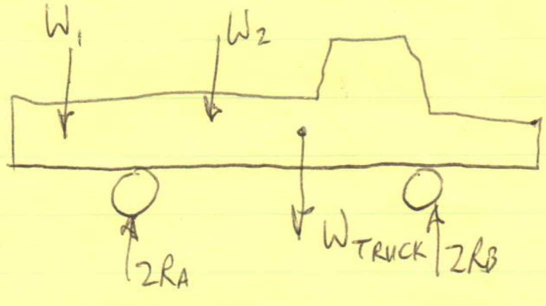
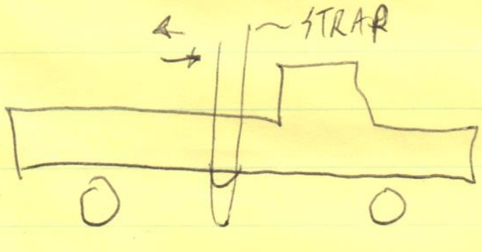
LECTURE 8

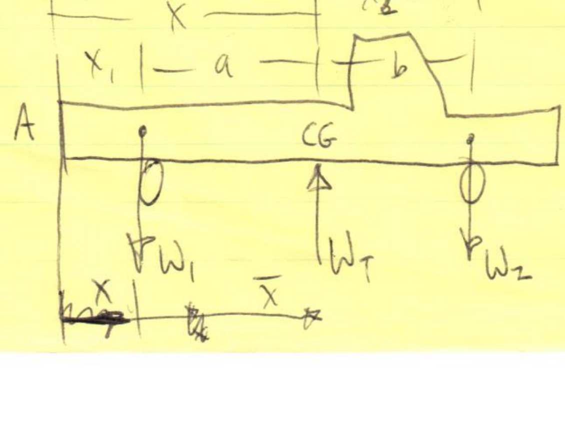
RECALL THE TRUCK PROBLEM: PROB. 4.1, p. 167



WE WERE GIVEN THE WEIGHT OF THE TRUCK AND THE POINT AT WHICH THIS WEIGHT CAN BE CONSIDERED TO ACT (CENTER OF GRAVITY). WHAT WOULD WE DO IF THIS INFORMATION WAS NOT GIVEN? EXPERIMENTALLY, WE COULD



FIND THE POINT WHERE THE TRUCK WOULD BALANCE (DANGEROUS!)



$$\sum M_{CG} = 0$$

IF WE KNEW THE LOCATION, CG AND WEIGHT OF EACH INDIVIDUAL PART OF THE TRUCK, WE COULD SAY

$$W_T = W_1 + W_2 \cancel{+ \cdots W_n} \qquad (\Sigma F_y)$$

LET'S SUM MOMENTS ABOUT ~~CG~~ A:

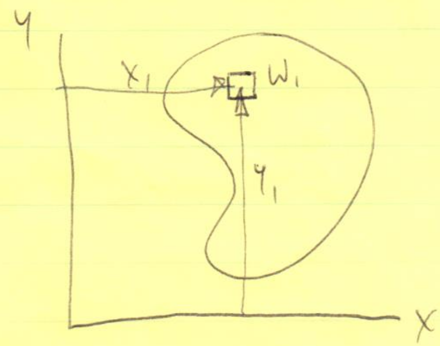
$$\cancel{xW_1 - bW_2 = a}$$

$$-X_1 W_1 + \bar{X} W_T - X_2 W_2 = 0$$

$$\bar{X} W_T = X_1 W_1 + X_2 W_2$$

$\bar{X}$ IS THE DISTANCE TO THE CENTER OF GRAVITY FROM THE Y AXIS.

NOW LET'S CONSIDER A FLAT PLATE:



LET'S DIVIDE THE PLATE INTO SMALL SQUARES.

WE CAN LOCATE THE CG FROM THE Y-AXIS BY SUMMING MOMENTS (ΣM_y)

$$\bar{x} W_p = x_1 W_1 + x_2 W_2 + \cdots x_n W_n = \sum_{i=1}^{n} x_i W_i$$

IN THE LIMIT, (RIEMANN SUM),

$$\bar{x} W_p = \int x \, dW$$

SUMMING MOMENTS ABOUT THE X-AXIS GIVES

$$\bar{y} W_p = \sum_{i=1}^{m} y_i W_i$$

IN THE LIMIT,

$$\bar{y} W_p = \int y \, dW$$

IF THE FLAT PLATE IS OF UNIFORM THICKNESS, WE CAN FIND THE ~~CENTER~~ CG IN TERMS OF AREA (CENTROID).

FOR A SMALL SQUARE ON THE PLATE,

$W_1 = \gamma t A_1$

γ = SPECIFIC WEIGHT $\left(\frac{\text{FORCE}}{\text{VOLUME}}, \frac{N}{m^3}\right)$ OR $\frac{LB}{FT^3}$

t = THICKNESS OF PLATE

A_1 = AREA OF SQUARE

NOW IF WE SUM MOMENTS,

$\bar{x}\gamma t A_T = \int x \gamma t \, dA$ OR

$\bar{x} A_t = \int x \, dA$

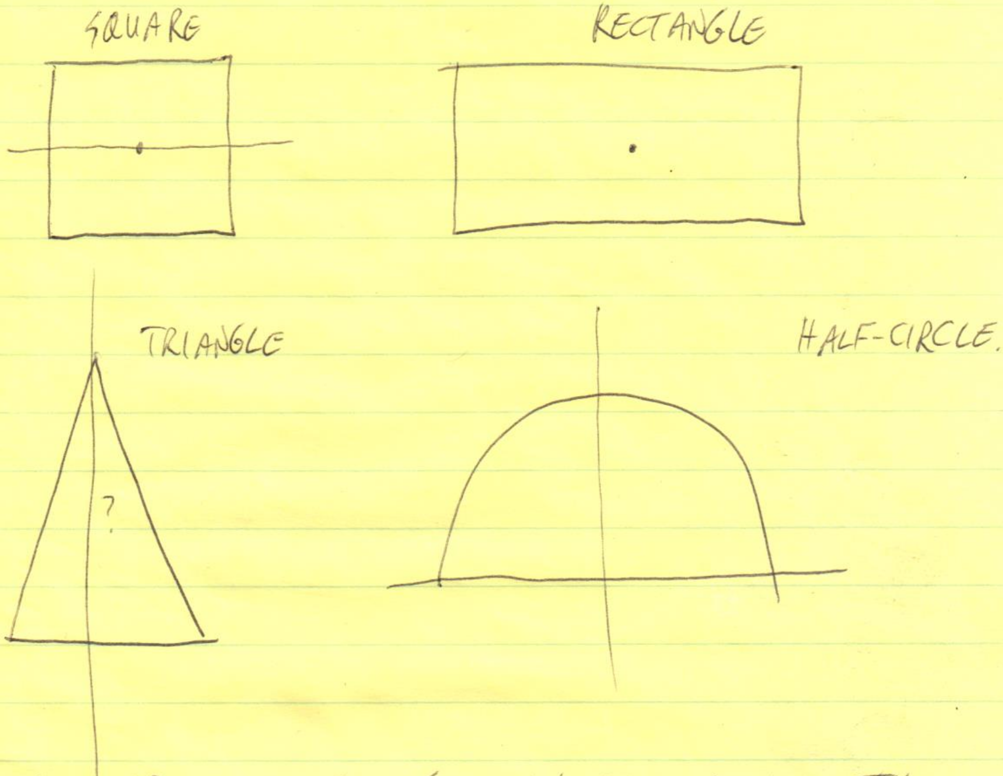
$\bar{y} A_t = \int y \, dA$

THE RIGHT-HAND SIDES ARE CALLED THE FIRST MOMENTS OF AREA A

$Q_y = \int x \, dA$ (W.R.T. Y-AXIS)

$Q_x = \int y \, dA$ (W.R.T. X-AXIS)

SYMMETRY CAN BE USED TO DETERMINE THE CENTROID.



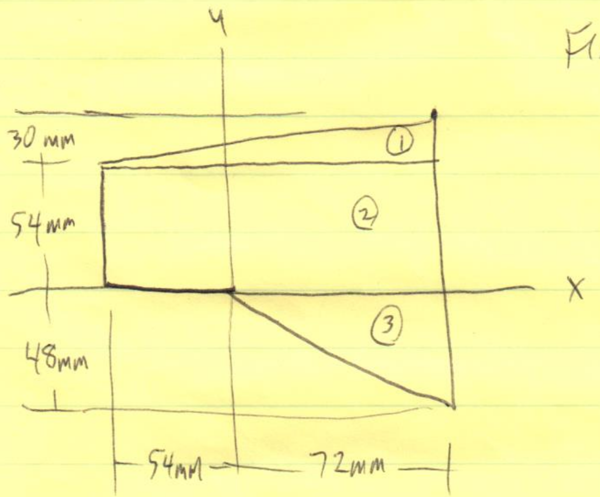
THE CENTROID LIES SOMEWHERE ALONG THE LINE OF SYMMETRY. IF THE AREA IS SYMMETRIC ALONG TWO LINES AND THE LINES ARE PERPENDICULAR, THEN THE CENTROID IS AT THE INTERSECTION.

P. ~~215~~ 225: CENTROID LOCATIONS FOR VARIOUS SHAPES ARE GIVEN. WE CAN USE THESE ELEMENTAL SHAPES TO FIND THE CENTROIDS OF MORE COMPLICATED SHAPES.

EXAMPLE PROB. 5.6

Given: Schematic

FIND CENTROID



SUM MOMENTS:

$$\Sigma M_y : \bar{X}(W_1 + W_2 + W_3) = \bar{X}_1 W_1 + \bar{X}_2 W_2 + \bar{X}_3 W_3$$

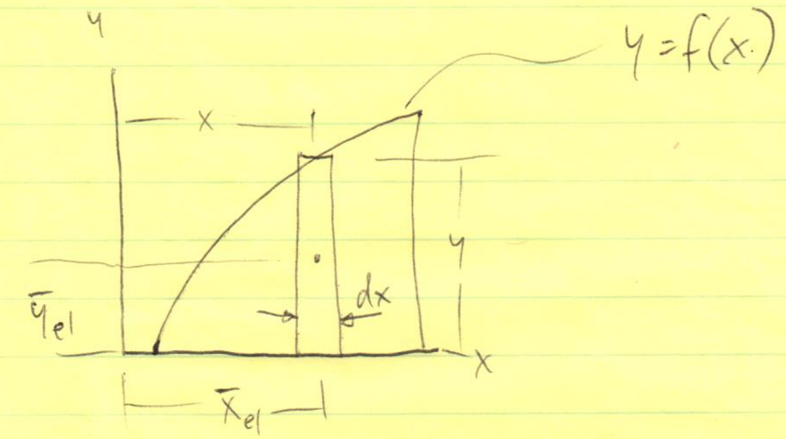
$$\Sigma M_x : \bar{Y}(W_1 + W_2 + W_3) = \bar{Y}_1 W_1 + \bar{Y}_2 W_2 + \bar{Y}_3 W_3$$

$\bar{X}$ = X LOCATION OF THE CENTROID OF THE COMPOSITE SHAPE

LECTURE 9

ON P. ~~215~~ 225, WE SAW THE CENTROIDAL LOCATIONS OF MANY SHAPES. HOW WERE THESE FOUND?

INTEGRATION.



DIVIDE THE AREA INTO RECTANGLES, FIND THE CENTROID OF EACH, THEN SUM THEM

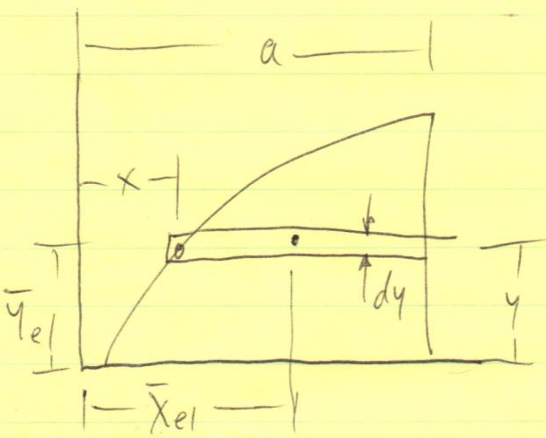
$$\bar{X} = \frac{\sum \bar{x}_i A_i}{\sum A_i} \quad , \quad \bar{Y} = \frac{\sum \bar{y}_i A_i}{\sum A_i}$$

TAKING THE LIMIT GIVES

$$\bar{X} = \frac{\int \bar{x}_{el} \, dA}{A} \quad , \quad \bar{Y} = \frac{\int \bar{y}_{el} \, dA}{A}$$

$$\bar{x}_{el} = x, \quad \bar{y}_{el} = \frac{y}{2}, \quad dA = y\,dx$$

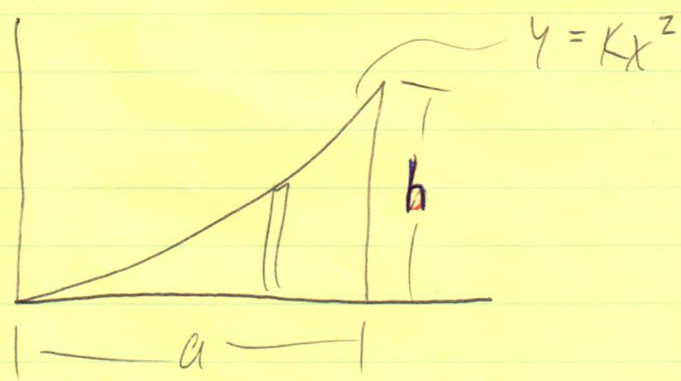
ANOTHER EQUIVALENT METHOD IS TO INTEGRATE IN THE Y-DIRECTION:



$$\bar{x}_{el} = \frac{a+x}{2}, \quad \bar{y}_{el} = y, \quad dA = (a-x)\,dy$$

EXAMPLE PROBLEM ex. 5.4 pg. 240

PARABOLIC SPANDREL



$$\bar{x} = \frac{3a}{4}, \quad \bar{y} = \frac{3h}{10}, \quad A = \frac{ah}{3}$$

EVALUATE K: @ $x=a$, $y=h$

$h = Ka^2$, $K=\frac{h}{a^2}$ $\therefore$ $y=\left(\frac{h}{a^2}\right)x^2$

$\bar{X} = \frac{\int \bar{X}_{el}\, dA}{A}$, $\bar{Y} = \frac{\int \bar{Y}_{el}\, dA}{A}$

$\bar{X}_{el} = X$, $\bar{Y}_{el} = \frac{y}{2}$, $dA = y\,dx$

$A = \int dA = \int_0^a y\,dx$

$= \int_0^a K x^2 dx$

$= \left[K\frac{x^3}{3}\right]_0^a$

$= \frac{1}{3}Ka^3$

$= \frac{1}{3}\left(\frac{h}{a^2}\right)a^3$

$\boxed{A = \frac{1}{3}ha}$

$\bar{X} = \frac{1}{A}\int \bar{X}_{el}\, dA$, $\bar{X}_{el} = X$, $dA = y\,dx$

$\bar{X} = \frac{1}{A}\int_0^a X\,(y\,dx)$

$y = \left(\frac{h}{a^2}\right)x^2$, $A = \frac{ha}{3}$

$= \frac{3}{ha}\int_0^a x\cdot\left(\frac{h}{a^2}\right)x^2 dx$

$$\bar{X} = \frac{3}{a^3}\left[\frac{x^4}{4}\right]_0^a$$

$$\boxed{\bar{X} = \frac{3}{4}a}$$

$$\bar{Y} = \frac{1}{A}\int_0^a \bar{Y}_{el}\, dA$$

$\bar{Y}_{el} = \frac{y}{2}$, $dA = y\,dx$

$y = \left(\frac{h}{a^2}\right)x^2$

$$= \frac{3}{ha}\int_0^a \frac{y}{2}\cdot y\,dx$$

$$= \cancel{\frac{3}{ha}}\ \frac{3}{2}\cdot\frac{1}{ha}\int_0^a y^2\,dx$$

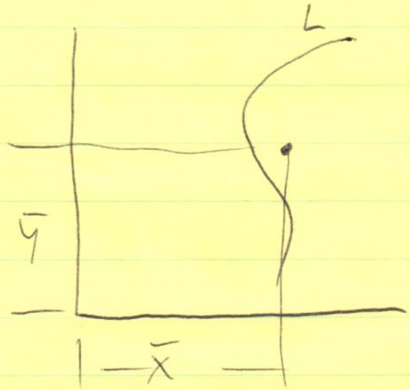
$$= \frac{3}{2}\cdot\frac{1}{ha}\int_0^a \left[\left(\frac{h}{a^2}\right)x^2\right]^2 dx$$

$$= \frac{3}{2}\cdot\frac{1}{ha}\int_0^a \left(\frac{h^2}{a^4}\right)x^4\,dx$$

$$= \frac{3}{2}\cdot\frac{h}{a^5}\left[\frac{x^5}{5}\right]_0^a$$

$$\boxed{\bar{Y} = \frac{3}{10}h}$$

THE CENTROID OF A LINE IN SPACE CAN BE FOUND USING THE METHODS WE'VE SEEN.



$$\bar{x} = \frac{1}{L}\int x\,dL \quad , \quad \bar{y} = \frac{1}{L}\int y\,dL$$

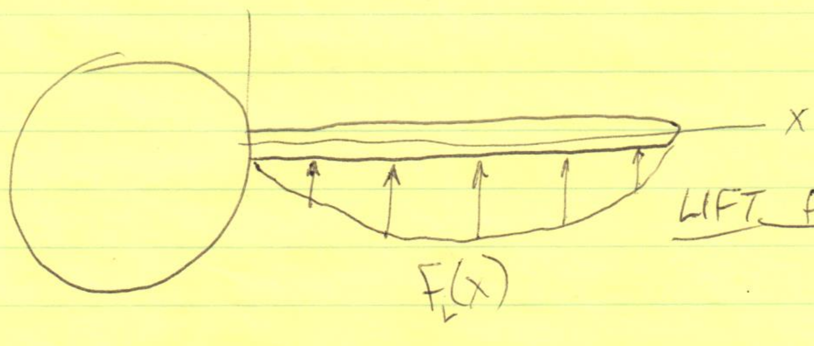
WHERE

$$dL = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \qquad \text{IF } y = f(x)$$

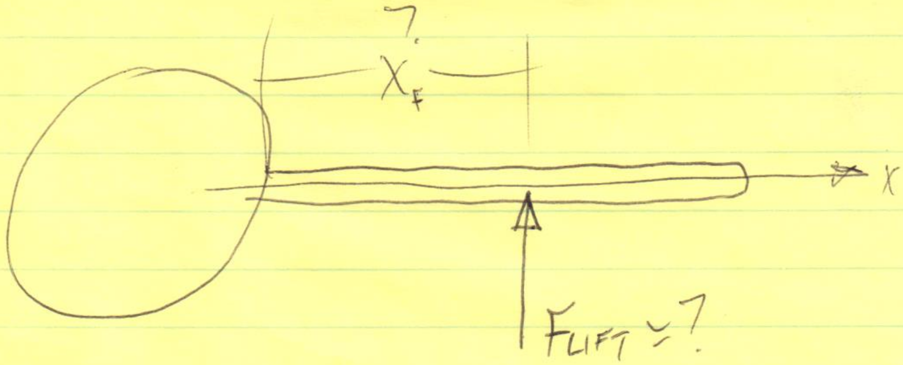
$$dL = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy \qquad \text{IF } x = g(y)$$

~~WE CAN USE CENTROIDS IN THE ANALYSIS OF DISTRIBUTED LOADS ON BEAMS.~~

AIRPLANE WING: 1450th test wing



LIFT ON A WING CAN BE MEASURED ALONG THE X-DIRECTION USING PRESSURE TAPS ON THE TOP AND BOTTOM SURFACES. WHAT IS THE EQUIVALENT FORCE AND WHERE DOES IT ACT?



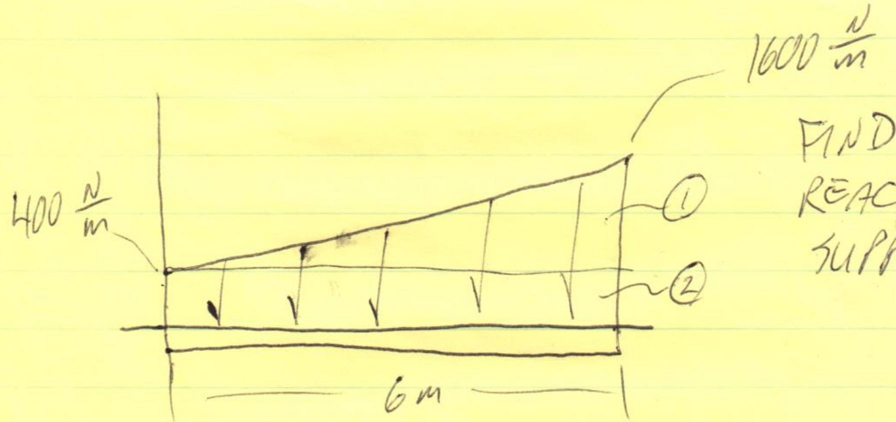
THE FORCE IS THE AREA UNDER THE CURVE.

$$F_{LIFT} = \int dA = \int F_2(x)\,dx$$

THIS FORCE IS LOCATED AT THE CENTROID OF THE AREA.

$$x_f = \overline{X}$$

EXAMPLE PROB. 5.75



FIND RESULTANT REACTIONS AT SUPPORT

AREA 1:

$$A_1 = \tfrac{1}{2}(6m)(1200\ \tfrac{N}{m}) = 3600\ N$$

$$\overline{X} = \frac{h}{3} = \frac{6}{3} = 2m$$

$$\overline{X}_1 = 6-2 = 4m$$

$$\overline{X}_1 A_1 = (4m)(3600\ N) = 1.44 \times 10^4\ N\text{-}m$$

THE ~~CENTROIDS~~ CENTER OF GRAVITY OF THREE-DIMENSIONAL OBJECTS CAN BE ~~DETERMINED~~ FOUND.

$\bar{x}W = \int x\,dW$ ETC.

FOR A HOMOGENEOUS BODY, $W = \gamma V$

(γ = SPECIFIC WEIGHT) V = La VOLUME

$$\bar{x} = \frac{1}{V}\int x\,dV$$

P. ~~[illegible]~~ 261

FOR A BODY COMPOSED OF SEVERAL SHAPES,

$$\bar{X} = \frac{\Sigma \bar{x} W}{\Sigma W}, \quad \bar{Y} = \frac{\Sigma \bar{y} W}{\Sigma W} \quad \text{ETC.}$$