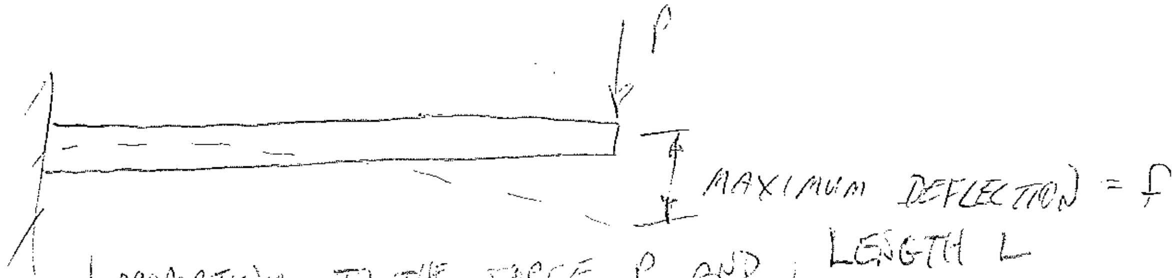
IN THE DESIGN OF STRUCTURES, IT IS IMPORTANT TO PREDICT HOW MUCH A MEMBER IS GOING TO DEFLECT.

CANTILEVERED BEAM:

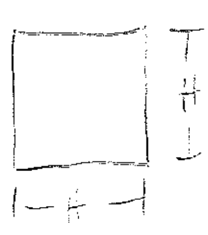


DEFLECTION IS PROPORTIONAL TO THE FORCE P AND LENGTH L INVERSELY PROPORTIONAL TO A PROPERTY OF THE MATERIAL (MODULUS OF ELASTICITY) AND THE SHAPE OF THE CROSS SECTION OF THE BEAM (MOMENT OF INERTIA OF AREA)

$$f \sim \frac{PL}{EI}$$

AS I INCREASES, THE DEFLECTION DECREASES.

LET'S CONSIDER A BEAM WITH A SQUARE CROSS SECTION:

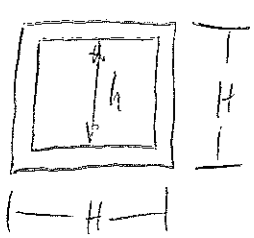


WE WILL FIND THAT THE MOMENT OF INERTIA OF AREA FOR THIS SHAPE IS

$$I_1 = \frac{H^4}{12}$$

FOR $H = 4\,\text{IN}$, $I_1 = 21.3\ \text{IN}^4$

NOW CONSIDER A ~~HOLLOW~~ BEAM WITH THE FOLLOWING CROSS SECTION:



$$I_2 = \frac{1}{12}\left(H^4 - h^4\right)$$

FOR $H = 4\,\text{IN}$, $h = 3.75\,\text{IN}$

$$I_2 = 4.85\ \text{IN}^4$$

$$I_2 = 0.228\, I_1$$

HOW MUCH WEIGHT DID WE SAVE? AREA ~ WEIGHT

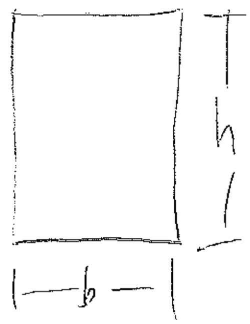
$A_1 = H^2 = \cancel{0}\ 16\ \text{IN}^2$ WEIGHT ~ MATERIAL COST

$A_2 = H^2 - h^2 = 1.93\ \text{IN}^2$ WEIGHT ~ ~~MAT~~ LABOR COST

$A_2 = 0.121\ \cancel{\text{in}}\ A_1$

WHILE THERE WAS A SIGNIFICANT REDUCTION IN I (INCREASE IN DEFLECTION), THE REDUCTION IN WEIGHT WAS MORE DRAMATIC.

LET'S MAKE ANOTHER COMPARISON

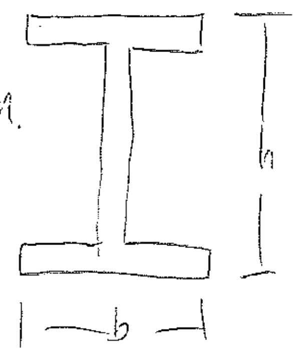


$$I_1 = \frac{bh^3}{12}, \quad A_1 = bh$$

LET $b = 11$ IN, $h = 18.2$ IN

$$I_1 = 5526\ \text{IN}^4, \quad A_1 = 200\ \text{IN}^2$$

I-BEAM.



$$I_2 = 1330\ \text{IN}^4, \quad A_2 = 22.3\ \text{IN}^2$$

$$I_2 = 0.24\, I_1$$

$$A_2 = 0.11\, A_1$$

THIS IS EVEN BETTER THAN SQUARE TUBING & THIS SHAPE HAS BEEN OPTIMIZED FOR MANY YEARS. THE HOUSING INDUSTRY IS STARTING TO REPLACE STANDARD 2×10 FLOOR JOISTS WITH MANUFACTURED I-BEAM LUMBER.

FROM THE DISCUSSION ON CENTROIDS, WE FOUND THE FIRST MOMENT OF AREA TO BE

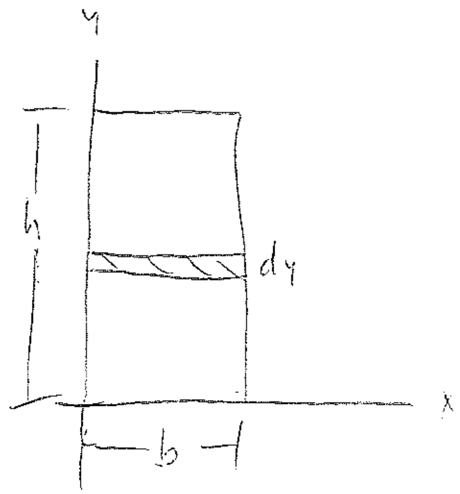
$$Q_Y = \int x\,dA \quad \text{AND} \quad Q_X = \int y\,dA$$

$$\text{AND} \quad \bar{X} = \frac{Q_Y}{A}, \quad \bar{Y} = \frac{Q_X}{A}$$

BY DEFINITION, THE SECOND MOMENT OF AREA (MOMENT OF INERTIA OF AREA) IS

ABOUT Y-AXIS ABOUT X-AXIS

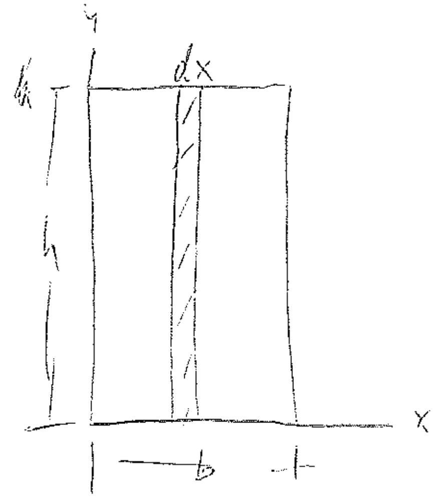
$$\cancel{I_x = \int y^2 dA} \quad I_Y = \int x^2 dA \quad \text{AND} \quad I_X = \int y^2 dA$$



RECTANGULAR CROSS SECTION (YARD STICK).

$$I_X = \int y^2 dA, \quad dA = b\,dy$$

$$I_X = \int_0^h b y^2 dy = b\left[\frac{y^3}{3}\right]_0^h = \frac{1}{3} b h^3$$



$I_y = \int x^2 dA$

$dA = h\,dx$

$I_y = \int_0^b x^2 h\,dx$

$= h\left[\frac{x^3}{3}\right]_0^b$

$I_y = \frac{1}{3} h b^3$

LET $h = 1$ IN, $b = \frac{5}{16}$ IN

$I_x = \frac{1}{3}\left(\frac{5}{16}\text{ IN}\right)\left(1.\text{ IN}\right)^3 = 0.104\ \text{IN}^4$

$I_y = \frac{1}{3}\left(1\text{ IN}\right)\left(\frac{5}{16}\text{ IN}\right)^3 = 0.0102\ \text{IN}^4$

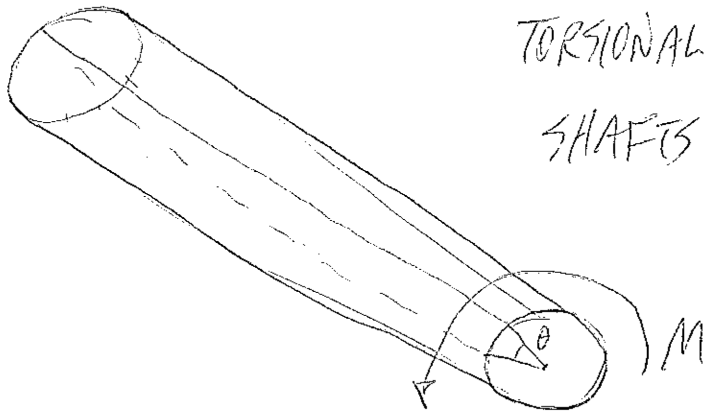
FOR THE SAME LOAD,
THE YARDSTICK WILL DEFLECT 10 TIMES MORE IN ONE DIRECTION THAN THE OTHER.

THE POLAR MOMENT OF INERTIA OF AREA IS IMPORTANT TO TORSION IN SHAFTS.

$J_0 = \int r^2 dA = \int (x^2 + y^2)\,dA = \int x^2 dA + \int y^2 dA$

$J_0 = I_y + I_x$

TORSIONAL DEFLECTION IN SHAFTS



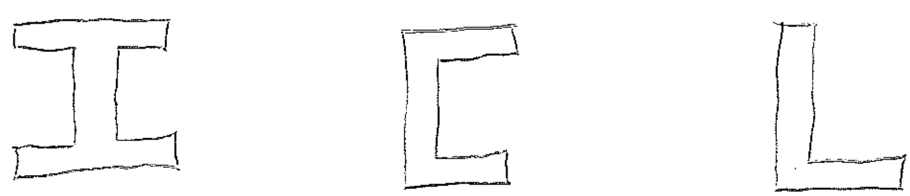
$$\theta = \frac{ML}{GI}$$

G = SHEARING MODULUS OF ELASTICITY

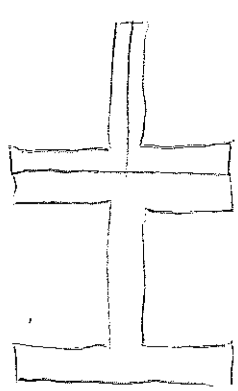
WHY ARE AUTOMOTIVE DRIVESHAFTS HOLLOW?

~~OUR FINAL OBJECTIVE IS~~ NEED TO BE ABLE TO DETERMINE THE MOMENT OF INERTIA OF COMPOSITE AREAS:

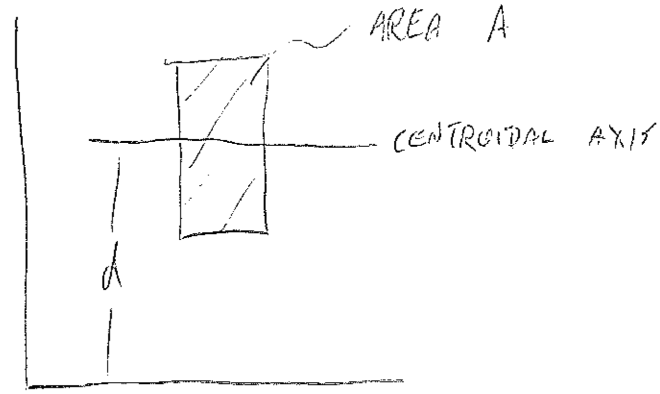
I-BEAMS CHANNELS ANGLES



COMBINATIONS



TO DO THIS, WE NEED THE PARALLEL AXIS THEORUM.



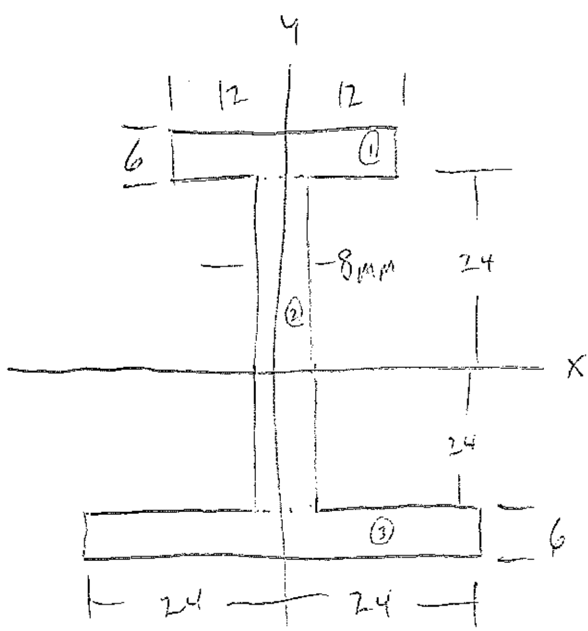
$$I = \bar{I} + Ad^2$$

$\bar{I}$ → MOMENT OF INERTIA ABOUT THE CENTROIDAL AXIS

EXAMPLE PROB. 9.31

Given: Schematic

FIND MOMENT OF INERTIA W.R.T. X AXIS



PROB. 9.31

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

FOR AREA 1

$$\bar{I} = \frac{1}{12} b h^3$$

TABLE p. ~~[illegible]~~ & 485

$$(I_x)_1 = \frac{1}{12} b h^3 + (bh) d^2$$

$$= \frac{1}{12}(24\,mm)(6\,mm)^3 + (24\,mm)(6\,mm)(24 + 3\,mm)^2$$

$$(I_x)_1 = 1.05 \times 10^5\ mm^4$$

FOR AREA 2:

$$(I_x)_2 = \frac{1}{12} b h^3$$

$$= \frac{1}{12}(8\,mm)(48\,mm)^3$$

$$(I_x)_2 = 7.37 \times 10^4\ mm^4$$

FOR AREA 3:

$$(I_x)_3 = \bar{I}_3 + Ad^2$$

$$= \frac{1}{12} b h^3 + bhd^2$$

$$= \frac{1}{12}(48)(6)^3 + (48)(6)(24+3)^2$$

PROB. 9.31

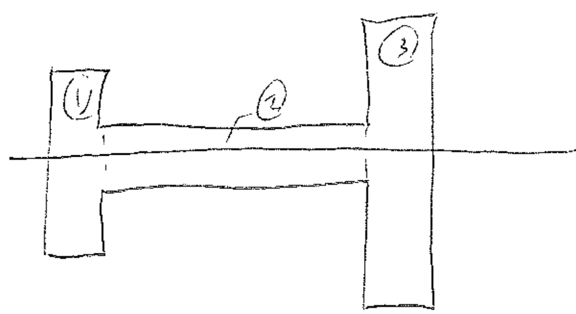
$(I_x)_3 = 2.11 \times 10^5 \ mm^4$

$$\boxed{I_x = 3.89 \times 10^5 \ mm^4}$$

HOMEWORK #8

PROBS. 9.50, 9.51, 9.54, 9.115, 9.127

IN-CLASS HOMEWORK: PROB. 9.33



$$(I_y) = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$(I_y)_1 = \frac{1}{12} b h^3 = \frac{1}{12}(6)(24)^3 = 6912 \ mm^4$$

$$(I_y)_2 = \frac{1}{12}(48)(8)^3 = 2048 \ mm^4$$

$$(I_y)_3 = \frac{1}{12}(6)(48)^3 = 5.53 \times 10^4 \ mm^4$$

$$\boxed{I_y = 6.42 \times 10^4 \ mm^4}$$

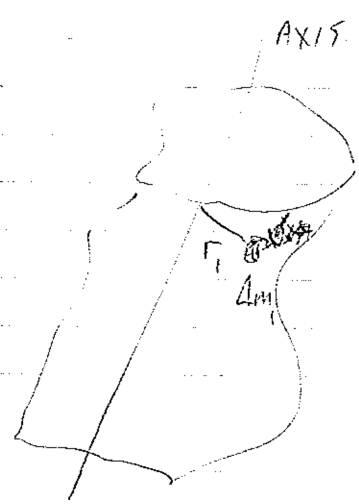
MOMENTS OF INERTIA OF MASSES

WHY ARE DRIVESHAFTS HOLLOW?

- WEIGHT PRIMARILY
- ALSO, THE MOMENT OF INERTIA IS HIGH FOR A SOLID SHAFT, WHICH WOULD SLOW DOWN ACCELERATION.
  FLYWHEEL ON AN ENGINE – SMOOTHS OUT ROTATION OF THE CRANKSHAFT

FOR A BODY, THE MASS MOMENT OF INERTIA ~~CAN BE~~ AROUND AN AXIS CAN BE FOUND BY INTEGRATION:

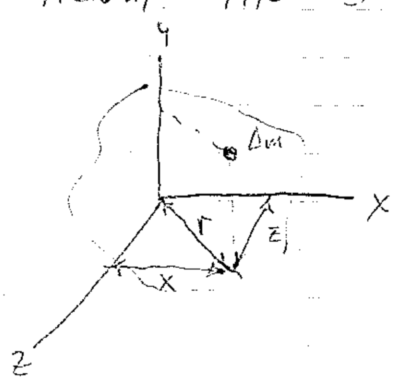
$$I = \int r^2 \, dm$$



AS THE RADIUS OF THE BODY INCREASES, $I \uparrow$

AS THE ~~MASS~~ MASS OF THE BODY INCREASES, $I \uparrow$

THE MASS MOMENTS OF INERTIA ABOUT THE 3 COORDINATE AXES ARE

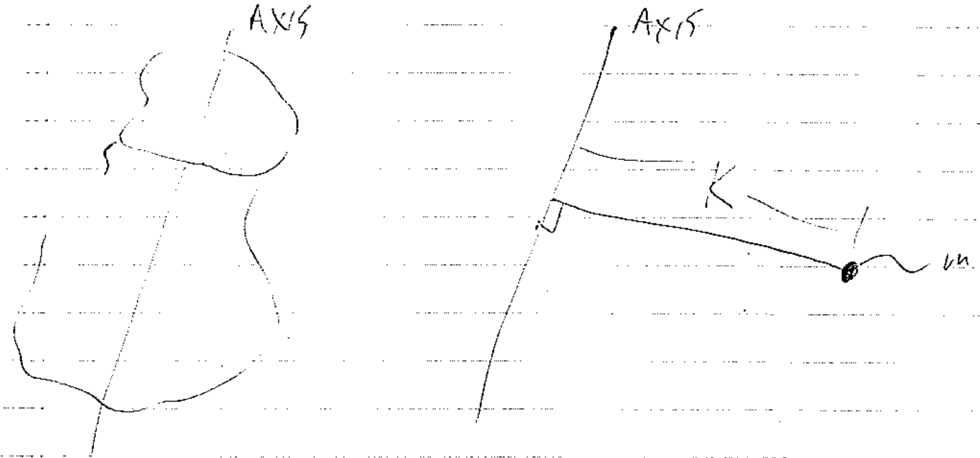


$$I_x = \int (y^2 + z^2) \, dm$$

$$I_y = \int (x^2 + z^2) \, dm$$

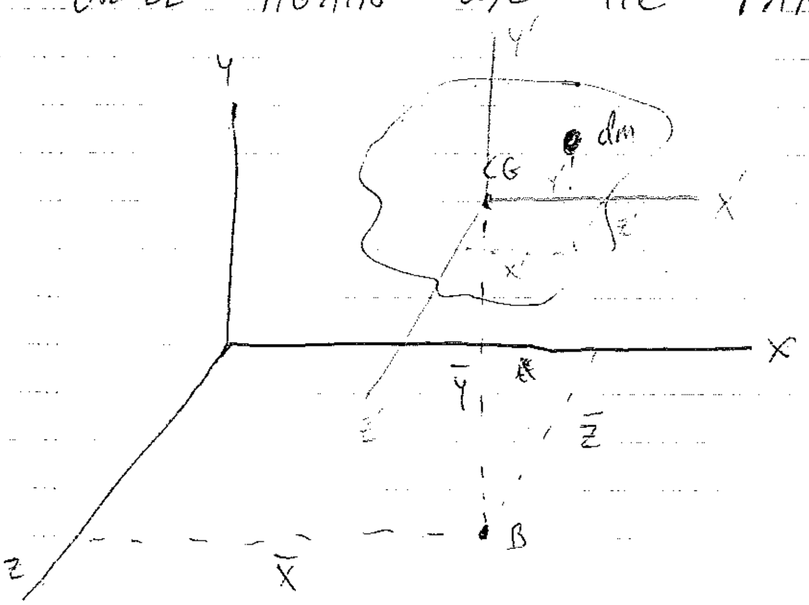
$$I_z = \int (x^2 + y^2) \, dm$$

THE RADIUS OF GYRATION IS THE DISTANCE THAT THE ENTIRE MASS OF A BODY WOULD RESIDE IF THE MASS MOMENT OF INERTIA WERE UNCHANGED.



$$I = k^2 m \quad \text{OR} \quad K = \sqrt{\frac{I}{m}}$$

JUST LIKE MOMENT OF INERTIA OF AREA, WE WILL NEED TO FIND THE MOMENT OF INERTIA OF MASS FOR COMPOSITE STRUCTURES. WE'LL AGAIN USE THE PARALLEL-AXIS THEORUM.



CG = ~~CENTROID~~ CENTER OF GRAVITY

$$X = X' + \bar{X}$$

$$Y = Y' + \bar{Y}$$

$$Z = Z' + \bar{Z}$$

MASS MOMENT OF INERTIA W.R.T. X-AXIS IS:

$$I_x = \int r^2 dm$$

$$= \int (y^2 + z^2)\, dm$$

$$= \int \left[(y' + \bar{y})^2 + (z' + \bar{z})^2\right] dm$$

$$I_x = \int (y'^2 + z'^2)\, dm + 2\bar{y}\int y'\, dm + 2\bar{z}\int z'\, dm$$

$$+ (\bar{y}^2 + \bar{z}^2)\int dm$$

$\int (y'^2 + z'^2)\, dm = \bar{I}_{x'}$ = MASS MOMENT OF INERTIA AROUND CENTROIDAL X′ AXIS

$\int y'\, dm$ = FIRST MOMENT OF AREA W.R.T. Z′X′-PLANE

REMEMBER FIRST MOMENT OF AREA:

$$Q_x = \int y\, dA = \bar{y} A$$

~~ALTERNATE~~ IF THE X-AXIS PASSES THROUGH THE CENTROID, $\bar{y} = 0 \therefore \int y\, dA = 0$

$\therefore \int y'\, dm = 0$ SINCE X′Z′ PLANE PASSES THROUGH CG

$\int z'\, dm = 0$ FOR THE SAME REASON.

$\int dm = m$ MASS OF OBJECT.

$$I_x = \bar{I}_{x'} + m(\bar{y}^2 + \bar{z}^2)$$

$I_y = \bar{I}_{y'} + m(\bar{z}^2 + \bar{x}^2)$

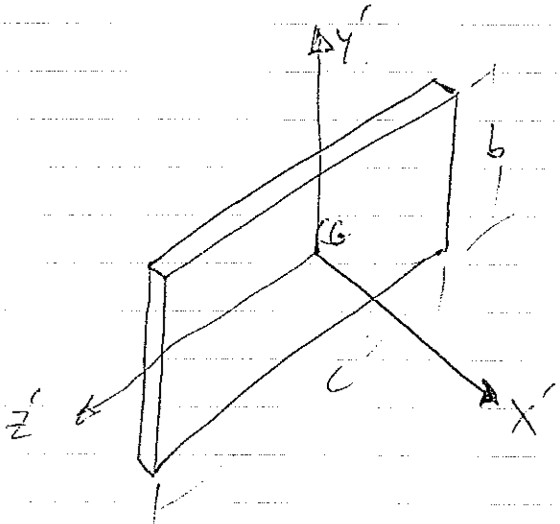
$I_z = \bar{I}_{z'} + m(\bar{x}^2 + \bar{y}^2)$

THE MASS MOMENT OF INERTIA ABOUT AN AXIS PARALLEL TO THE CENTROIDAL AXIS IS

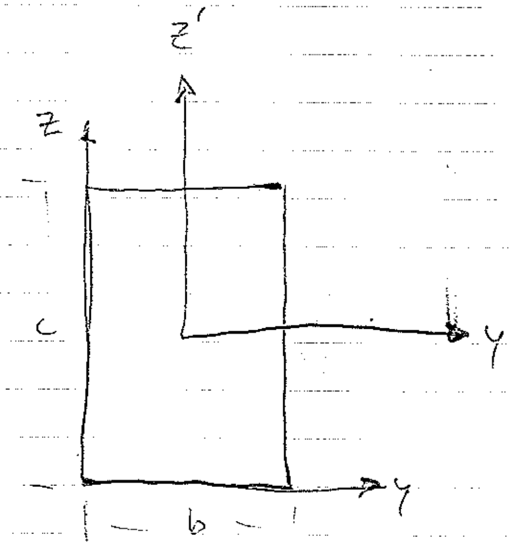
$I = \bar{I} + md^2$ d = DISTANCE BETWEEN AXES

EX. PROB.

FIND MASS MOMENTS OF INERTIA OF A THIN RECTANGULAR PLATE.



p. ~~801~~ 517



$\cancel{\bar{I}_{y'} = \int (x'^2 + z'^2)\, dm}$

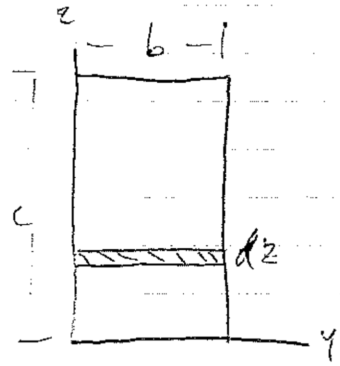
$\cancel{dm = \rho t\, dA}$

~~LET X = 0 (THIN PLATE)~~

PARALLEL-AXIS ~~THEOR~~ THEORUM:

$I_y = \bar{I}_{y'} + md^2$

$\bar{I}_{y'} = I_y - md^2 \quad , \quad d = \frac{c}{2}$



$I_y = \int (x^2 + z^2)\,dm$

$dm = \rho t\,dA$

LET $x \to 0$ (THIN PLATE)

$I_y = \rho t \int z^2\,dA \qquad dA = b\,dz$

$I_y = \rho t \int_0^c z^2 (b)\,dz$

$= \rho t b \left[\frac{z^3}{3}\right]_0^c$

$I_y = \rho t b \frac{c^3}{3}$

$m = \rho t b c$

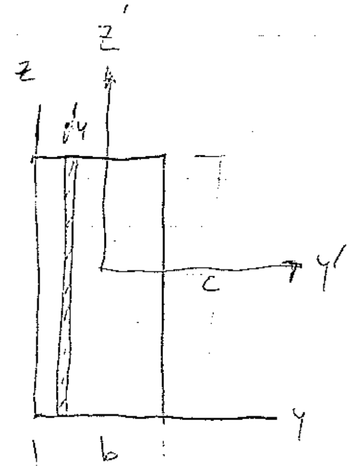
$I_y = m\frac{c^2}{3}$

$\bar{I}_{y'} = I_y - m\left(\frac{c}{2}\right)^2$

$= m\frac{c^2}{3} - m\frac{c^2}{4}$

$\boxed{\bar{I}_{y'} = \frac{mc^2}{12}}$

SIMILARLY,



$\bar{I}_{z'} = I_z - md^2 \quad , \quad d = \frac{b}{2}$

$$I_z = \int (x^2 + y^2)\, dm$$ LET $x \to 0$ THIN PLATE

$$I_z = \rho t \int_0^b y^2 \cdot c \; dy$$

$$= \rho t c \left[\frac{y^3}{3} \right]_0^b$$

$$= \rho t c \frac{b^3}{3}$$ $m = \rho t b c$

$$I_z = m \frac{b^2}{3}$$

$$\bar{I}_{z'} = m \frac{b^2}{3} - m \left(\frac{b}{2}\right)^2$$

$$\boxed{\bar{I}_{z'} = \frac{mb^2}{12}}$$

$$I_x = \int (y^2 + z^2)\, dm$$

$$= \int y^2\, dm + \int z^2\, dm$$

$$= I_z + I_y$$

$$= m \frac{b^2}{3} + m \frac{c^2}{3}$$

$$I_x = \frac{m}{3}(b^2 + c^2)$$

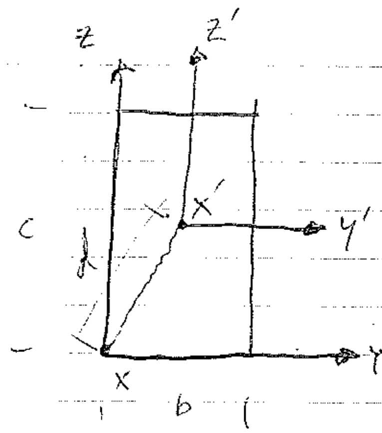
PARALLEL-AXIS THEORUM:

$$I_x = \bar{I}_{x'} + md^2$$

$$\bar{I}_{x'} = I_x - md^2$$

$$d^2 = \left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2$$

$$d^2 = \frac{1}{4}\left(b^2 + c^2\right)$$



$$\bar{I}_{x'} = \frac{m}{3}\left(b^2 + c^2\right) - \frac{m}{4}\left(b^2 + c^2\right)$$

$$\boxed{\bar{I}_{x'} = \frac{m}{12}\left(b^2 + c^2\right)}$$

NOTICE FOR A THIN PLATE:

$$I_{x,\,mass} = \rho t \, I_{x,\,AREA}$$

FOR A 3-D BODY, (HOMOGENEOUS MATL)

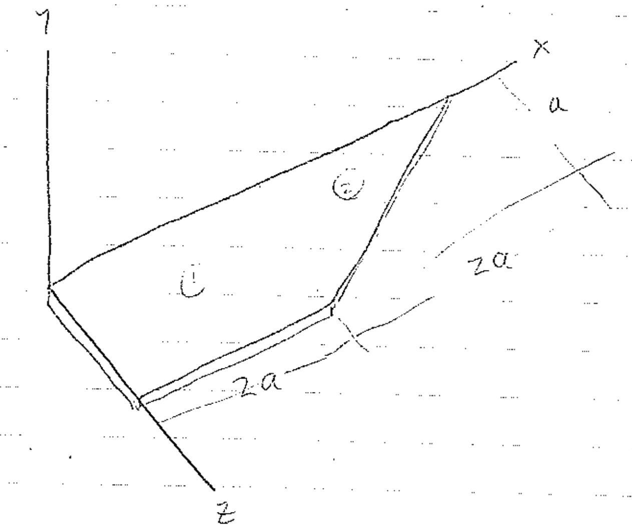
$$I = \rho \int r^2 \, dV$$

FOR COMPOSITE BODIES,

$$I_x = \Sigma\left[\bar{I}_{x'} + m\left(\bar{y}^2 + \bar{z}^2\right)\right]$$

EX.

PROB. 9.117



a) X-AXIS:

$I_x = I_{x,1} + I_{x,2}$

AREA 1: $m_1 = \rho t(a)(2a) = 2\rho t a^2$

$I_{x,1} = \bar{I}_{x,1} + m_1 d^2$

$= \frac{1}{12} m_1 a^2 + m_1 \left(\frac{a}{2}\right)^2$

$= (2\rho t a^2)\left[a^2\left(\frac{1}{12} + \frac{1}{4}\right)\right]$

$I_{x,1} = \frac{2}{3} \rho t a^4$

OR

$I_{x,1} = \rho t \cdot I_{x,1,AREA}$

$= \rho t \cdot \frac{1}{3} b h^3$

$= \rho t \cdot \frac{1}{3}(2a)(a)^3$

$= \frac{2}{3} \rho t a^4$

AREA 2: $m_2 = \rho t \cdot \frac{1}{2}(2a)(a) = \rho t a^2$

$I_{x,2} = \rho t \; I_{x,2,AREA}$

$= \rho t\left[\frac{1}{12}(2a)(a)^3\right]$

$I_{x,2} = \frac{1}{6} \rho t a^4$

9.117 CONT.

$$I_x = I_{x,1} + I_{x,2}$$
$$= \frac{2}{3}\rho t a^4 + \frac{1}{6}\rho t a^4$$
$$I_x = \frac{5}{6}\rho t a^4$$
$$m = m_1 + m_2$$
$$= 2\rho t a^2 + \rho t a^2$$
$$m = 3\rho t a^2$$
$$I_x = (3\rho t a^2)\left(\frac{1}{3}\cdot\frac{5}{6}a^2\right)$$
$$\boxed{I_x = \frac{5}{18} m a^2}$$ IN TERMS OF MASS

(b) Y-AXIS:

$$I_y = I_x + I_z$$
$$I_z = I_{z,1} + I_{z,2}$$

AREA 1: $m_1 = 2\rho t a^2$

$$I_{z,1} = \bar{I}_{z,1} + m_1 d^2$$
$$= \frac{1}{12} m_1 (2a)^2 + m_1 (a)^2 = (2\rho t a^2)\left(\frac{4a^2}{12} + a^2\right)$$
$$= (2\rho t a^4)\left(\frac{1}{3} + 1\right)$$
$$I_{z,1} = \frac{8}{3}\rho t a^4$$

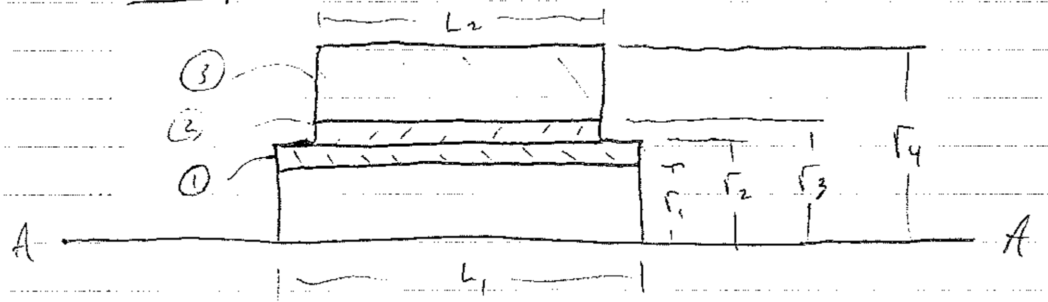
OR
$$I_{z,1} = \rho t \cdot I_{z,1,AREA}$$
$$= \rho t \cdot \frac{1}{3} b h^3$$
$$= \rho t \cdot \frac{1}{3}(a)(2a)^3$$
$$= \frac{8}{3}\rho t a^4$$

AREA 2: $m_2 = \rho t a^2$

9.117 CONT.

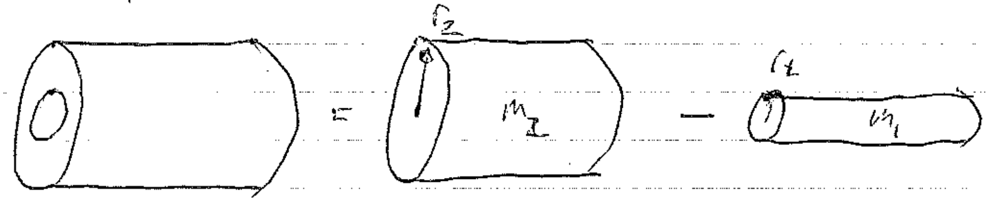
$$I_{z,2} = \rho t \cdot I_{z,2,AREA}$$
$$= \rho t (\bar{I}_{z,2} + Ad^2)$$
$$= \rho t \left\{ \tfrac{1}{36}(a)(2a)^3 + \tfrac{1}{2}(a)(2a) \cdot \left[(2a) + \tfrac{1}{3}(2a)\right]^2 \right\}$$
$$I_{z,2} = \frac{22}{3} \rho t a^4$$
$$I_z = I_{z,1} + I_{z,2}$$
$$= \frac{8}{3}\rho t a^4 + \frac{22}{3}\rho t a^4$$
$$I_z = 10 \rho t a^4$$
$$m = 3\rho t a^2$$
$$I_z = (3\rho t a^2)\left(\tfrac{1}{3} \cdot 10 a^2\right)$$
$$I_z = \frac{10}{3} m a^2$$
$$I_y = I_x + I_z$$
$$= \frac{5}{18} m a^2 + \frac{10}{3} m a^2$$
$$\boxed{I_y = \frac{65}{18} m a^2}$$

Ex. PROB. 9.128



$$\gamma_B = 0.310 \ \frac{\text{LBF}}{\text{IN}^3}, \quad \gamma_A = 0.100 \ \frac{\text{LBF}}{\text{IN}^3}, \quad \gamma_N = 0.0452 \ \frac{\text{LBF}}{\text{IN}^3}$$

$$I_{AA} = I_1 + I_2 + I_3$$



$$I_1 = \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2$$

$$= \frac{1}{2}\left(\gamma_B V_2 r_2^2 - \gamma_B V_1 r_1^2\right) \cdot \frac{1}{g}$$

$$= \frac{\frac{1}{2}\gamma_B}{g}\left(\pi r_2^2 L_1 \cdot r_2^2 - \pi r_1^2 L_1 \cdot r_1^2\right)$$

$$I_1 = \frac{\pi \gamma_B L_1}{2g}\left(r_2^4 - r_1^4\right)$$

$$= \frac{\pi}{2} \frac{\left(0.310 \ \frac{\text{LBF}}{\text{IN}^3}\right)\left(\frac{13}{16} \ \text{IN}\right)}{\left(32.2 \ \frac{\text{ft}}{\text{s}^2}\right)} \left[\left(\frac{3/8}{2} \ \text{IN}\right)^4 - \left(\frac{1/4}{2} \ \text{IN}\right)^4\right]\left(\frac{\text{ft}}{12 \ \text{IN}}\right)$$

$$I_1 = 1.015 \times 10^{-6} \ \text{LBF} \cdot \text{IN} \cdot \text{s}^2$$

$$I_2 = \frac{\pi \gamma_A L_2}{2g}\left(r_3^4 - r_2^4\right)$$

$$= \frac{\pi}{2} \frac{(0.100)\left(\frac{11}{16}\right)}{(32.2)} \left[\left(\frac{1/2}{2}\right)^4 - \left(\frac{3/8}{2}\right)^4\right]\left(\frac{1}{12}\right)$$

$$I_2 = 7.463 \times 10^{-7} \ \text{LBF} \cdot \text{IN} \cdot \text{s}^2$$

9.128 CONT.

$$I_3 = \frac{\pi \gamma_N L_2}{2g}\left(r_4^4 - r_3^4\right)$$

$$= \frac{\pi}{2}\,\frac{(0.0452)\left(\frac{11}{16}\right)}{(32.2)}\left[\left(\frac{1.125}{2}\right)^4 - \left(\frac{1/2}{2}\right)^4\right]\left(\frac{1}{12}\right)$$

$$I_3 = 1.215 \times 10^{-5}\ \text{LBF}\cdot\text{IN}\cdot\text{S}^2$$

$$\boxed{I_{AA} = 1.391 \times 10^{-5}\ \text{LBF}\cdot\text{IN}\cdot\text{S}^2 = 1.16 \times 10^{-6}\ \text{LBF}\cdot\text{FT}\cdot\text{S}^2}$$

RADIUS OF GYRATION:

$$K = \sqrt{\frac{I}{m}}$$

$$m = m_1 + m_2 + m_3$$

$$m_1 = \frac{\gamma_B}{g}\left(\pi r_2^2 L_1 - \pi r_1^2 L_1\right)$$

$$= \frac{\pi \gamma_B L_1}{g}\left(r_2^2 - r_1^2\right)$$

$$= \frac{\pi\left(0.310\ \frac{\text{LBF}}{\text{IN}^3}\right)\left(\frac{13}{16}\ \text{IN}\right)}{\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)}\left[\left(\frac{3/8\ \text{IN}}{2}\right)^2 - \left(\frac{1/4\ \text{IN}}{2}\right)^2\right]\left(\frac{\text{ft}}{12\ \text{IN}}\right)$$

$$m_1 = 4.00 \times 10^{-5}\ \frac{\text{LBF}\cdot\text{S}^2}{\text{IN}}$$

$$m_2 = \frac{\pi \gamma_A L_2}{g}\left(r_3^2 - r_2^2\right)$$

$$= \frac{\pi (0.1)\left(\frac{11}{16}\right)}{(32.2)}\left[\left(\frac{1/2}{2}\right)^2 - \left(\frac{3/8}{2}\right)^2\right]\left(\frac{1}{12}\right)$$

$$m_2 = 1.528 \times 10^{-5}\ \frac{\text{LBF}\cdot\text{S}^2}{\text{IN}}$$

9.128 CONT.

$$M_3 = \frac{\pi \gamma_N L_2}{g}\left(r_4^2 - r_3^2\right)$$

$$= \frac{\pi (0.0452)\left(\frac{11}{16}\right)}{(32.2)}\left[\left(\frac{1.125}{2}\right)^2 - \left(\frac{1/2}{2}\right)^2\right]\left(\frac{1}{12}\right)$$

$$M_3 = 6.415 \times 10^{-5} \ \frac{\text{LBF}\cdot s^2}{\text{IN}}$$

$$M = 1.194 \times 10^{-4} \ \frac{\text{LBF}\cdot s^2}{\text{IN}}$$

$$K = \sqrt{\frac{\left(1.391 \times 10^{-5} \ \text{LBF}\cdot\text{IN}\cdot s^2\right)}{\left(1.194 \times 10^{-4} \ \frac{\text{LBF}\cdot s^2}{\text{IN}}\right)}}$$

$$\boxed{K = 0.3413 \ \text{IN}}$$

Statics Homework Handout 9: Assignment #9: 9.49, 9.50, 9.51, 9.54, 9.115, 9.127

9.31 and 9.32 Determine the moment of inertia and the radius of gyration of the shaded area with respect to the x axis.

9.33 and 9.34 Determine the moment of inertia and the radius of gyration of the shaded area with respect to the y axis.

9.49 Two 20-mm steel plates are welded to a rolled S section as shown. Determine the moments of inertia and the radii of gyration of the section with respect to the centroidal x and y axes.

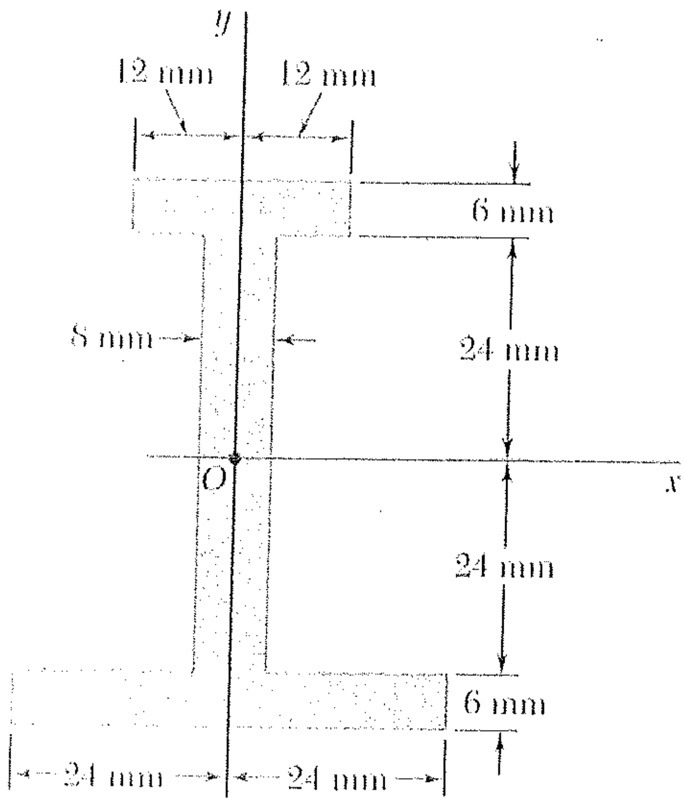


Fig. P9.31 and P9.33

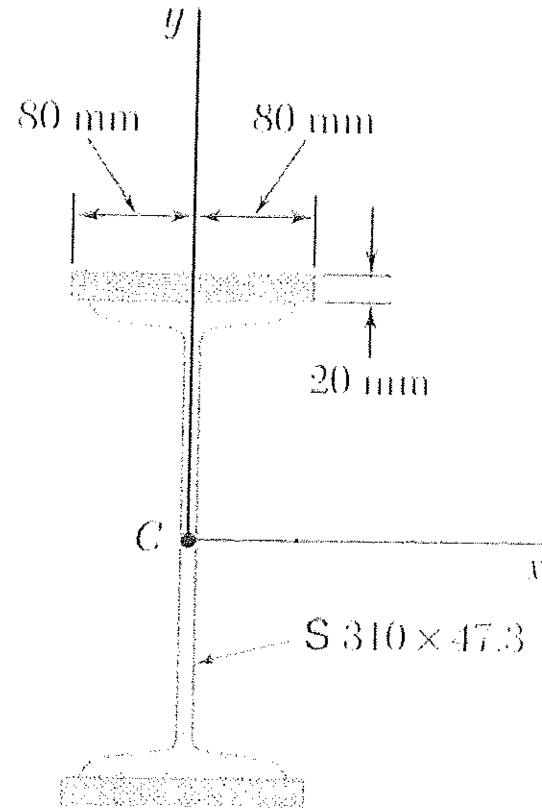


Fig. P9.49

9.50 To form a reinforced box section, two rolled W sections and two plates are welded together. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal axes shown.

9.51 Four $3 \times 3 \times \frac{1}{4}$-in. angles are welded to a rolled W section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to its centroidal x and y axes.

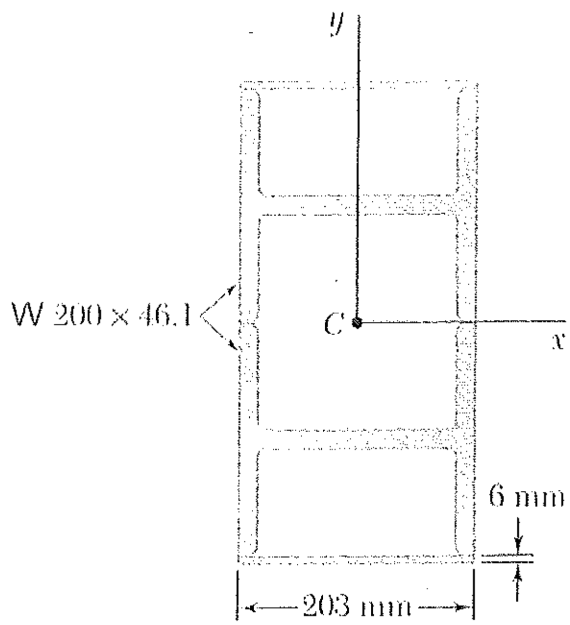


Fig. P9.50

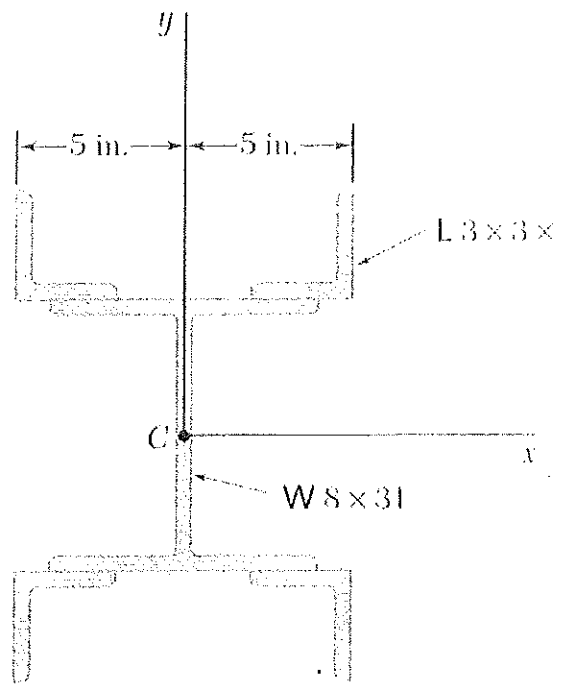


Fig. P9.51

9.54 To form an unsymmetrical girder, two 76 × 76 × 6.4-mm angles and two 152 × 102 × 12.7-mm angles are welded to a 16-mm steel plate as shown. Determine the moments of inertia of the combined section with respect to its centroidal x and y axes.

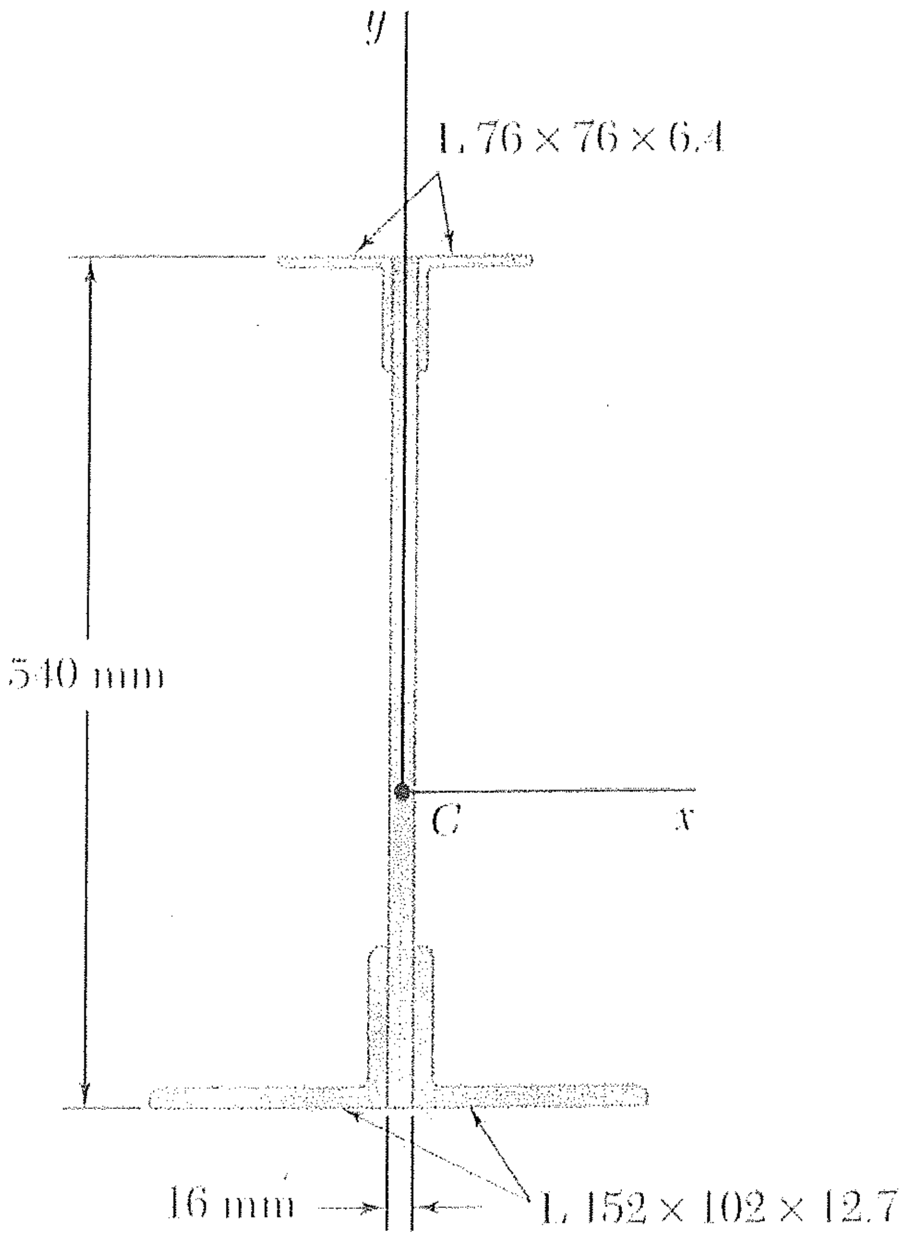


Fig. P9.54

9.115 A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m, determine its moment of inertia with respect to (a) the x axis, (b) the y axis.

9.117 A thin plate of mass m has the trapezoidal shape shown. Determine the mass moment of inertia of the plate with respect to (a) the x axis, (b) the y axis.

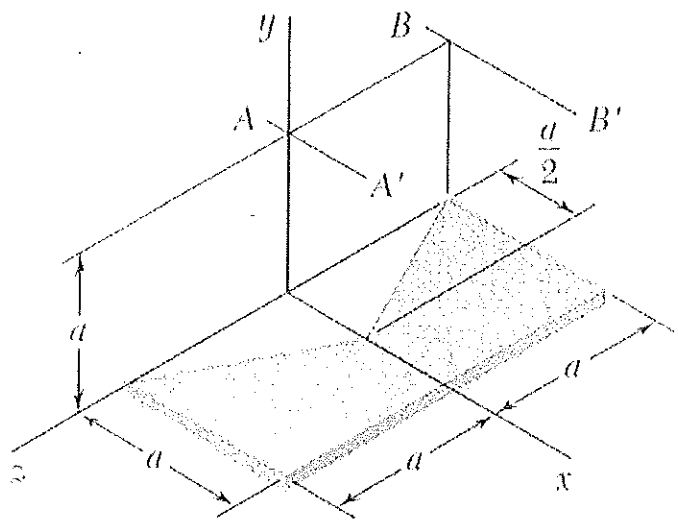


Fig. P9.115 and P9.116

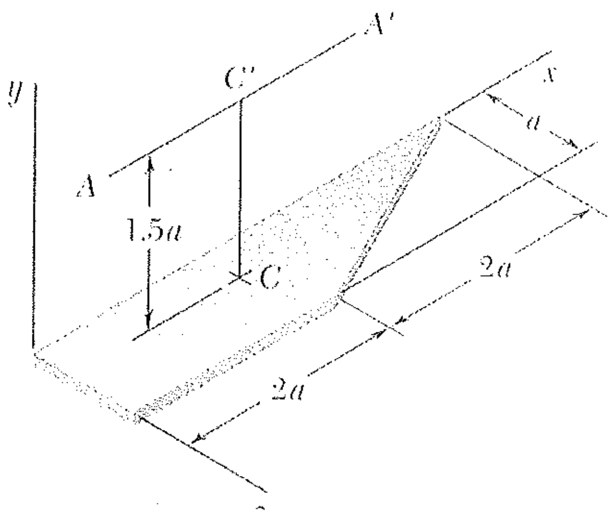


Fig. *P9.117* and *P9.118*

9.127 Shown is the cross section of a molded flat-belt pulley. Determine its moment of inertia and its radius of gyration with respect to the axis AA'. (The density of brass is 8650 kg/m^3 and the density of the fiber-reinforced polycarbonate used is 1250 kg/m^3.)

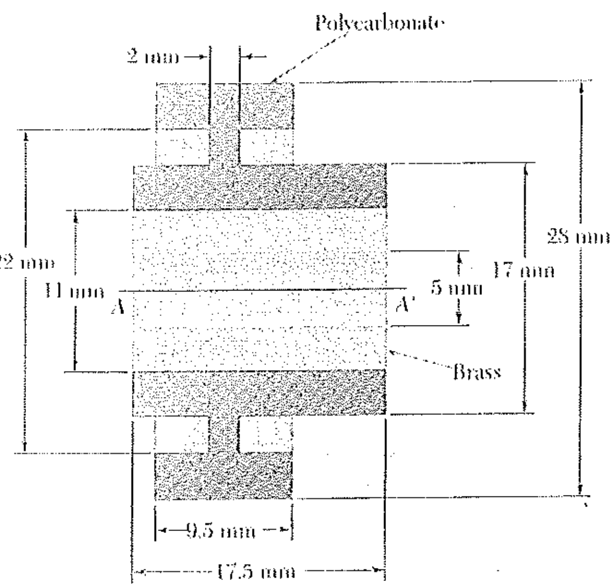


Fig. P9.127

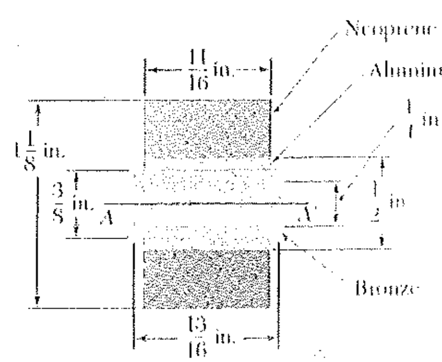


Fig. P9.128

9.128 Shown is the cross section of an idler roller. Determine its mass moment of inertia and its radius of gyration with respect to the axis AA'. (The specific weight of bronze is 0.310 lb/in^3; of aluminum, 0.100 lb/in^3; and of neoprene, 0.0452 lb/in^3.)

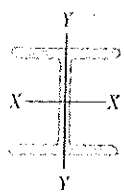
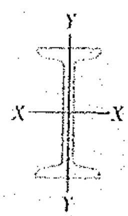
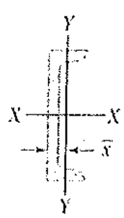
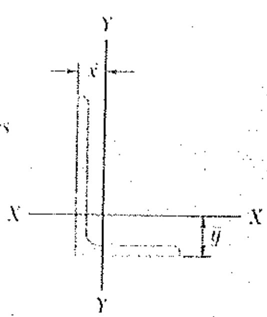
9.31 $390 \times 10^3\ \text{mm}^4$; 21.9 mm.

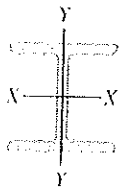
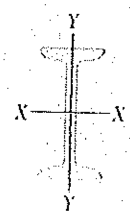
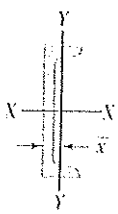
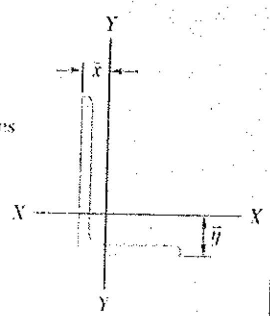
9.32 $64.3 \times 10^3\ \text{mm}^4$; 8.87 mm.

9.49 $\bar{I}_x = 260 \times 10^6\ \text{mm}^4$, $\bar{I}_y = 17.55 \times 10^6\ \text{mm}^4$;
$\bar{k}_x = 144.6$ mm, $\bar{k}_y = 37.6$ mm.
9.50 $\bar{I}_x = 256 \times 10^6\ \text{mm}^4$, $\bar{I}_y = 100.0 \times 10^6\ \text{mm}^4$;
$\bar{k}_x = 134.1$ mm, $\bar{k}_y = 83.9$ mm.
9.51 $\bar{I}_x = 250\ \text{in}^4$, $\bar{I}_y = 141.6\ \text{in}^4$;
$\bar{k}_x = 4.10$ in., $\bar{k}_y = 3.08$ in.

9.115 (a) $7ma^2/18$. (b) $0.819ma^2$.

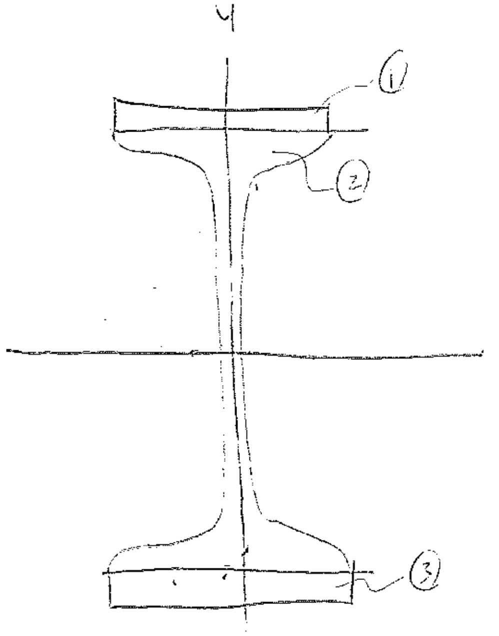
9.127 $837 \times 10^{-9}\ \text{kg} \cdot \text{m}^2$; 6.92 mm.
9.128 $1.160 \times 10^{-6}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$; 0.341 in.

		Designation	Area, in²	Depth, in.	Width, in.	Axis X-X: $\bar{I}_x$, in⁴	Axis X-X: $\bar{k}_x$, in.	Axis X-X: $\bar{y}$, in.	Axis Y-Y: $\bar{I}_y$, in⁴	Axis Y-Y: $\bar{k}_y$, in.	Axis Y-Y: $\bar{x}$, in.
W Shapes (Wide-Flange Shapes)		W18 × 76†	22.3	18.21	11.035	1330	7.73		152	2.61	
		W16 × 57	16.8	16.43	7.120	758	6.72		43.1	1.60	
		W14 × 38	11.2	14.10	6.770	385	5.88		26.7	1.55	
		W8 × 31	9.13	8.00	7.995	110	3.47		37.1	2.02	
S Shapes (American Standard Shapes)		S18 × 55.7†	16.1	18.00	6.001	804	7.07		20.8	1.14	
		S12 × 31.8	9.35	12.00	5.000	218	4.83		9.36	1.00	
		S10 × 25.4	7.46	10.00	4.661	124	4.07		6.79	0.954	
		S6 × 12.5	3.67	6.00	3.332	22.1	2.45		1.82	0.705	
C Shapes (American Standard Channels)		C12 × 20.7†	6.09	12.00	2.942	129	4.61		3.88	0.799	0.698
		C10 × 15.3	4.49	10.00	2.600	67.4	3.87		2.28	0.713	0.634
		C8 × 11.5	3.38	8.00	2.260	32.6	3.11		1.32	0.625	0.571
		C6 × 8.2	2.40	6.00	1.920	13.1	2.34		0.692	0.537	0.512
Angles		L6 × 6 × 1†	11.00			35.5	1.80	1.86	35.5	1.80	1.86
		L4 × 4 × $\frac{1}{2}$	3.75			5.56	1.22	1.18	5.56	1.22	1.18
		L3 × 3 × $\frac{1}{4}$	1.44			1.24	0.930	0.842	1.24	0.930	0.842
		L6 × 4 × $\frac{1}{2}$	4.75			17.4	1.91	1.99	6.27	1.15	0.987
		L5 × 3 × $\frac{1}{2}$	3.75			9.45	1.59	1.75	2.58	0.829	0.750
		L3 × 2 × $\frac{1}{4}$	1.19			1.09	0.957	0.993	0.392	0.574	0.493

		Designation	Area mm²	Depth mm	Width mm	Axis X-X: $\bar{I}_x$ 10^6 mm⁴	Axis X-X: $\bar{k}_x$ mm	Axis X-X: $\bar{y}$ mm	Axis Y-Y: $\bar{I}_y$ 10^6 mm⁴	Axis Y-Y: $\bar{k}_y$ mm	Axis Y-Y: $\bar{x}$ mm
W Shapes (Wide-Flange Shapes)		W460 × 113†	14400	463	280	554	196.3		63.3	66.3	
		W410 × 85	10800	417	181	316	170.7		17.94	40.6	
		W360 × 57	7230	358	172	160.2	149.4		11.11	39.4	
		W200 × 46.1	5890	203	203	45.8	88.1		15.44	51.3	
S Shapes (American Standard Shapes)		S460 × 81.4†	10390	457	152	335	179.6		8.66	29.0	
		S310 × 47.3	6032	305	127	90.7	122.7		3.90	25.4	
		S250 × 37.8	4806	254	118	51.6	103.4		2.83	24.2	
		S150 × 18.6	2362	152	84	9.2	62.2		0.758	17.91	
C Shapes (American Standard Channels)		C310 × 30.8†	3929	305	74	53.7	117.1		1.615	20.29	17.73
		C250 × 22.8	2897	254	65	28.1	98.3		0.949	18.11	16.10
		C200 × 17.1	2181	203	57	13.57	79.0		0.549	15.88	14.50
		C150 × 12.2	1548	152	48	5.45	59.4		0.288	13.64	13.00
Angles		L152 × 152 × 25.4†	7100			14.78	45.6	47.2	14.78	45.6	47.2
		L102 × 102 × 12.7	2420			2.31	30.9	30.0	2.31	30.9	30.0
		L76 × 76 × 6.4	929			0.516	23.6	21.4	0.516	23.6	21.4
		L152 × 102 × 12.7	3060			7.24	48.6	50.5	2.61	29.2	25.1
		L127 × 76 × 12.7	2420			3.93	40.3	44.5	1.074	21.1	19.05
		L76 × 51 × 6.4	768			0.454	24.3	25.2	0.163	14.58	12.52

HOMEWORK #9 : 9.49, 9.50, 9.51, 9.54, 9.115, 9.127

PROB. 9.49



FIND I_x, I_y

FOR I_x

$$(I_x)_1 = \bar{I}_x + Ad^2$$
$$= \tfrac{1}{12} bh^3 + bhd^2$$
$$= \tfrac{1}{12}(160\,mm)(20\,mm)^3 + (160)(20)\left(\tfrac{305}{2} \cancel{+10}\right)^2$$

$$(I_x)_1 = 8.46 \times 10^7\ mm^4$$

$$(I_x)_3 = 8.46 \times 10^7\ mm$$

$$(I_x)_2 = 90.7 \times 10^6\ mm^4$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 = 2.6 \times 10^8\ mm^4$$

PROB. 9.49

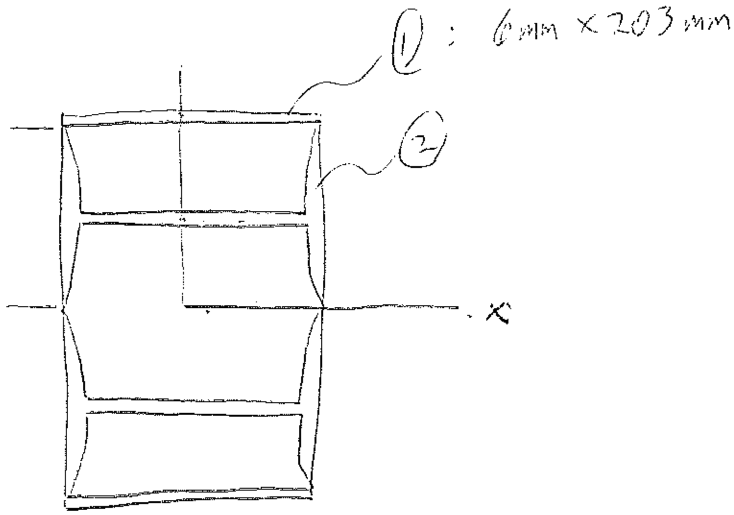
$$(I_y)_1 = \frac{1}{12}b^3h = \frac{1}{12}(160)^3(20) = 6.83 \times 10^6 \text{ mm}^4$$

$$(I_y)_3 = 6.83 \times 10^6 \text{ mm}^4$$

$$(I_y)_2 = 3.9 \times 10^6 \text{ mm}^4$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 = 1.70 \times 10^7 \text{ mm}^4$$

PROB. 9.50



$$(I_x)_1 = 2\left[\frac{1}{12}bh^3 + Ad^2\right]$$

$$= 2\left[\frac{1}{12}(203\text{mm})(6\text{ mm})^3 + (203)(6)\left(\frac{203}{2} + 3\right)^2\right]$$

$$(I_x)_1 = 1.03 \times 10^8 \text{ mm}^4$$

$$(I_x)_2 = 2\left[\bar{I}_x + Ad^2\right]$$

$$= 2\left[15.44 \times 10^6 + (5890)\left(\frac{203}{2}\right)^2\right]$$

PROB. 9.50

$$(I_x)_2 = 1.52 \times 10^8 \text{ mm}^4$$

$$I_x = (I_x)_1 + (I_x)_2 = 2.55 \times 10^8 \text{ mm}^4$$

$$(I_y)_1 = 2\left[\frac{1}{12} b^3 h\right]$$

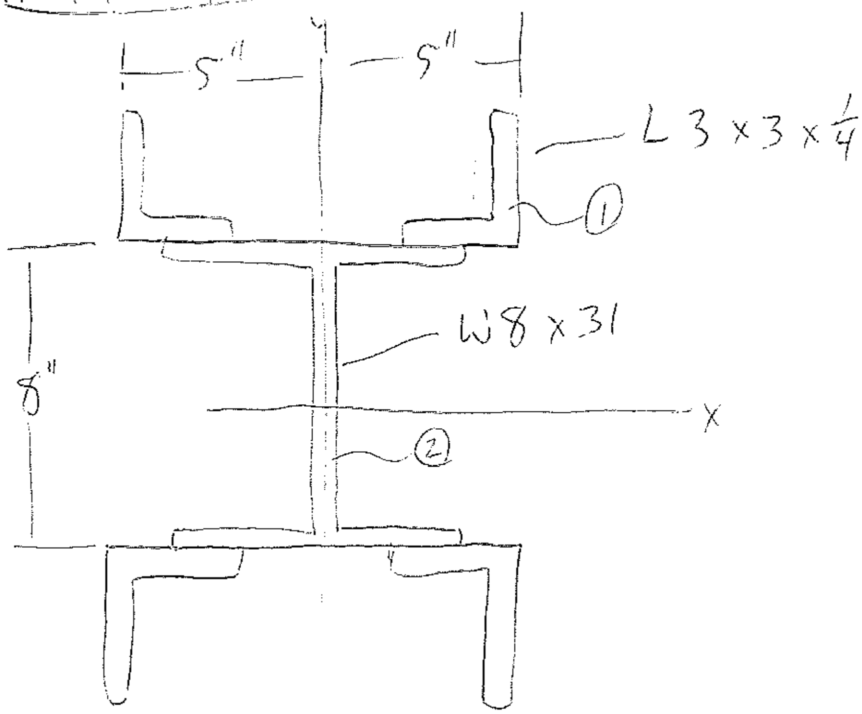
$$= 2\left[\frac{1}{12}(203)^3(6)\right]$$

$$(I_y)_1 = 8.36 \times 10^6 \text{ mm}^4$$

$$(I_y)_2 = \cancel{48}\ 2(45.8 \times 10^6) = 9.16 \times 10^7 \text{ mm}^4$$

$$I_y = (I_y)_1 + (I_y)_2 = 10^8 \text{ mm}^4$$

PROB. 9.57



$$(I_x)_1 = 4\left[\bar{I}_x + Ad^2\right]$$

$$= 4\left[(1.24\text{ IN}^4) + (1.44\text{ IN}^2)(4\text{ IN} + 0.842\text{ IN})^2\right]$$

$$(I_x)_1 = 1.4 \times 10^2\text{ IN}^4$$

$$(I_x)_2 = 110\text{ IN}^4$$

$$I_x = (I_x)_1 + (I_x)_2 = 250\text{ IN}^4$$

$$(I_y)_1 = 4(\bar{I}_y + Ad^2)$$

$$= 4\left[(1.24) + (1.44)(5 - 0.842)^2\right]$$

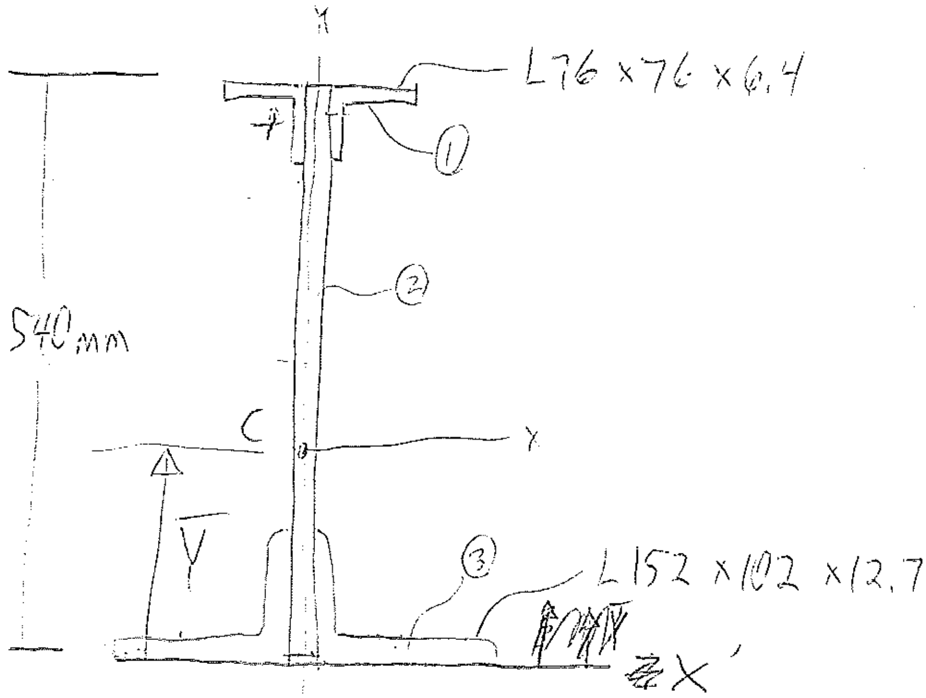
$$(I_y)_1 = 104.5\text{ IN}^4$$

PROB. 9.51

$$(I_y)_2 = 37.1 \text{ IN}^4$$

$$(I_y) = (I_y)_1 + (I_y)_2 = 142 \text{ IN}^4$$

PROB. 9.54



FIRST, FIND CENTROID OF BEAM.

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

FOR AREA 1: $\bar{y}_1 = 540 - 21.4 = 519 \text{ mm}$

$A_1 = 2(929 \text{ mm}^2) = 1860 \text{ mm}^2$

FOR AREA 2: $\bar{y}_2 = \frac{1}{2}(540 \text{ mm}) = 270 \text{ mm}$

$A_2 = (16 \text{ mm})(540 \text{ mm}) = 8640 \text{ mm}^2$

PROB. 9.54

~~FOR AREA 3: $\bar{y}_3 = 50.5\,mm$,~~

~~$A_3 = 2(3060\,mm^2) = 6120\,mm^2$~~

$$\cancel{\bar{Y} = \frac{(519)(1860) + (270)(8640) + (50.5)(6120)}{(1860) + (8640) + (6120)}}$$

$$\cancel{\bar{Y} = 217\,mm}$$

FOR AREA 3: $\bar{y}_3 = 25.1\,mm$

$A_3 = 2(3060\,mm^2) = 6120\,mm^2$

$$\bar{Y} = \frac{(519)(1860) + (270)(8640) + (25.1)(6120)}{(1860) + (8640) + (6120)}$$

$$\bar{Y} = 208\,mm$$

$$(I_x)_1 = 2(\bar{I}_x + Ad^2)$$

$$= 2\left[(0.516 \times 10^6\,mm^4) + (929\,mm^2)(540 - 208 - 21.4)^2\right]$$

$$(I_x)_1 = 1.8 \times 10^8\,mm^4$$

$$(I_x)_3 = 2(\bar{I}_x + Ad^2)$$

$$= 2\left[(2.61 \times 10^6\,mm^4) + (3060)(208 - 25.1)^2\right]$$

$$(I_x)_3 = 2.1 \times 10^8\,mm^4$$

PROB. 9.54

$$(I_x)_{2^+} = \frac{1}{3} b h^3$$

$$= \frac{1}{3}(16)(540-208)^3$$

$$(I_x)_{2^+} = 1.95 \times 10^8 \ mm^4$$

$$(I_x)_{2^-} = \frac{1}{3} b h^3$$

$$= \frac{1}{3}(16)(208)^3$$

$$(I_x)_{2^-} = 4.8 \times 10^7 \ mm^4$$

$$\boxed{I_x = 6.33 \times 10^8 \ mm^4}$$

$$(I_y)_1 = 2(\bar{I}_y + Ad^2)$$

$$= 2\left[(0.516 \times 10^6) + (929)(8 + 21.4)^2\right]$$

$$(I_y)_1 = 2.64 \times 10^6 \ mm^4$$

$$(I_y)_2 = \frac{1}{12} b^3 h$$

$$= \frac{1}{12}(16)^3(540)$$

$$(I_y)_2 = 1.84 \times 10^5 \ mm^4$$

PROB. 9.54

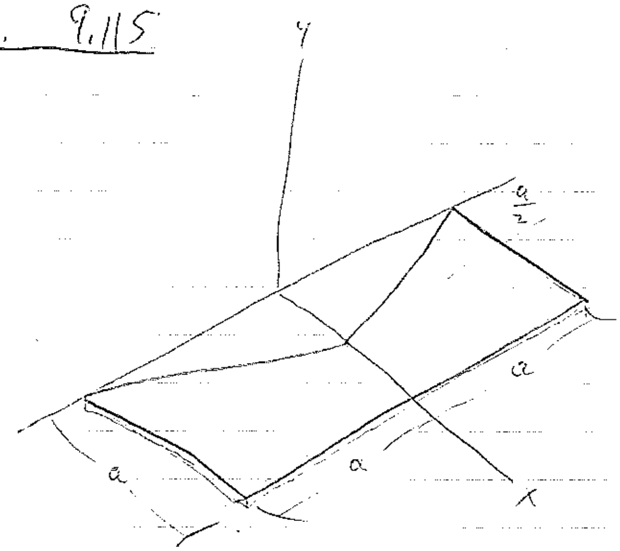
$$(I_y)_3 = 2\left[\bar{I}_y + Ad^2\right]$$

$$= 2\left[(7.24 \times 10^6) + (3060)(8 + 50.5)^2\right]$$

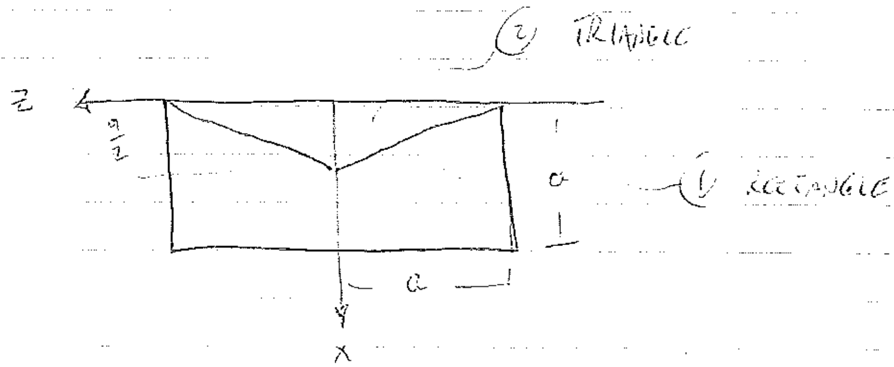
$$(I_y)_3 = 3.54 \times 10^7 \text{ mm}^4$$

$$\boxed{I_y = 3.82 \times 10^7 \text{ mm}^4}$$

PROB. 9.115



a) X-AXIS:



AREA 1: $m_1 = \rho t (a)(2a) = 2\rho t a^2$

$$I_{x,1} = \frac{1}{12} m_1 (2a)^2 = \frac{1}{3} m_1 a^2$$

$$= \frac{1}{3}(2\rho t a^2)a^2$$

$$I_{x,1} = \frac{2}{3}\rho t a^4$$

AREA 2: $m_2 = -\rho t\left(\frac{a}{2}\right)(a) = -\frac{1}{2}\rho t a^2$

$$I_{x,MASS} = -2\rho t \cdot I_{x,AREA}$$

$$= -2\rho t\left[\frac{1}{12}\left(\frac{a}{2}\right)(a)^3\right]$$

$$I_{x,MASS} = -\frac{1}{12}\rho t a^4$$

$$I_x = I_{x,1} + I_{x,2}$$

$$= \frac{2}{3}\rho t a^4 - \frac{1}{12}\rho t a^4$$

$$I_x = \frac{7}{12}\rho t a^4$$

$$m = m_1 + m_2$$

$$= 2\rho t a^2 - \frac{1}{2}\rho t a^2$$

$$m = \frac{3}{2}\rho t a^2$$

$$I_x = \left(\frac{3}{2}\rho t a^2\right)\left(\frac{2}{3}\cdot\frac{7}{12}a^2\right)$$

$$\boxed{I_x = \frac{7}{18} m a^2}$$

b) Y-AXIS:

$$I_y = I_{y,1} + I_{y,2}$$

AREA 1: $m_1 = 2\rho t a^2$

$$I_{y,1} = \bar{I}_{y,1} + m_1 d^2$$

$$= \frac{1}{12} m_1\left[(a)^2 + (2a)^2\right] + m_1\left(\frac{a}{2}\right)^2$$

$$= \frac{1}{12} m_1\cdot(5a^2) + m_1\cdot\frac{1}{4}a^2$$

$$I_{y,1} = \frac{2}{3} m_1 a^2$$

$$= \frac{2}{3}(2\rho t a^2)a^2$$

$$I_{y,1} = \frac{4}{3}\rho t a^4$$

9.115 CONT.

AREA 2:

$$I_{y,2} = I_{x,2} + I_{z,2}$$

$$I_{x,2} = \rho t \; I_{x,2,AREA}$$

$$= \rho t \left[2 \cdot \tfrac{1}{12} \left(\tfrac{a}{2}\right)(a)^3 \right]$$

$$I_{x,2} = \tfrac{1}{12} \rho t a^4$$

$$I_{z,2} = \rho t \; I_{z,2,AREA}$$

$$= \rho t \left[\tfrac{1}{12} (2a)\left(\tfrac{a}{2}\right)^3 \right]$$

$$I_{z,2} = \tfrac{1}{48} \rho t a^4$$

$$I_{y,2} = I_{x,2} + I_{z,2}$$

$$= \tfrac{1}{12} \rho t a^4 + \tfrac{1}{48} \rho t a^4$$

$$I_{y,2} = \tfrac{5}{48} \rho t a^4$$

$$I_y = I_{y,1} - I_{y,2}$$

$$= \tfrac{4}{3} \rho t a^4 - \tfrac{5}{48} \rho t a^4$$

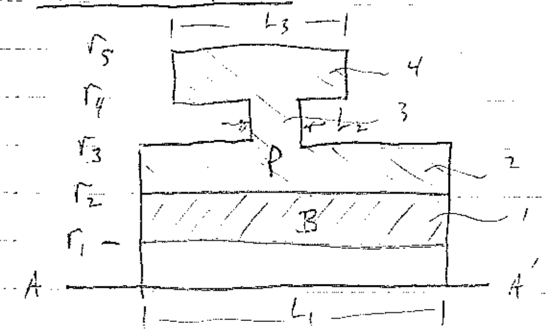
$$I_y = \tfrac{59}{48} \rho t a^4$$

$$m = \tfrac{3}{2} \rho t a^2$$

$$I_y = \left(\tfrac{3}{2} \rho t a^2\right)\left(\tfrac{2}{3} \cdot \tfrac{59}{48} a^2\right)$$

$$\boxed{I_y = \frac{59}{72} m a^2 = 0.8194 m a^2}$$

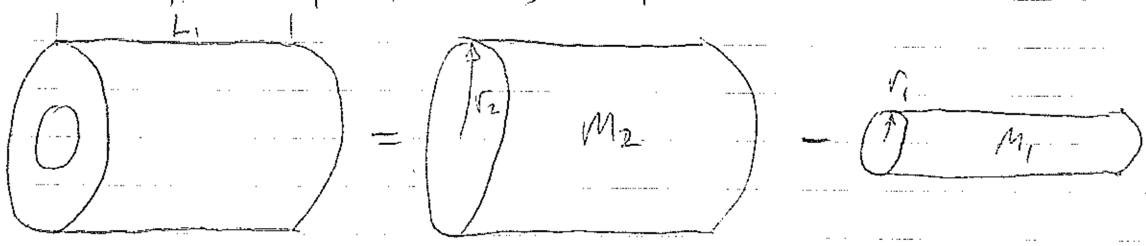
PROB. 9.127



$$\rho_B = 8650 \ \frac{kg}{m^3}$$

$$\rho_P = 1250 \ \frac{kg}{m^3}$$

$$I_{AA} = I_1 + I_2 + I_3 + I_4$$



$$I_1 = \frac{1}{2} m_2 r_2^2 - \frac{1}{2} m_1 r_1^2$$

$$= \frac{1}{2}\left(\rho_B V_2 r_2^2 - \rho_B V_1 r_1^2\right)$$

$$= \frac{1}{2} \rho_B \left(\pi r_2^2 L \cdot r_2^2 - \pi r_1^2 L \cdot r_1^2\right)$$

$$I_1 = \frac{\pi}{2} \rho_B L_1 \left(r_2^4 - r_1^4\right)$$

$$I_1 = \frac{\pi}{2}\left(8650 \ \frac{kg}{m^3}\right)(17.5 \ mm)\left[\left(\frac{11}{2} \ mm\right)^4 - \left(\frac{5}{2} \ mm\right)^4\right]\left(\frac{m}{1000 \ mm}\right)^5$$

$$I_1 = 2.083 \times 10^{-7} \ kg\text{-}m^2$$

$$I_2 = \frac{\pi}{2} \rho_P L_1 \left(r_3^4 - r_2^4\right)$$

$$= \frac{\pi}{2}(1250)(17.5)\left[\left(\frac{17}{2}\right)^4 - \left(\frac{11}{2}\right)^4\right]\left(\frac{1}{1000}\right)^5$$

$$I_2 = 1.479 \times 10^{-7} \ kg\text{-}m^2$$

9.127 CONT.

$$I_3 = \frac{\pi}{2} \rho_p L_2 (r_4^4 - r_3^4)$$

$$= \frac{\pi}{2}(1250)(2)\left[\left(\frac{22}{2}\right)^4 - \left(\frac{17}{2}\right)^4\right]\left(\frac{1}{1000}\right)^5$$

$$I_3 = 3.7 \times 10^{-8} \text{ Kg-m}^2$$

$$I_4 = \frac{\pi}{2} \rho_p L_3 (r_5^4 - r_4^4)$$

$$= \frac{\pi}{2}(1250)(9.5)\left[\left(\frac{28}{2}\right)^4 - \left(\frac{22}{2}\right)^4\right]\left(\frac{1}{1000}\right)^5$$

$$I_4 = 4.435 \times 10^{-7} \text{ Kg-m}^2$$

$$\boxed{I_{AA} = 8.367 \times 10^{-7} \text{ Kg-m}^2}$$