the tension in the cable.

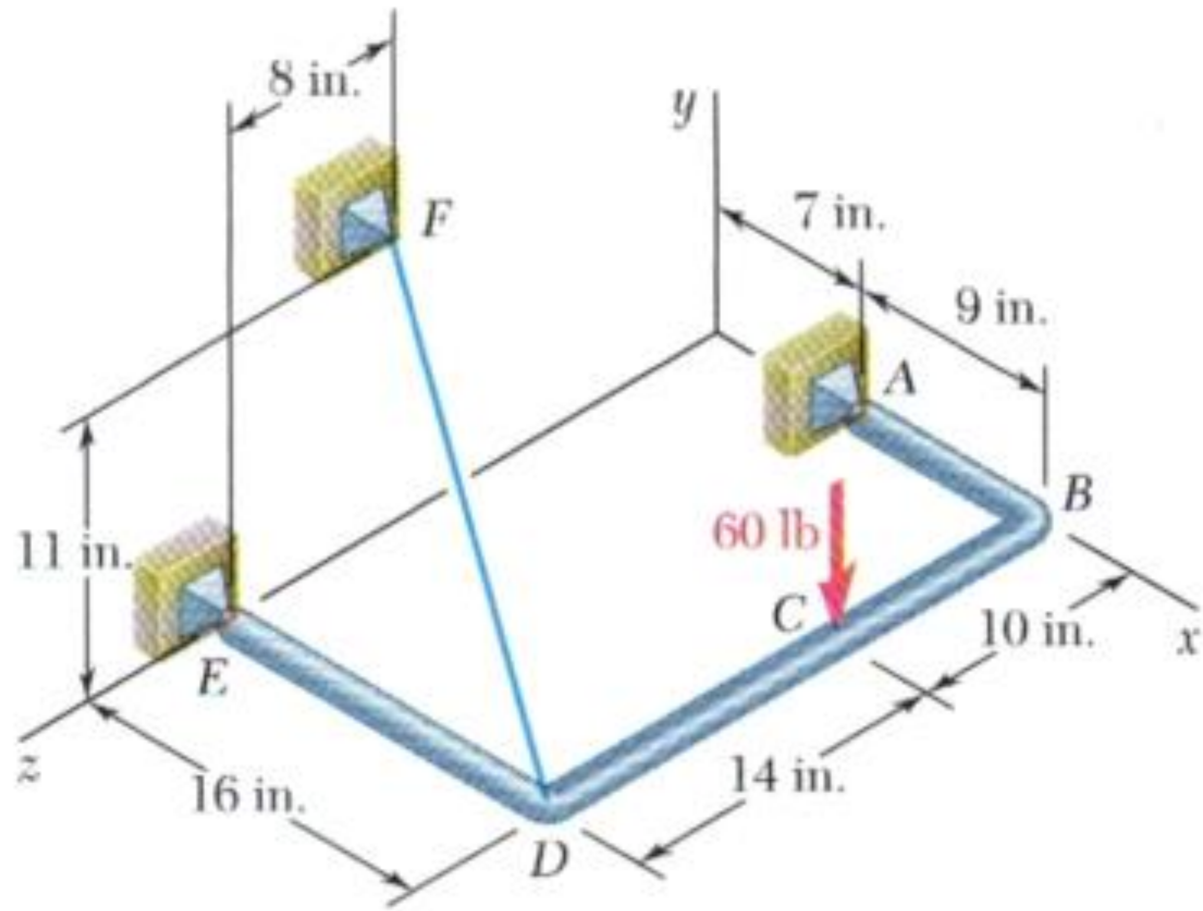


Fig. P4.135

Prob. 4.136

4.136 Solve Prob. 4.135, assuming that cable *DF* is replaced by a cable connecting *B* and *F*.

4.135 The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 60-lb load is applied at *C* as shown, determine the tension in the cable.

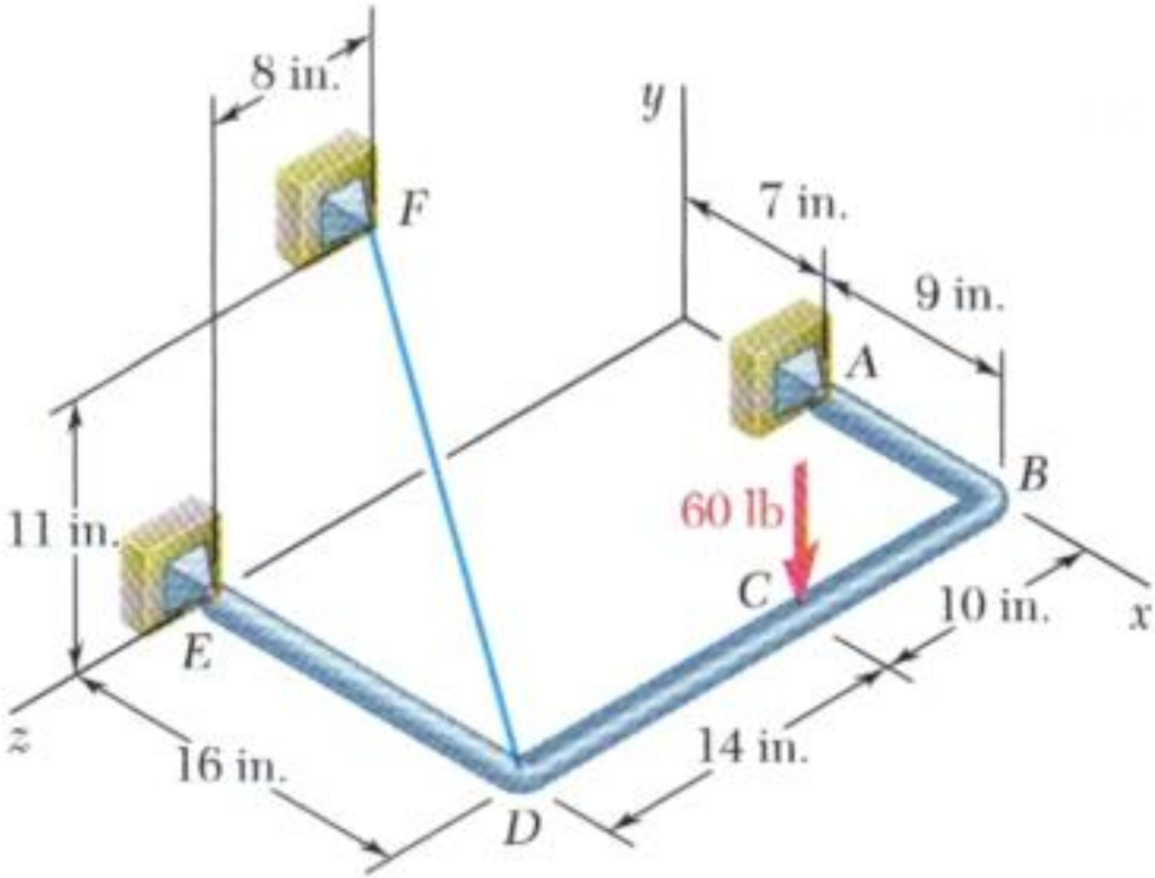


Fig. P4.135