Surface Approximation to Scattered Lines

Abstract–A method for fitting a surface to line data is introduced. A weighted-mean approach to the problem is described using rational Gaussian (RaG) weights. The Gaussians are stretched along the lines and their widths are selected to reproduce sharp edges and corners in the surface. When the length of the lines becomes zero, the formulation converges to a weighted sum of points. Therefore, traditional weighted mean approximations that are defined in terms of rational weights become a special case of the proposed method. Implementation strategies are detailed and examples of the method in approximation of range data and polygon meshes are presented.

Index Terms–Computational geometry and object modeling, surface representation, approximation, rational approximation, shape

1 Introduction

A surface is often defined as a weighted sum of points [3, 4, 5, 8, 10, 14, 19]. Examples defining a surface by a weighted sum of polynomials are also available in the literature. Catmull and Rom [1] defined a surface by a blending of polynomials, and as a special case, Goshtasby [7] defined a surface by a weighted sum of planes (polynomials of degree 1). Surfaces that are defined in terms of points can also be considered a weighted sum of polynomials where the degree of the polynomials is zero. If a surface is defined parametrically, a component of the surface is defined by a weighted sum of a component of the points, which can be considered planes of slope zero in the parameter space.

In this paper, a surface that is defined by a weighted sum of lines is introduced. The motivation behind this development is to provide the ability to reproduce sharp edges and corners in a surface without sacrificing the degree of continuity of the surface. The proposed surface formulation has a very high degree of continuity even at very sharp edges and corners.

Current representations are not designed to model sharp edges and corners. To produce sharp edges in a shape, subdivision methods have been used. Subdivision rules have been changed at the edges to avoid smoothing of the edges [2, 9].

Forcing a surface that is defined in terms of points to produce a sharp edge or corner in one place may result in high fluctuations somewhere else in the surface. The formulation proposed here is specifically developed to produce sharp edges and corners in a surface. The formulation is written as a weighted sum of lines, enabling the surface to make a sharp turn in the vicinity of a line approximating it. With a smoothness parameter that can be varied interactively or through an automated process, the surface can be made to get as close as desired to the lines and reproduce them, or smooth the lines as desired and produce a very smooth surface.

In the following sections, first, the problem to be solved is described. Then, a solution to the problem using rational Gaussian (RaG) weights [6, 12] is proposed. Next, applications of the method in smooth approximation of polygon meshes are presented and concluding remarks are made.

2 Problem Description

A set of scattered line segments $\{L_i(x, y) : i = 1, \ldots, N\}$ defined by their endpoints: $\{(x_{1_i}, y_{1_i}), (x_{2_i}, y_{2_i}) : i = 1, \ldots, N\}$ is given. If data at the end points of line $L_i(x, y)$ are (Z_{i_1}, Z_{i_2}), we will have $L_i(x_{i_1}, y_{i_1}) = Z_{i_1}$ and $L_i(x_{i_2}, y_{i_2}) = Z_{i_2}$. We would like to find a surface $f(x, y)$ that satisfies

$$f(x, y) \approx L_i(x, y), \qquad i = 1, \ldots, N, \tag{1}$$

at points on the line segments.

3 Approach

The surface for approximation of scattered lines is obtained by extending a surface formulation that approximates scattered points. Consider fitting a single-valued surface to scattered points $\{(x_i, y_i) : i = 1, \ldots N\}$ with associating data values $\{Z_i : i = 1, \ldots N\}$. An example of scattered points is given in Fig. 1a. If we use the single-valued RaG surface [6] as the approximating surface, we will have

$$f(x, y) = \frac{\sum_{i=1}^{N} Z_i W_i G_i(x, y)}{\sum_{i=1}^{N} W_i G_i(x, y)}. \tag{2}$$

This is a weighted mean method with rational weight functions defined by

$$\frac{W_i G_i(x, y)}{\sum_{i=1}^{N} W_i G_i(x, y)}, \qquad i = 1, \ldots, N. \tag{3}$$

$G_i(x, y)$ is a 2-D Gaussian centered at the ith data point, (x_i, y_i), and W_i is the weight or the importance of that data point with respect to other data points in determining the surface. If the data value at (x_i, y_i) is Z_i, surface $f(x, y)$ will approximate Z_i at (x_i, y_i). The standard deviations of the Gaussians are free parameters that can be varied to produce desired levels of detail in the obtained surface. The standard deviations of all Gaussians may be set to the same value σ to enable control of the global smoothness/detailedness of the surface.

Now, consider using a data line in place of a data point. For the sake of simplicity, let's first assume that data along a line do not vary and all lines are parallel to the x-axis. An example of such data lines is given in Fig. 1b. Therefore, instead of point (x_i, y_i) we will have a line with end points (x_{i_1}, y_i) and (x_{i_2}, y_i) and the same data value Z_i everywhere along the line. To fit a surface to these lines, we will stretch a Gaussian horizontally proportional to the length of the associating line.

Assuming the coordinates of the midpoint of the ith line are (x_i, y_i), since a 2-D Gaussian can be decomposed into two 1-D Gaussians, we have

$$\begin{aligned} G_i(x,y) &= e^{-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma^2}}, \\ &= e^{-\frac{(x-x_i)^2}{2\sigma^2}} e^{-\frac{(y-y_i)^2}{2\sigma^2}}, \\ &= G_i(x)G_i(y). \end{aligned} \tag{4}$$

To stretch $G_i(x)$ along the x-axis, we scale down the x coordinates by m_i, or equivalently, increase σ by a factor of m_i to obtain

$$H_i(x) = e^{-\frac{(x-x_i)^2}{2(m_i\sigma)^2}}, \tag{5}$$

Note that $m_i > 1$. We let $m_i = (1 + \epsilon_i)$ and set ϵ_i proportional to the length of the ith line. Therefore, relation (2) becomes

$$f(x,y) = \frac{\sum_{i=1}^{N} Z_i W_i H_i(x) G_i(y)}{\sum_{i=1}^{N} W_i H_i(x) G_i(y)}. \tag{6}$$

Now, let's suppose data along a line vary linearly, but the lines are still parallel to the x-axis. Fig. 1c shows a change in the height of one line segment as x is changed. To fit a surface to the new set of lines, instead of using a Gaussian of a fixed height Z_i, we let the height of the Gaussian vary with data along the line. Assuming data at the endpoints of the ith line are Z_{i_1} and Z_{i_2}, and Z_i is the data value at the line midpoint, in equation (6), we will replace Z_i with

$$Z_i(x) = Z_i + (x - x_i)(Z_{i_2} - Z_i)/(x_{i_2} - x_i) \tag{7}$$

to change the height of the Gaussian along the line proportional to data to obtain

$$f(x,y) = \frac{\sum_{i=1}^{N} Z_i(x) W_i H_i(x) G_i(y)}{\sum_{i=1}^{N} W_i H_i(x) G_i(y)}. \tag{8}$$

Finally, to adapt the approximation to lines with arbitrary orientations like the example given in Fig. 1d, first, each line is rotated about its center so it becomes parallel to the x-axis. Then, the above formulation is used to find its influence on the surface. Finally, the computed values are rotated back to their original positions. The process is carried out line by line and the contributions from the lines are added together to create the approximating surface.

Suppose the ith line makes angle θ_i with the x-axis. We then rotate the coordinate system about the line's midpoint by θ_i clockwise so that it becomes parallel to the x-axis. If the coordinates of points on the line before and after this rotation are (X, Y) and (x, y), respectively, we will have

$$x = (X - X_i)\cos\theta_i - (Y - Y_i)\sin\theta_i + x_i \tag{9}$$

$$y = (X - X_i)\sin\theta_i + (Y - Y_i)\cos\theta_i + y_i. \tag{10}$$

Substituting relations (9) and (10) into the right side of equation (8), we obtain a relation in (X, Y), showing the surface value at (X, Y) in the approximation domain. Therefore,

$$f(X,Y) = \frac{\sum_{i=1}^{N} Z_i(X,Y) W_i H_i(X,Y) G_i(X,Y)}{\sum_{i=1}^{N} W_i H_i(X,Y) G_i(X,Y)} \tag{11}$$

where

$$Z_i(X,Y) = Z_i + ((X - X_i)\cos\theta_i - (Y - Y_i)\sin\theta_i)(Z_{i_2} - Z_i)/D_i, \tag{12}$$

$$H_i(X,Y) = e^{-\frac{[(X-X_i)\cos\theta_i - (Y-Y_i)\sin\theta_i]^2}{2(m\sigma)^2}}, \tag{13}$$

and

$$G_i(X,Y) = e^{-\frac{[(X-X_i)\sin\theta_i + (Y-Y_i)\cos\theta_i]^2}{2\sigma^2}}. \tag{14}$$

Quantity

$$D_i = (x_{i_2} - x_i) = \sqrt{(X_{i_2} - X_i)^2 + (Y_{i_2} - Y_i)^2} \tag{15}$$

is half the length of the ith line in the xy or XY domain. To ensure that longer line segments have more influence on the created surface than shorter ones, weight W_i of line L_i is set equal to its length. Therefore, $W_i = 2D_i$. A more appropriate weight would calculate the length of the line using data values as well as the coordinates of the line endpoints. That is,

$$W_i = \sqrt{(x_{i_2} - x_{i_1})^2 + (y_{i_2} - y_{i_1})^2 + (Z_{i_2} - Z_{i_1})^2}. \tag{16}$$

Substituting (12)–(14) and (16) into (11) we obtain a single-valued surface approximating line data.

4 Examples

A simple example demonstrating the proposed idea is given in Fig. 2. Fig. 2a shows seven line segments. The coordinates of the line endpoints as well the associating data values are shown in Table 1. Fig. 2b shows the surface approximating the lines according to formula (11). Although the surface approximates the lines, flat spots are obtained along the lines. This is a known property of weighted mean methods. Since the sum of the weights is required to be 1 everywhere in the approximation domain, when the weight functions are monotonically decreasing and have rather small widths, flat spots appear at and in the neighborhood of the points, or in our case, at and in the neighborhood of the line segments. To avoid the creation of such flat spots, instead of single-valued surfaces, we will use parametric surfaces. Therefore, if instead of the single-valued surface given by (11), we use the parametric surface given by

$$f_x(u,v) = \frac{\sum_{i=1}^{N} W_i Z_i(u,v) H_i(u,v) G_i(u,v)}{\sum_{i=1}^{N} W_i H_i(u,v) G_i(u,v)}, \tag{17}$$

$$f_y(u,v) = \frac{\sum_{i=1}^{N} W_i X_i H_i(u,v) G_i(u,v)}{\sum_{i=1}^{N} W_i H_i(u,v) G_i(u,v)}, \tag{18}$$

$$f_z(u,v) = \frac{\sum_{i=1}^{N} W_i Y_i H_i(u,v) G_i(u,v)}{\sum_{i=1}^{N} W_i H_i(u,v) G_i(u,v)}, \tag{19}$$

we will obtain the surface shown in Fig. 2c. f_x, f_y, f_z are the X, Y, and Z components of the surface, each obtained by varying u and v from 0 to 1. Parameter coordinates at the line midpoints and line endpoints are set proportional to the XY coordinates of the line midpoints and line endpoints, respectively. That is,

$$u_i = (X_i - X_{min})/(X_{max} - X_{min}) \tag{20}$$

$$u_{i_1} = (X_{i_1} - X_{min})/(X_{max} - X_{min}) \tag{21}$$

$$u_{i_2} = (X_{i_2} - X_{min})/(X_{max} - X_{min}) \tag{22}$$

$$v_i = (Y_i - Y_{min})/(Y_{max} - Y_{min}) \tag{23}$$

$$v_{i_1} = (Y_{i_1} - Y_{min})/(Y_{max} - Y_{min}) \tag{24}$$

$$v_{i_2} = (Y_{i_2} - Y_{min})/(Y_{max} - Y_{min}) \tag{25}$$

where X_{min}, X_{max}, Y_{min}, and Y_{max} define the approximation domain.

Formulas (17)–(19) describe a parametric surface approximating the lines. The larger the flat spots in the single-valued surface, the higher the density of points will be in the parametric surface near the data lines and the closer the surface will be to the lines. Fig. 2c shows the parametric surface approximating the lines in Fig. 2a. Increasing the standard deviation of the Gaussians (the smoothness parameter of the surface), a smoother surface is obtained as shown in Fig. 2d, and decreasing the smoothness parameter produces a surface that passes closer to the lines and reproduces more details as can be observed in Fig. 2e. Fig. 2f shows a view of this surface from the opposite side. Portions of the lines hidden by the surface from the front view are visible in the rear view.

A more complex example of surface fitting by the proposed method is given in Fig. 3. Fig. 3a shows a range map of a room scene. Closer points are shown brighter. Linear features in the image are detected and shown in Fig 3b. The line segments are then used to recover the scene geometry using parameter coordinates of line end points proportional to the image coordinates of the line end points. The geometry recovered by formulas (17)–(19) is shown in Fig. 3c. The lines are overlaid with the recovered surface to show the quality of approximation. Parts of line segments in front of the surface are visible in this image. The remainder of the line segments are behind the surface and so are not visible.

An example of the proposed method in approximation of triangle meshes is given in Fig. 4. Fig. 4a shows a triangle mesh obtained by triangulating a digital elevation map of an area over the Grand Canyon. The triangle mesh in shaded form is shown in Fig. 4b. Using the edges of the triangles as input to the proposed surface formulation, and using uv parameters proportional to the xy coordinates of the mesh points, the approximating surface shown in Fig. 4c is obtained. The triangle mesh is overlaid with the surface to show the quality of approximation. Increasing the smoothness parameters, the surface shown in Fig. 4d is obtained. The surfaces closely follow the mesh smoothing mesh edges and vertices.

Since the proposed surface is in parametric form, the lines are not required to represent single-valued data. Rather, they can represent any geometry as long as parameter coordinates at the line endpoints are known. Examples of cylindrical surfaces are given in Fig. 5. Table 2 shows the

XYZ coordinates as well as the uv coordinates at the twelve line endpoints. These line segments represent four sides of a cube, defining a cylindrical surface. The lines are shown in Fig. 5a and the surface obtained from the lines is depicted in Fig. 5b. Reducing the smoothness parameter σ pulls the surface closer to the lines as depicted in Fig. 5c. Another example of a cylindrical surface is depicted in Figs. 5d–f. Eight more line segments are added to the set of lines used in Fig. 5a, connecting the center of the top and bottom faces of the cube to the four corners as depicted in Fig. 5d. More specifically, the XYZ and uv coordinates at the twenty line end points are given in Table 3. There are two very small holes on the top and bottom of the cylindrical surface, which become invisible after quantization. The approximating surface reproduces the surface of the cube closely as depicted in Fig. 5f.

A more complex example of a cylindrical surface is depicted in Fig. 6. Fig. 6a shows triangulation of a range scan of a person's knee area. The mesh in shaded form is shown in Fig. 6b. Using the edges of the mesh as input to the proposed weighted linear method and using cylindrical parametrization, the surface shown in Fig. 6c was obtained. Increasing the smoothness parameter reduces the noisy details in the surfaces and captures the global shape of the knee as shown in Fig. 6d.

The proposed method can, in general, be used to fit a surface to scattered lines. For single-valued data, parameters u and v of the line endpoints can be set proportional to the x and y coordinates of the points. For more complex geometries, paramerization can be a challenge. Methods to parametrize points in geometries that can be mapped to a plane, cylinder, sphere, or torus have been proposed [11, 16]. The weakness of the proposed method compared to subdivision methods [13, 17, 18] can be considered in requiring parameters at the mesh points. The proposed method, however, has advantages over subdivision methods. For instance, it is not required to have a complete triangle mesh to obtain a surface. Some of the triangle edges or vertices can be missing and the process will still work. The proposed method also has a smoothness parameter that can be changed to suppress noisy details in the constructed surface. This function, if not impossible, is difficult to achieve in subdivision methods.

5 Conclusions

If scattered data in a plane are triangulated, approximation to the data can be achieved by 1) approximating the triangle vertices, 2) approximating the triangle faces, and 3) approximating the triangle edges. This paper described a method for approximating the triangle edges. Methods that approximate triangle faces can reproduce nearly planar faces in the surface [7]. Methods that approximate triangle edges can reproduce sharp edges. The method proposed here does not require that all edges in a polygon mesh be provided or that entire edge segments be given. Rather, partial edge information is sufficient to create a surface that approximates the mesh.

If a polygon mesh is parametrized, the proposed method can be considered an alternative to subdivision methods for smoothly approximating the mesh. The surface obtained by the proposed method has a high degree of continuity everywhere. Moreover, very accurate surface normals can be calculated, creating very smooth surfaces at very high resolutions. The proposed method is particularly suitable in applications where user interaction is allowed to change the smoothness parameter until a desired level of detail is reproduced in the created surface.

References

[1] E. Catmull and R. Rom, A class of local interpolating splines, in *Computer Aided Geometric Design*, R. Barnhill and R. Riesenfeld (eds.), Academic Press, New York, 1974, pp. 317–326.

[2] T. DeRose, M. Kass, and T. Truong, Subdivision surfaces in character animation, *Proc. ACM SIGGRAPH*, 1998, pp. 85–94.

[3] G. Farin, *Curves and Surfaces for Computer Aided Geometric Design*, Academic Press, New York, 1988.

[4] R. Franke, Scattered data interpolation: Test of some methods, *Mathematics of Computation*, vol. 38, no. 157, 1982, pp. 181–200.

[5] R. Franke and L. L. Schumaker, A bibliography of multivariate approximation, *Topics in Multivariate Approximation*, Academic Press, 1987, pp. 275–335.

[6] A. Goshtasby, Design and recovery of 2-D and 3-D shapes using rational Gaussian curves and surfaces, *Int'l J. Computer Vision*, 1993, pp. 233–256.

[7] A. Goshtasby, A weighted mean approach to smooth parametric representation of polygon meshes, *The Visual Computer*, vol. 20, no. 5, 2004, pp. 344–359.

[8] E. Grosse, A catalogue of algorithms for approximation, *Algorithms for Approximation II*, J. Mason and M. Cox (eds.), Chapman and Hall, 1990, pp. 479–514.

[9] H. Hoppe, T. DeRose, T. Duchamp, M. Halstead, H. Jin, J. McDonald, J. Schweitzer, and W. Stuetzler, Piecewise smooth surface recosntruction, *Computer Graphics*, vol. 28, no. 3, 1994, pp. 295–302.

[10] J. Hoschek and D. Lasser, *Fundamentals of Computer aAided Deometric Design*, Translated by L. L. Schumaker, A. K. Peters, Wellesley, MA, 1993.

[11] A. Khodakovsky, N. Litke, and P. Schröder, Globally smooth parametrization with low distortion, *Proc. SIGGRAPH*, ACM Press, 2003, pp. 350–357.

[12] M. Jackowski, M. Satter, and A. Goshtasby, Approximating digital 3-D shapes by rational Gaussian surfaces, *IEEE Trans. Visualization and Computer Graphics*, vol. 9, no. 1, 2003, pp. 56–69.

[13] W. Ma, Subdivision surfaces for CAD–An overview, *Computer Aided Design*, vol. 37, 2005, pp. 693–709.

[14] M. E. Mortenson, *Geometric Modeling*, 2nd Edition, Wiley Computer Publishing, 1997.

[15] P. Oswald, Designing composite triangular subdivision schemes, *Computer Aided Geometric Design*, vol. 22, 2005, pp. 659–679.

[16] E. Praun and H. Hoppe, Spherical parametrization and remeshing, *Proc. SIGGRAPH*, ACM Press, 2003, pp. 340–349.

[17] S. Schaefer and J. Warren, On C^2 triangle/quad subdivision, *ACM Trans. Graphics*, vol. 24, no. 1, 2005, pp. 28–36.

[18] J. Warren and H. Weimer, *Subdivision Methods for Geometric Design*, Morgan Kaufmann Publishers, San Francisco, 2002.

[19] V. Weiss, L. Andor, G. Renner, and T. Varady, Advanced surface fitting techniques, *Computer Aided Geometric Design*, vol. 19, 2002, pp. 19–42.

Table 1: Data for the line segments depicted in Fig. 2.

i	X_{i_1}	Y_{i_1}	Z_{i_1}	X_{i_2}	Y_{i_2}	Z_{i_2}
1	-50	-50	0	50	-50	0
2	50	-50	0	50	50	0
3	50	50	0	-50	50	0
4	-50	50	0	-50	-50	0
5	1	1	50	50	50	0
6	-1	1	50	-50	50	0
7	0	-10	40	0	-20	20

Table 2: The twelve line segments defining the cylindrical surface depicted in Figs. 4b and 4c.

i	X_{i_1}	Y_{i_1}	Z_{i_1}	u_{i_1}	v_{i_1}	X_{i_2}	Y_{i_2}	Z_{i_2}	u_{i_2}	v_{i_2}
1	0	0	0	0	0	0	1	0	0.25	0
2	0	1	0	0.25	0	1	1	0	0.5	0
3	1	1	0	0.5	0	1	0	0	0.75	0
4	1	0	0	0.75	0	0	0	0	1	0
5	0	0	1	0	1	0	1	1	0.25	1
6	0	1	1	0.25	1	1	1	1	0.5	1
7	1	1	1	0.5	1	1	0	1	0.75	1
8	1	0	1	0.75	1	0	0	1	1	1
9	0	0	0	0	0	0	0	1	0	1
10	0	1	0	0.25	0	0	1	1	0.25	1
11	1	1	0	0.5	0	1	1	1	0.5	1
12	1	0	0	0.75	0	1	0	1	0.75	1

Table 3: The twenty line segments defining the cube depicted in Figs 4e and 4f.

i	X_{i_1}	Y_{i_1}	Z_{i_1}	u_{i_1}	v_{i_1}	X_{i_2}	Y_{i_2}	Z_{i_2}	u_{i_2}	v_{i_2}
1	0	0	0	0	0.33	0	1	0	0.25	0.33
2	0	1	0	0.25	0.33	1	1	0	0.5	0.33
3	1	1	0	0.5	0.33	1	0	0	0.75	0.33
4	1	0	0	0.75	0.33	0	0	0	1	0.33
5	0	0	1	0	0.66	0	1	1	0.25	0.66
6	0	1	1	0.25	0.66	1	1	1	0.5	0.66
7	1	1	1	0.5	0.66	1	0	1	0.75	0.66
8	1	0	1	0.75	0.66	0	0	1	1	0.66
9	0	0	0	0	0.33	0	0	1	0	0.66
10	0	1	0	0.25	0.33	0	1	1	0.25	0.66
11	1	1	0	0.5	0.33	1	1	1	0.5	0.66
12	1	0	0	0.75	0.33	1	0	1	0.75	0.66
13	0.5	0.5	0	0	0	0	0	0	0	0.33
14	0.5	0.5	0	0.25	0	0	1	0	0.25	0.33
15	0.5	0.5	0	0.5	0	1	1	0	0.5	0.33
16	0.5	0.5	0	0.75	0	1	0	0	0.75	0.33
17	0.5	0.5	1	0	1	0	0	1	0	0.66
18	0.5	0.5	1	0.25	1	0	1	1	0.25	0.66
19	0.5	0.5	1	0.5	1	1	1	1	0.5	0.66
20	0.5	0.5	1	0.75	1	1	0	1	0.75	0.66

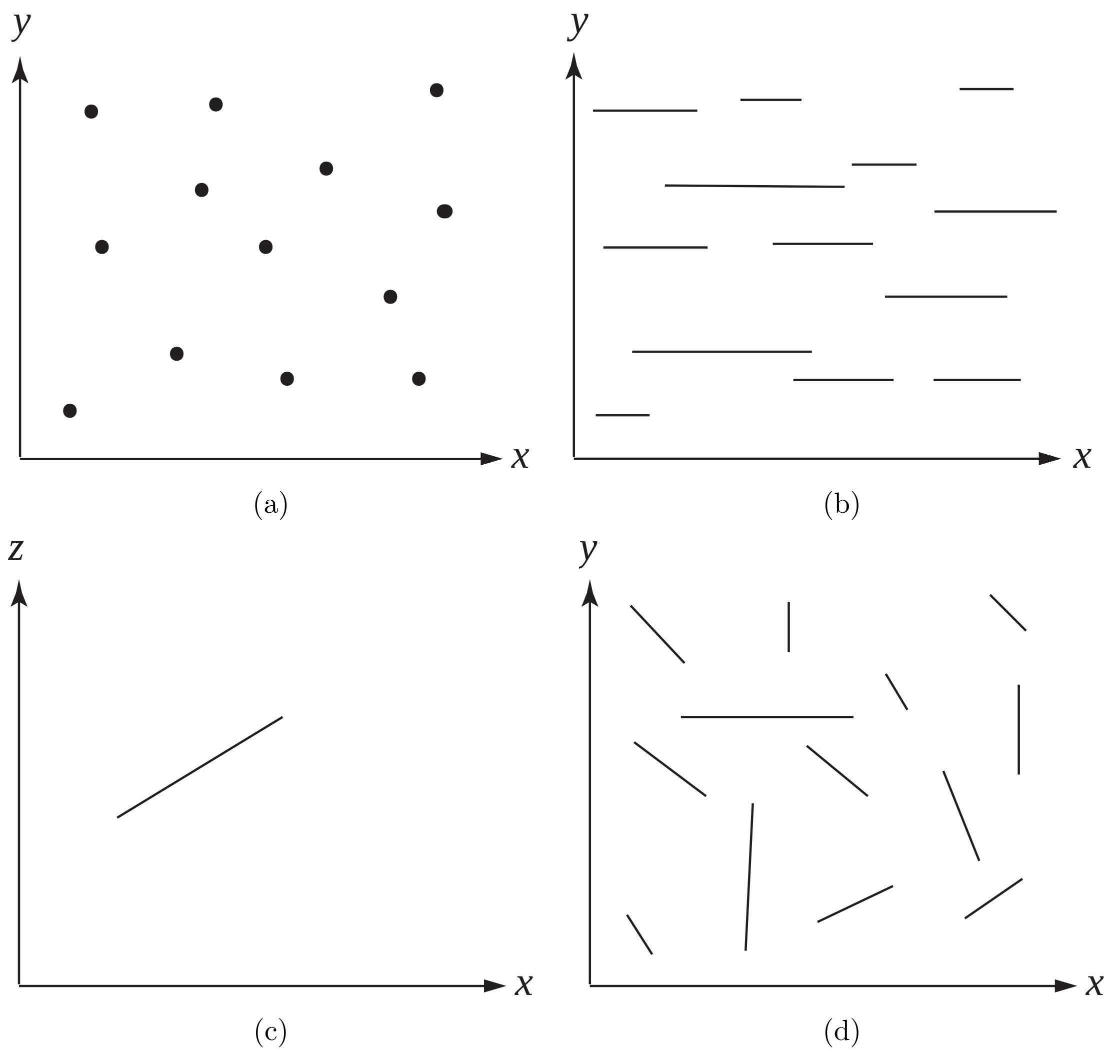

Figure 1: (a) An example of scattered data points in 2-D. (b) Scattered lines parallel to the x-axis with uniform data along each line. (c) Same as (b), but allowing data values to change linearly along the lines. The figure shows only one of the lines. (d) Scattered lines with data varying linearly along the lines.

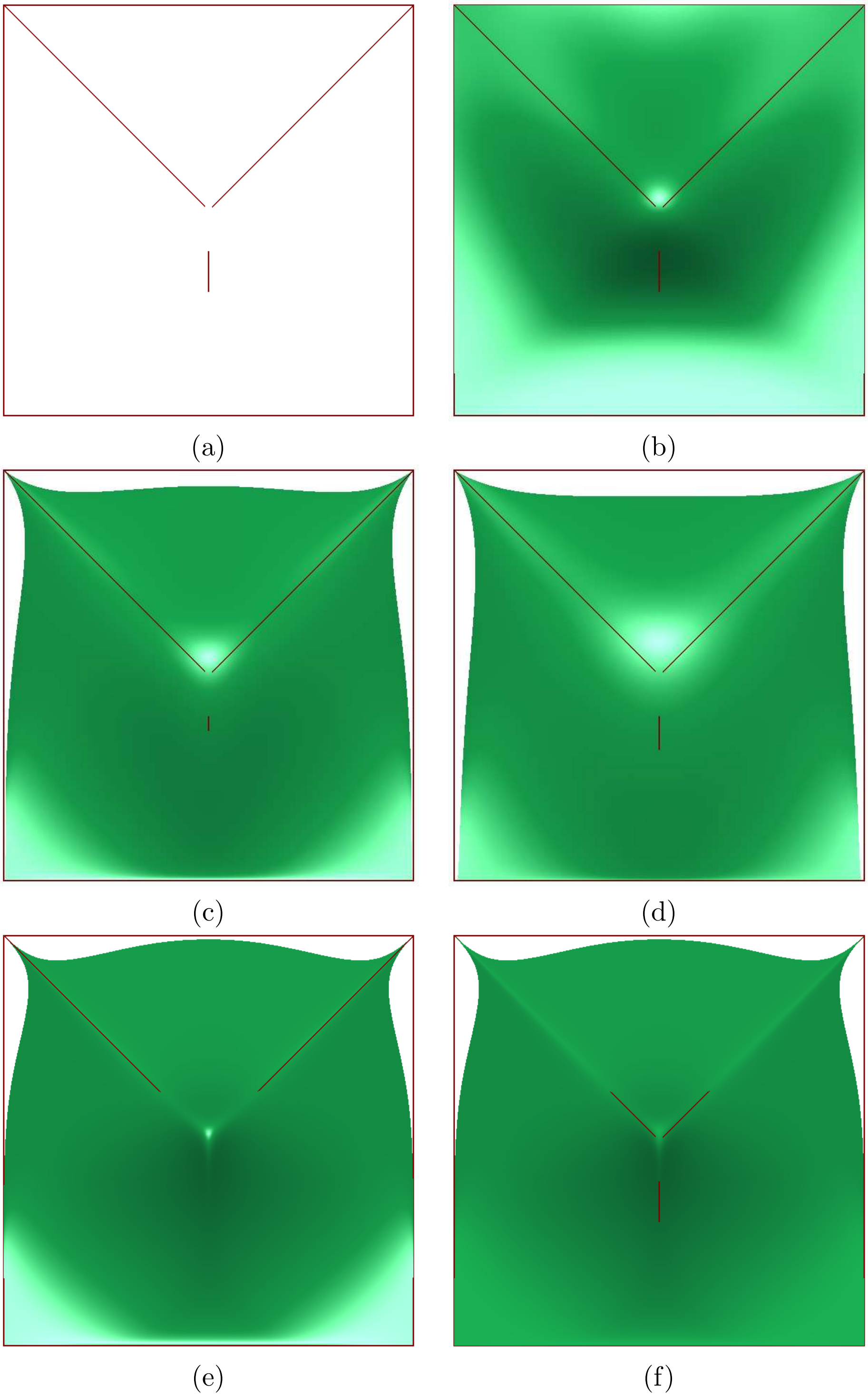

Figure 2: (a) The line segments with associating data given in Table 1. (b) The single-valued surface defined by (11) approximating the lines. (c) The parametric surface defined by (17)–(19) approximating the lines. (d) Same as (c) but with a larger σ. (e) Same as (c) but with a smaller σ. (f) View of (e) from the opposite side.

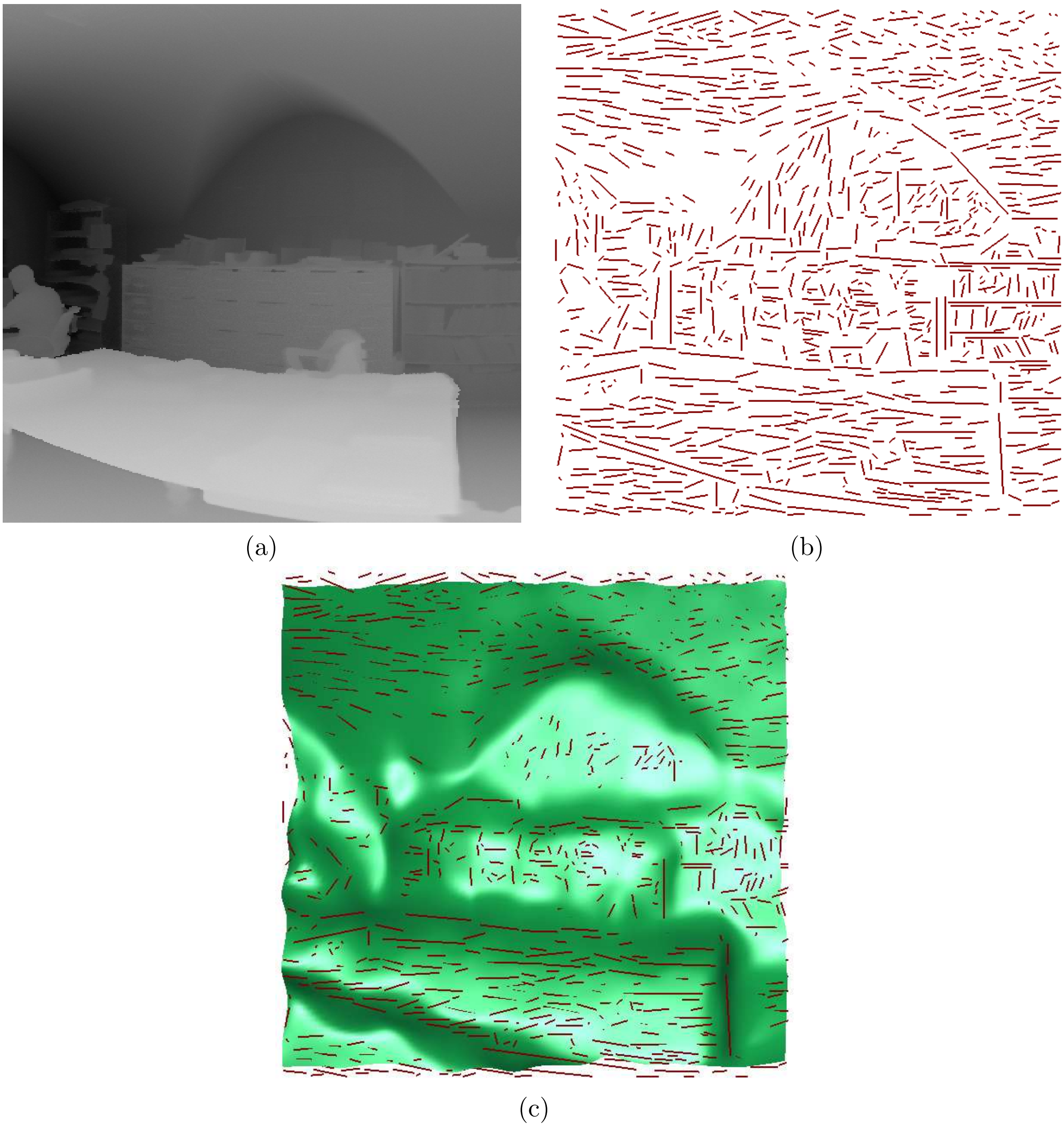

Figure 3: (a) A depth map of a room scene. This data set is courtesy of the University of California at Los Angeles. (b) Line segments corresponding to straight edges in (a). Depths along these lines were used to estimate the scene geometry depicted in (c) according to formulas (17)–(19).

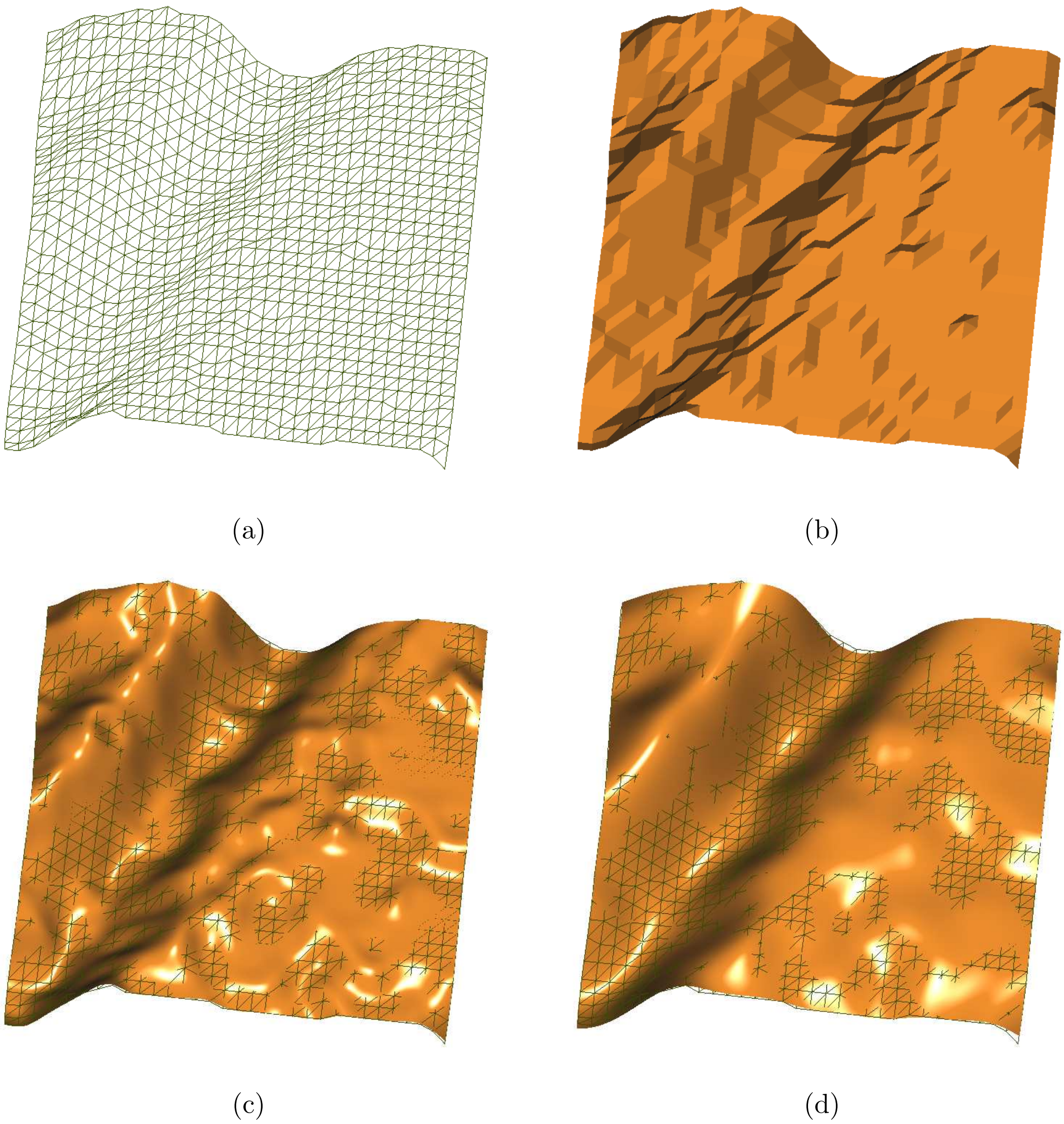

Figure 4: (a) Triangulation of a digital elevation map of an area over the Grand Canyon. (b) The triangle mesh in shaded form. This data set is courtesy of the U.S. Geological Survey. (c) Approximation of the mesh by the proposed method using the triangle edges. (d) Approximation with a larger smoothness parameter.

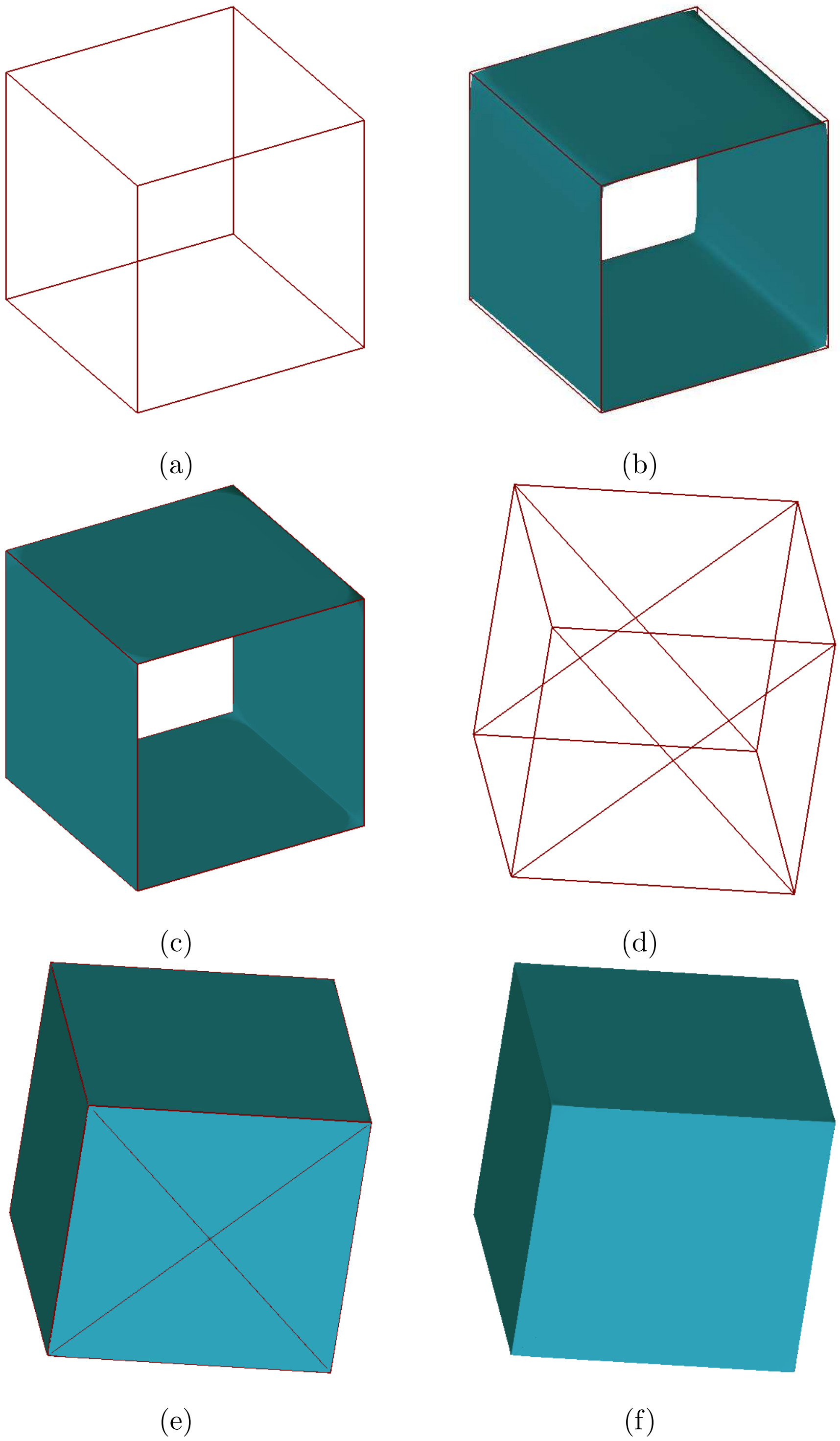

Figure 5: (a) The line segments given in Table 2. (b) The surface approximating the lines. (c) Same as (b) but using a smaller σ. (d) The line segments given in Table 3. (e) The surface approximating the lines. (f) Same as (e) but without showing the lines.

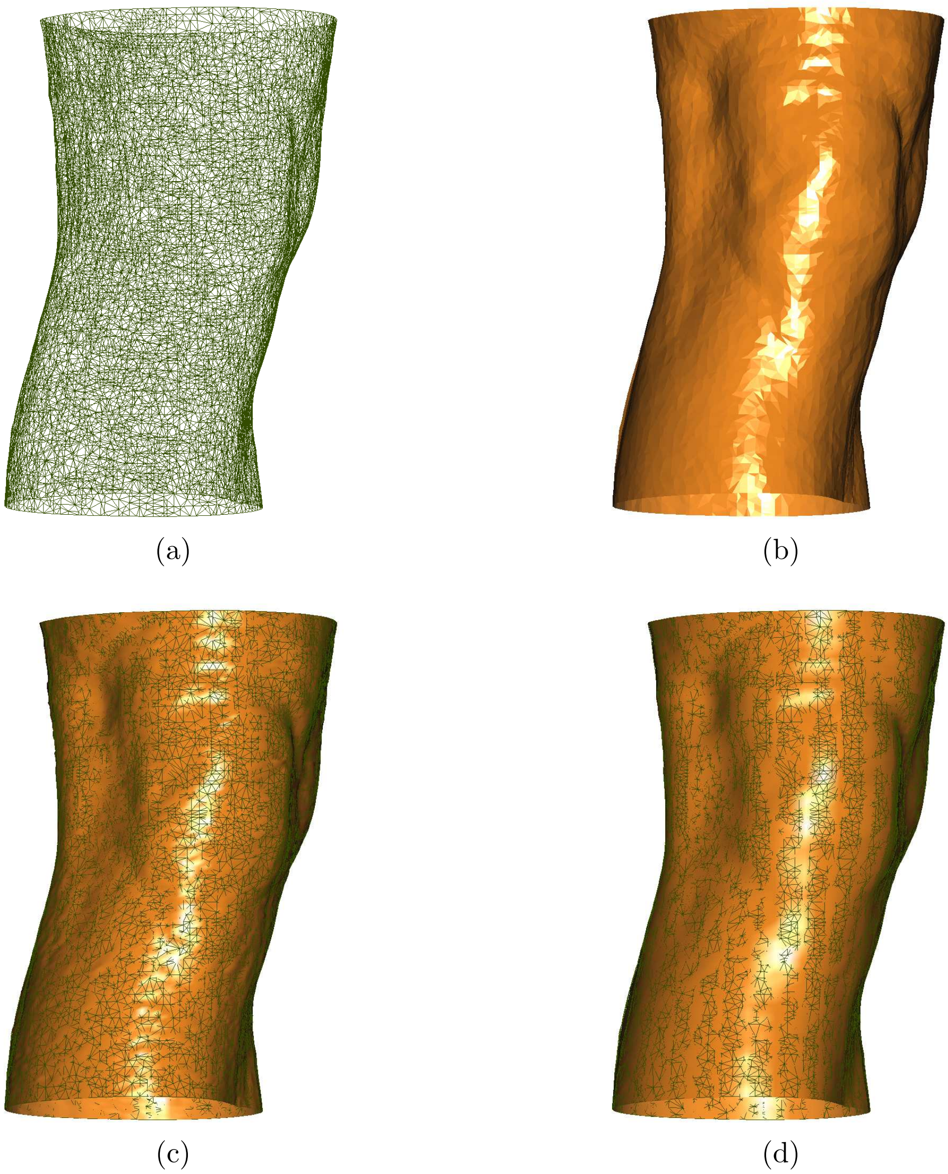

Figure 6: (a) Triangulation of range data of a knee. This data set is courtesy of SGI. (b) The triangle mesh in shaded form. (c) Approximation of the mesh by the proposed method using the triangle edges. (d) The same surface with a large smoothness parameter.