

Control Flow Analysis

Adapted from Lectures by
Prof. Saman Amarasinghe (MIT)

Outline

- Control-Flow Analysis
- Dominators
- Graph Traversal
- Reducible Graphs
- Interval Analysis

L3ICFA

Constant Propagation

```
int sumcalc(int a, int b, int N)
{
    int i;
    int x, y;
    x = 0;
    y = 0;
    for(i = 0; i <= N; i++) {
        x = x + (4*a/b)*i + (i+1)*(i+1);
        x = x + b*y;
    }
    return x;
}
```

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        x = x + b*0;
    }
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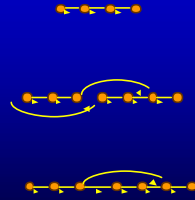
Implementing constant propagation

- Find an RHS expression which is a constant
- Replace the use of the LHS variable with the RHS constant given that:
 - All paths to the use of LHS variable passes the assignment of the LHS variable with the constant
 - There are no intervening definition of the LHS variable
- Need to know the “control-flow” of the program

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Representing the control flow of the program

- Most instructions
 - execute the next instruction
 - straight line control-flow
- Jump instructions
 - execute from different location
 - jump in control-flow
- Branch instructions
 - execute either the next instruction or from a different location
 - fork in the control-flow



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Representing the control flow of the program

- Forms a graph
-
- The diagram shows a complex graph of nodes (yellow dots) and edges (yellow arrows). The nodes are arranged in a roughly horizontal line, but many edges loop back or branch to other nodes, creating a dense network of connections.
- Observation
 - a very large graph
 - lot of straight-line connections
 - simplify the graph by grouping some instructions

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Basic Blocks

- A **basic block** is a maximal sequence of instructions such that
 - Only the first instruction can be reached from outside the basic block
 - All the instructions are executed consecutively if the first instruction is executed
 - No branch or jump instructions in the basic block, except the last instruction
 - No labels within the basic block, except before the first instruction

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Control Flow Graph (CFG)

- Control-Flow Graph $G = \langle N, E \rangle$
- Nodes(N): Basic Blocks
- Edges(E): $(x,y) \in E$ iff first instruction in the basic block y follows the last instruction in the basic block x
 - First instruction in y is the target of branch or jump instruction (last instruction) in the basic block x
 - first instruction of y is next after the last instruction of x in memory and the last instruction of x is not a jump instruction

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Control Flow Graph (CFG)

- Block with the first instruction of the procedure is the entry node (block with the procedure label)
- The blocks with the return instruction (jsr) are the exit nodes.
 - Can make a single exit node by adding a special node

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Why do we perform Control-flow Analysis

- Loops are important to optimize
 - Programs spend a lot of times in loops and recursive cycles
 - Many special optimizations can be done on loops
- Programmers organize code using structured control-flow (if-then-else, for-loops etc.)
 - Optimizer can exploit this, once they are discovered

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Challenges in Control-Flow Analysis

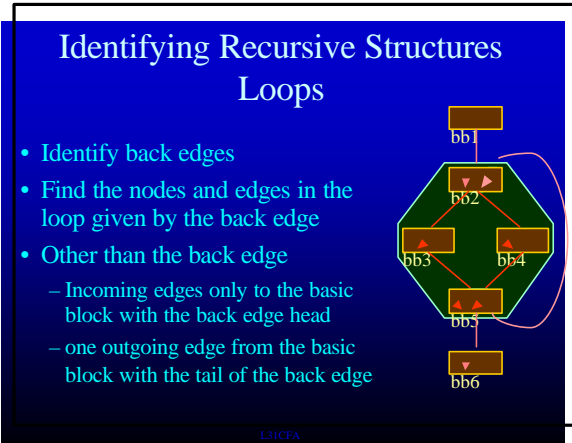
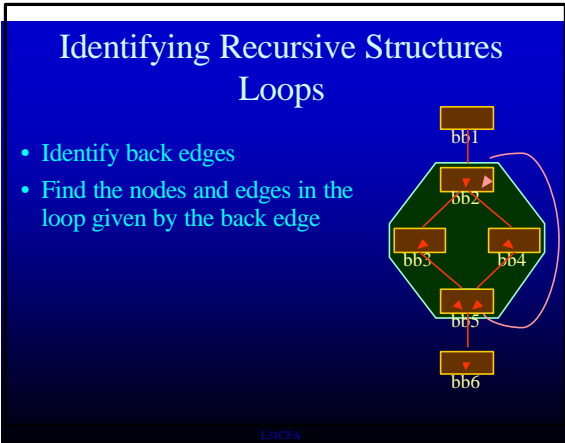
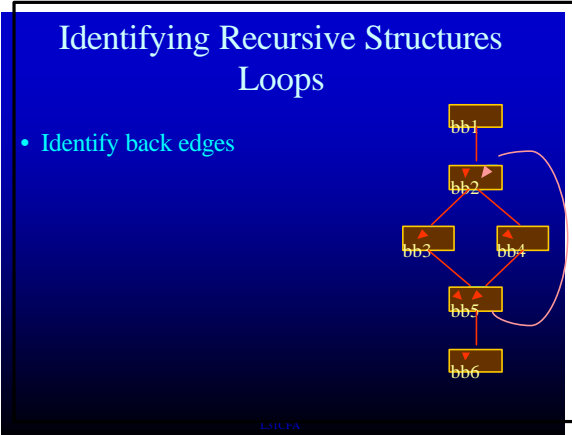
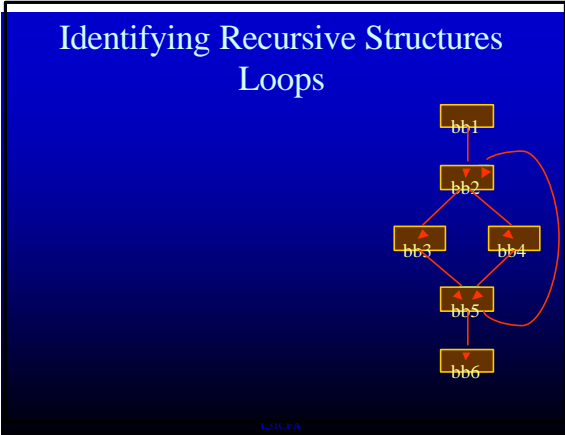
- Unstructured Control Flow
 - Use of `goto`'s by the programmer
 - Only way to build certain control structures
- Obscured control flow
 - method invocations
 - procedure variables
 - higher-order functions
 - jump tables

```

L1: x = 0
   if (y > 0) goto L3
L2: if (y < 0) goto L1
L3: y = y + z
   goto L2

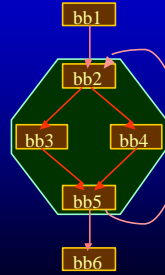
Myobject->run()
  
```

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Identifying Recursive Structures Loops

- Identify back edges
- Find the nodes and edges in the loop given by the back edge
- Other than the back edge
 - Incoming edges only to the basic block with the back edge head
 - one outgoing edge from the basic block with the tail of the back edge
- How do I find the back edges?



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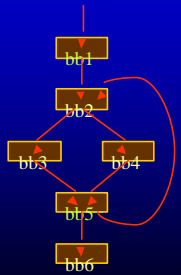
Dominators

- Node x dominates node y ($x \text{ dom } y$) if every possible execution path from entry to node y includes node x

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Dominators

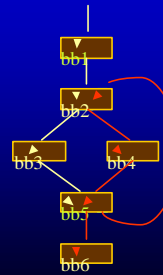
- Is $bb1 \text{ dom } bb5$?



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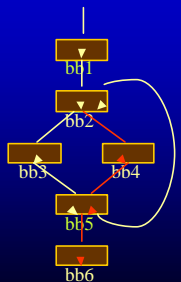
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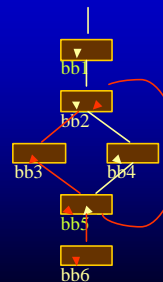
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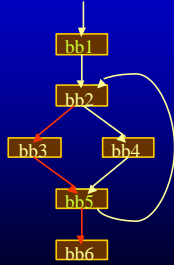
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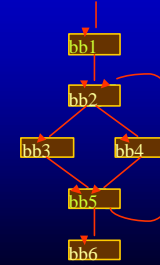
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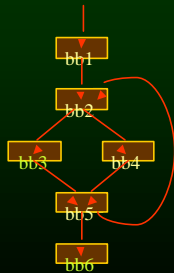
- Is bb1 dom bb5? **Yes!**



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QUESTION

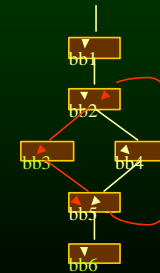
- Is bb1 dom bb5? **Yes!**
- Is bb3 dom bb6?



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QUESTION

- Is bb1 dom bb5? **Yes!**
- Is bb3 dom bb6? **No!**



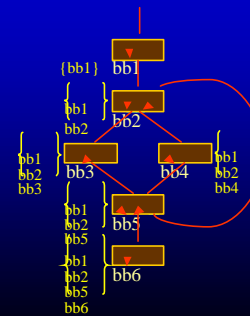
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Computing Dominators

- $a \text{ dom } b$ iff
 - $a = b$ or
 - a is the unique immediate predecessor of b or
 - a is a dominator of all immediate predecessor of b
- **Algorithm**
 - Make dominator set of the entry node have itself
 - Make dominator set of the rest have all the nodes
 - Visit the nodes in any order
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 - Repeat until no change

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Dominators (Desired Solution)

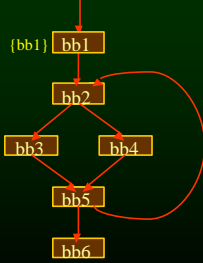


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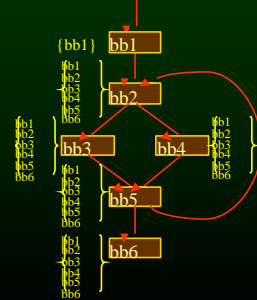


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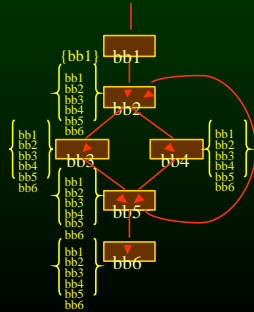


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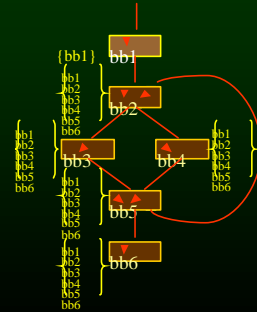


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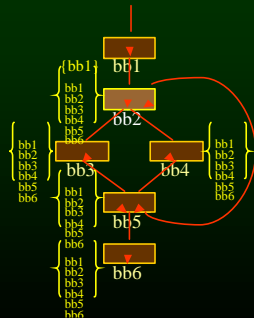


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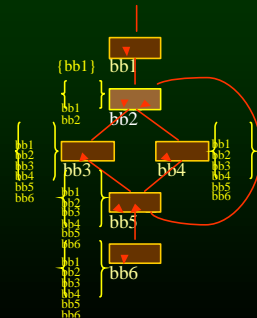


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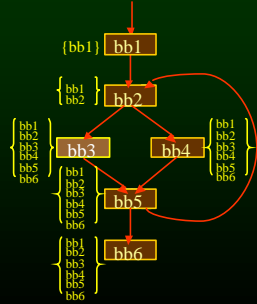


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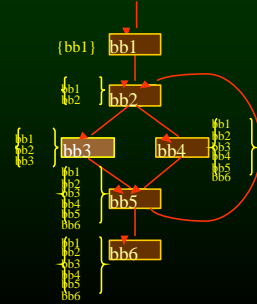


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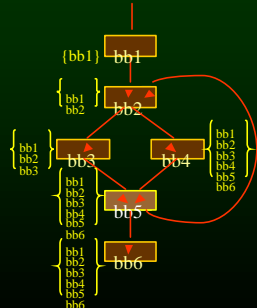


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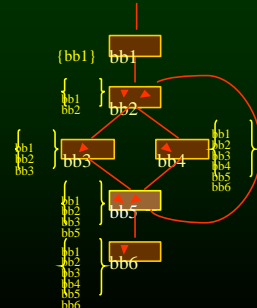


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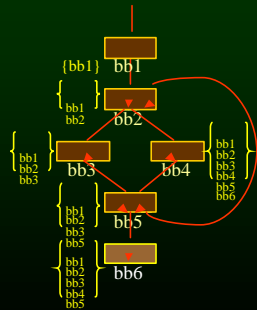


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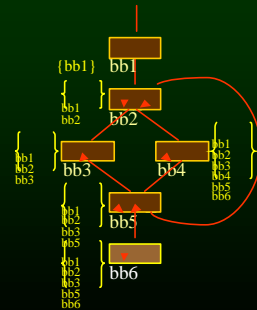


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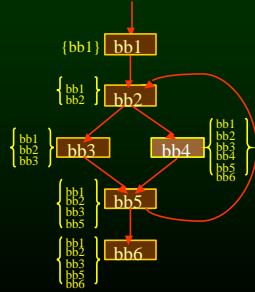


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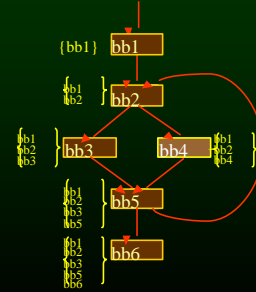


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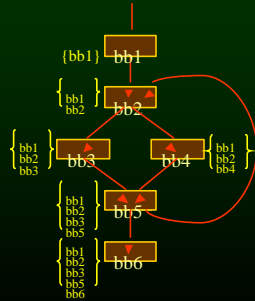


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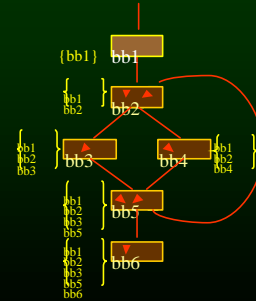


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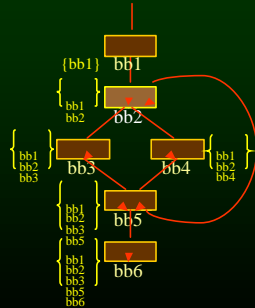


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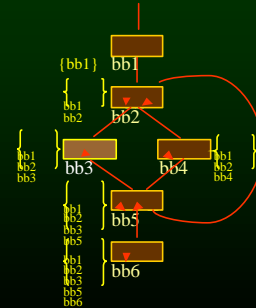


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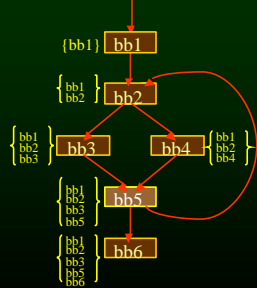


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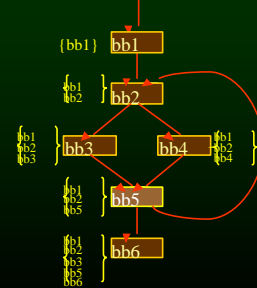


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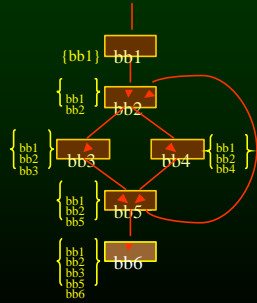


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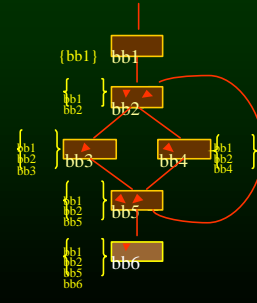


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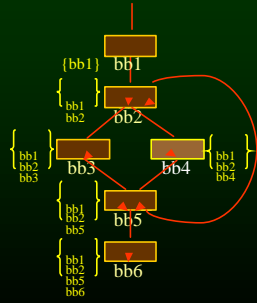


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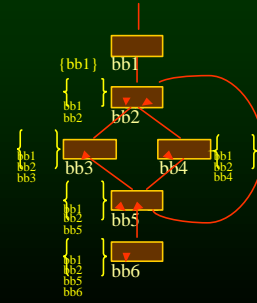


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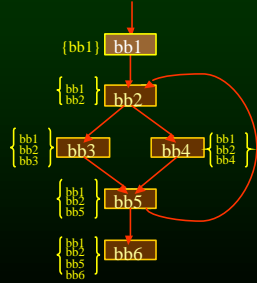


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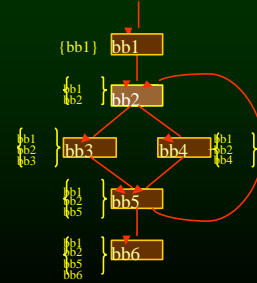


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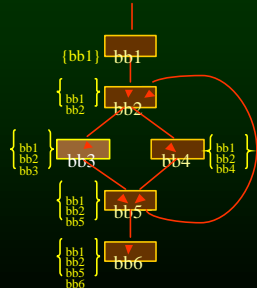


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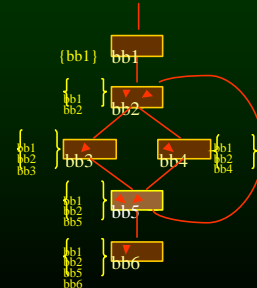


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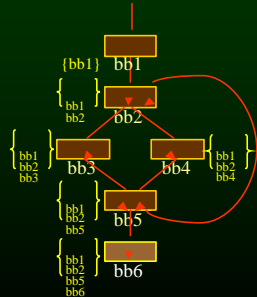


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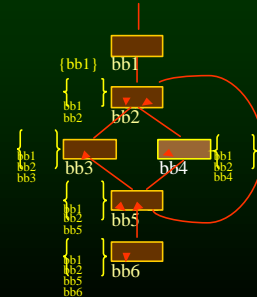


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Computing Dominators

- Algorithm

- Make dominator set of the entry node have itself
- Make dominator set of the rest have all the nodes
- Visit the nodes in any order
- Make dominator set of the current node intersection of the dominator sets of the predecessor nodes + the current node
- Repeat until no change

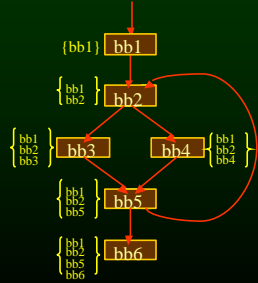


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Computing Dominators

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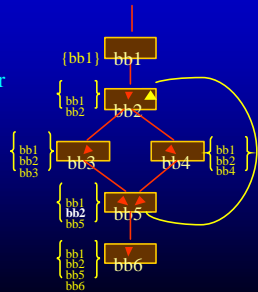
Computing Dominators

- What we just witnessed was an iterative data-flow analysis algorithm in action
 - Initialize all the nodes to a given value
 - Visit nodes in some order
 - Calculate the node's value
 - Repeat until no value changes

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What is a back edge?

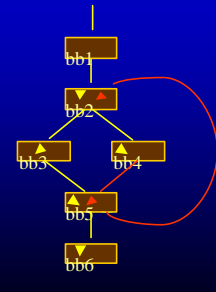
- An edge $(x, y) \in E$ is a back edge iff $y \text{ dom } x$
 - is node y in the dominator set of node x



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Traversing the CFG

- **Depth-First Traversal**
 - Visit all the descendants of a node before visiting any siblings
- **Depth-first spanning tree**
 - a set of edges corresponding to a depth-first visitation of CFG



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Preorder and Postorder

- In preorder traversal, each node is processed before its descendants in the depth-first tree
- In postorder traversal, each node is processed after its descendants in the depth-first tree

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Outline

- Control-Flow Analysis
- Dominators
- Graph Traversal
- **Reducible Graphs**
- Interval Analysis

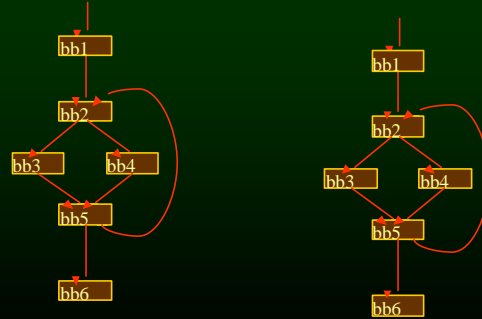
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Reducible CFGs

- Reducibility formalizes well structured-ness of a program
- A graph is **reducible** iff repeated application of the following two actions yields a graph with only one node
 - replace self loop by a single node
 - replace a sequence of nodes such that all the incoming edges are to the first node and all the outgoing edges are to the last node

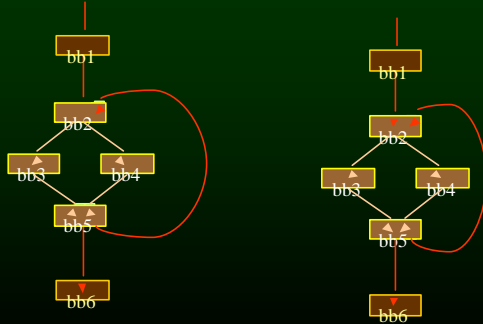
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Reducible CFGs



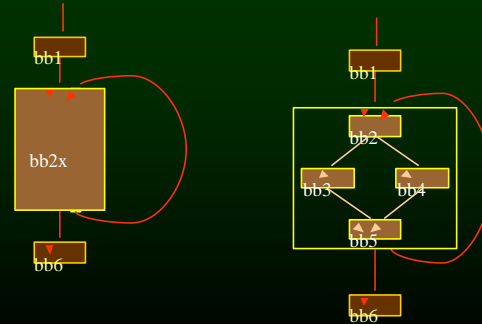
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Reducible CFGs



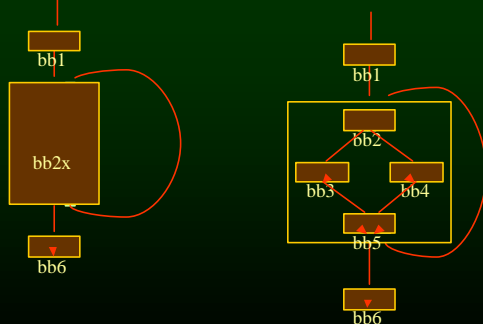
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Reducible CFGs



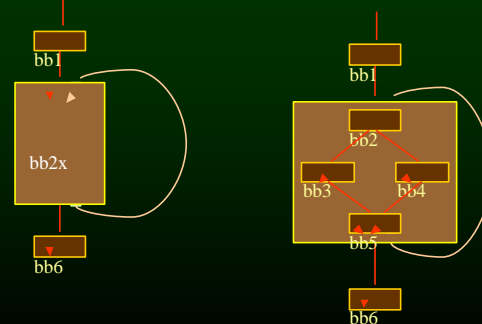
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Reducible CFGs

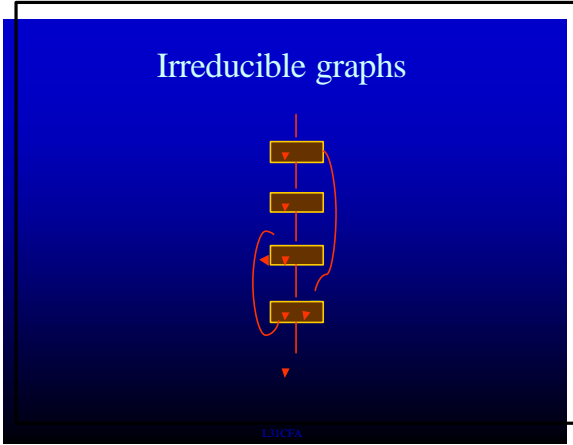
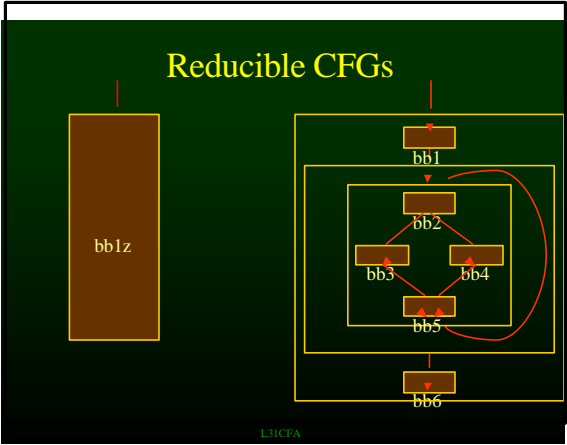
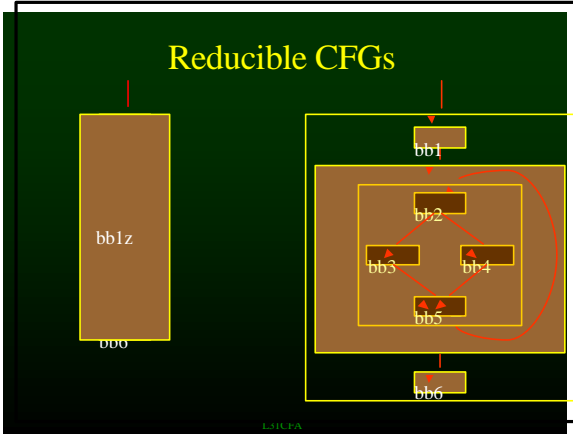
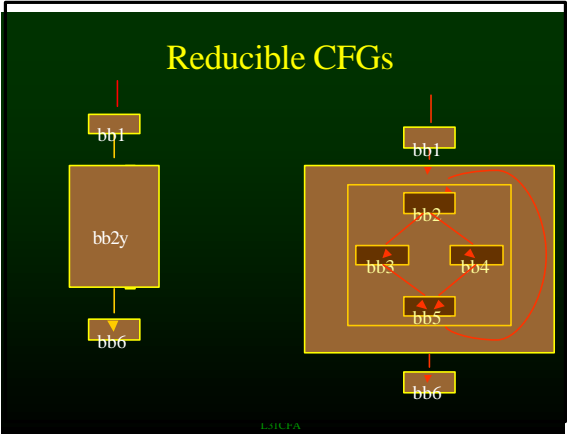
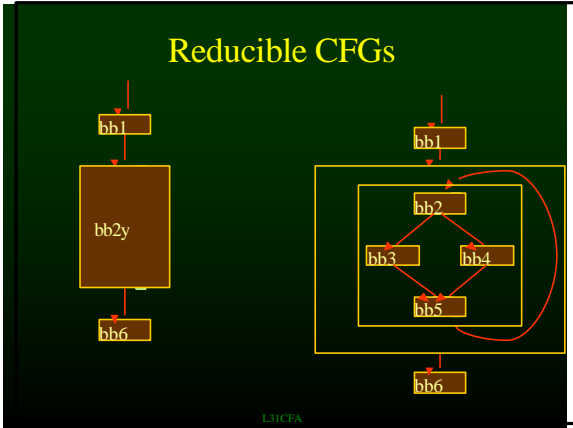
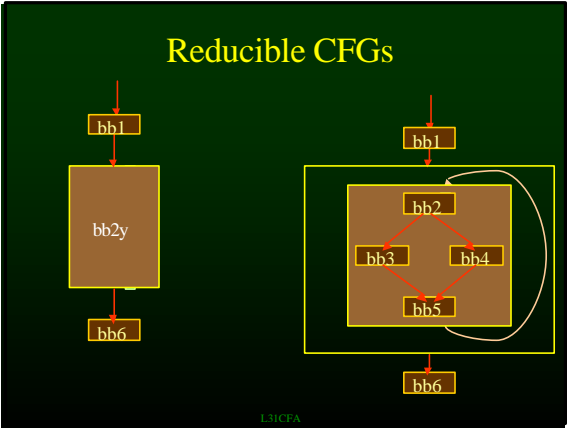


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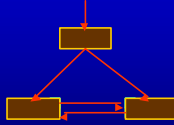
Reducible CFGs



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Irreducible graphs



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Approaches of Control-Flow Analysis

- Iterative Analysis
 - Use a CFG
 - Propagate values
 - Iterate until no change
- Interval Based Analysis
 - Use a reducible CFG
 - Calculate in hierarchical graphs
 - No iterations (faster)

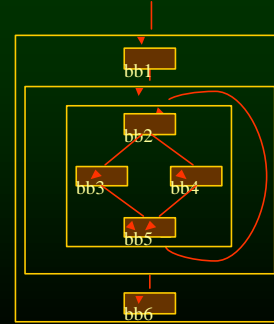
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Interval Based Analysis

- If a node does not include a graph:
 - calculate value
- If a node includes a graph
 - Calculate values of the nodes in that graph
 - Propagate values (no back edges \Rightarrow no iteration)
 - Use entry (or exit) value as the value of the enclosing node

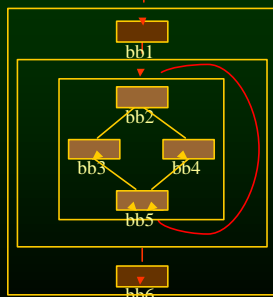
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Interval Analysis



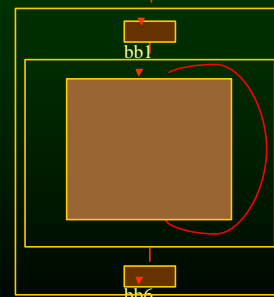
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Interval Analysis

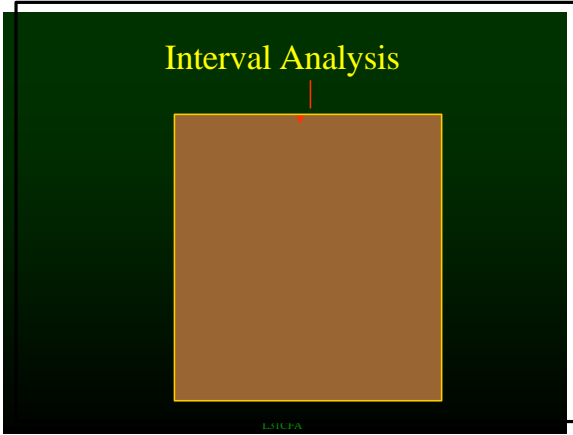
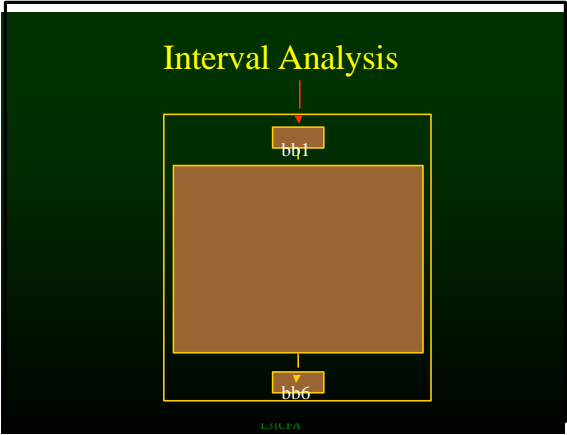
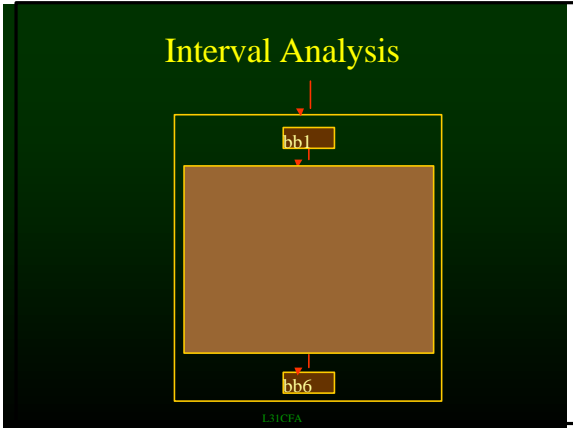
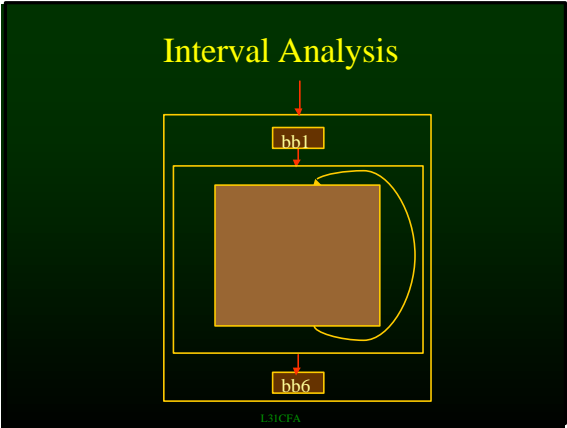


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Interval Analysis



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- ### Outline
- Control-Flow Analysis
 - Dominators
 - Graph Traversal
 - Reducible Graphs
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- L3ICFA