

IMAGE MATCHING BY A
PROBABILISTIC RELAXATION LABELING PROCESS

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ABSTRACT

Correspondence between closed-boundary regions in two segmented images of the same scene is determined by a probabilistic relaxation labeling process. Region shape similarities are used to determine the initial probability that two regions correspond to each other. Distances between centers of gravity of the regions are then used to update the correspondence probabilities until a globally consistent match is obtained or computation time exceeds its limit. This technique can be used on images with translational, rotational, and scaling differences.

computed using information from all object labels, rather than the immediate neighbors, to achieve a globally consistent labeling.

The assumptions are that we are given two segmented images from the same scene. The images may have translational, rotational, and scaling differences, and there might be some regions in one image which do not exist in the other image. The problem is to determine correspondence for those regions which exist in both images and identify those regions which exist in only on image.

2. PROBABILISTIC RELAXATION LABELING

1. INTRODUCTION

Image matching is the process of determining the correspondence between symbolic features such as regions, line segments, line intersections, etc., in two segmented images of the same scene. Probabilistic relaxation labeling is a cooperative process for reducing ambiguity between labels of the features by iteratively updating the label probabilities using initial observation and world knowledge.

Let $A=\{a_1, a_2, \dots, a_m\}$ be a set of objects (the regions in image 2), and let $B=\{b_1, b_2, \dots, b_n, b^*\}$ be a set of labels (the regions in image 1). From local measurements (shown in the next section), we can estimate the initial probability that object a_i has label b_j , $P_i^{(0)}(b_j)$. b^* is the "undefined object" label and $P_i^{(0)}(b^*)$ is the probability that object a_i is undefined (region a_i is not present in image 1). The initial probabilities satisfy the condition $\sum_j P_i^{(0)}(b_j)=1.0$ for all $a_i \in A$.

The label probabilities are updated according to the following iterative formula.

$$P_i^{(k+1)}(b_j) = \frac{P_i^{(k)}(b_j)[1+q_i^{(k)}(b_j)]}{\sum_j P_i^{(k)}(b_j)[1+q_i^{(k)}(b_j)]} \quad (1)$$

Image matching has been approached by Price using local information in images such as regions' area size, roundness, length to width ratio, and color⁹, and by Zahn using global information in the images such as inter-region distances and relative positions of regions¹². In this paper a procedure for image matching is developed which uses both local and global information in the images. Local information was region shape similarities, and were used to determine the initial probability that two regions correspond to each other. Global information was inter-region distances (distances between centers of gravity of regions), and were used to update the correspondence (label) probabilities based on a probabilistic relaxation labeling process.

where $q_i^{(k)}(b_j)$ is the neighbor contribution to $P_i^{(k)}(b_j)$, in the k th iteration, and is in the range $[-1.0, 1.0]$. The label probabilities at any iteration satisfy $\sum_j P_i^{(k)}(b_j)=1.0$.

There are two sources of information for this relaxation process, the initial probability estimates and the neighbor contribution factors. In the following, each is discussed in detail.

3. INITIAL LABEL PROBABILITY ESTIMATION

Probabilistic relaxation labeling was first described by Rosenfeld, Hummel, and Zucker¹⁰ and has since been used in many computer vision applications^{1,2,4,11,13}. Different variations of the probabilistic relaxation labeling process have also been suggested^{5,6,14}. The relaxation process which is used in the following is based on the original relaxation labeling process of Rosenfeld-Hummel-Zucker but the neighbor contribution factors are

The initial probability that region a_i has label b_j is computed using the similarity between the two regions. Since image 1 and image 2 may have translational, rotational, and scaling differences, region b_j and region a_i may have translational, rotational, and scaling differences. We should therefore use the properties of the regions which are invariant under translation, rotation, and

scaling.

Some of these properties are average intensity, color, and shape. We will be using the shapes of regions to determine the similarity between them. There are shape similarity measurement techniques which can measure shape similarities irrespective of their translational, rotational, and scaling differences^{7,8}. We will be using the shape similarity measurement technique of reference 3 to measure shape similarities. The similarity between two shapes is represented by a number between 0.0 and 1.0, 1.0 showing complete similarity and 0.0 showing complete dissimilarity (a line and a circle has been defined as two completely dissimilar shapes).

We define weight associated with the label b_j of region a_i to be $W_i(b_j)$ = the similarity between regions a_i and b_j . $W_i(b_j)$ changes between 0.0 and 1.0, and the more similar the two regions a_i and b_j , the closer the value of $W_i(b_j)$ to 1.0. We also define $W_i(b^*) = 1.0 - \max W_i(b_j)$. This is natural because if all the label weights of a region are small, it shows that there is no region in image 1 similar to a_i , and therefore $W_i(b^*)$ (the weight of the undefined region label) should be large. If one of the labels has a high weight, it shows that a similar region exists in image 1, and so $W_i(b^*)$ should be small. We use weight function $W_i(b_j)$ to estimate the initial probability that region a_i has label b_j as below.

$$P_i^{(0)}(b_j) = W_i(b_j) / \sum W_i(b_j)$$

4. NEIGHBOR CONTRIBUTION FACTORS

Neighbor contribution factors are tools by which the label probabilities can be updated so that the label assignments become consistent with the world knowledge. World knowledge can be, for example, relative sizes of the regions, distances of the regions from each other, and relative positions of the regions in the first image. We represent world knowledge in matrix form, calling it a knowledge matrix, and denoting it by V . An element of knowledge matrix $V(b, b')$ may show,

1. the size (perimeter size or area size) ratio of regions b and b' ,
 2. the distance between regions b and b' ,
 3. the position of region b relative to region b' (above, below, to the right, to the left),
 4. the shape similarity of regions b and b' ,
 5. etc.,
- in image 1.

To find how compatible label b_j of object a_i is with labels of other objects, we pursue as follows. Let $V(b_j) = \{V(b_j, b_1), V(b_j, b_2), \dots, V(b_j, b_n)\}$ show the distances between region b_j and other regions in image 1. Then for region a_i in image 2 we determine the distances of a_i to every other region. Let $D(a_i) = \{D(a_i, a_1), D(a_i, a_2), \dots, D(a_i, a_m)\}$ denote this. Every object in image 2 has a set of labels, each with a different probability. Assuming $c_i \in B$ is that label of a_i which has the largest probability, then we determine another set

of distances,
 $U^{(k)}(b_j) = \{V(b_j, c_1), V(b_j, c_2), \dots, V(b_j, c_m)\}$ where $V(b_j, c_i)$ shows the distance between regions b_j and c_i in the first image. Now the cross-correlation between $D(a_i)$ and $U^{(k)}(b_j)$,

$$q_i^{(k)}(b_j) = \frac{E[U^{(k)}(b_j)D(a_i)] - E[U^{(k)}(b_j)]E[D(a_i)]}{\sigma[U^{(k)}(b_j)]\sigma[D(a_i)]}$$

shows how much labels of other objects support object a_i to have label b_j in the k th step, where

$$\begin{aligned} E[U^{(k)}(b_j)] &= (1/(m-1)) \sum V(b_j, c_i) \\ E[D(a_i)] &= (1/(m-1)) \sum D(a_i, a_i') \\ E[U^{(k)}(b_j)D(a_i)] &= (1/(m-1)) \sum [V(b_j, c_i')D(a_i, a_i')] \\ \sigma[U^{(k)}(b_j)] &= [(1/(m-1)) \sum (U^{(k)}(b_j, c_i) - E[U^{(k)}(b_j)])^2]^{1/2} \\ \sigma[D(a_i)] &= [(1/(m-1)) \sum (D(a_i, a_i') - E[D(a_i)])^2]^{1/2} \end{aligned}$$

$q_i^{(k)}(b_j)$ is a measure in the range $[-1.0, 1.0]$, and if region a_i truly corresponds to region b_j , then regardless of some mistakes in the label of other regions, $q_i^{(k)}(b_j)$ will be high. On the other hand, if region a_i truly does not correspond to region b_j , then even though all other regions in image 2 are labeled correctly, the value of $q_i^{(k)}(b_j)$ will be low. This is a very desirable property, and we use $q_i^{(k)}(b_j)$ as the neighbor contribution factor in formula (1).

Neighbor contribution factors for the undefined region labels $q_i^{(k)}(b^*)$ cannot be determined in this manner because correlation of distances that do not exist does not make sense ($V(b^*)$ is undefined). To determine the neighbor contributions for object a_i having label b^* (the neighbor support for region a_i of image 2 having no correspondence in image 1), we observe the following. If object a_i truly has label b^* , then assigning label b^* to a_i should increase the true label probabilities for other objects, while if b^* is not the true label of a_i , assigning b^* to a_i should decrease the true label probabilities of other objects.

Assuming that most of the highest probability labels of objects show their true labels, we 1) determine the largest probability label of each object when b^* is assigned to a_i (except when the largest probability label of objects is already b^*), and 2) determine the largest probability label of objects this time by assigning the largest probability label of a_i to a_i (again, except when the largest probability label of objects is b^*). If there are M objects with increased value for their largest label probabilities in case 1 and N objects with increased value for their largest label probabilities in case 2 then we should,

- i) increase the label probability for object a_i having label b^* if $M > N$,
- ii) decrease the label probability for object a_i having label b^* if $M < N$, and
- iii) not change the label probability for object a_i having label b^* if $M = N$.

Now, if m' is the number of objects that have largest probability labels other than b^* , then $(M-N)/m'$ simulates the neighbor support for object a_i having label b_j , and we will use $(M-N)/m'$ as the neighbor contribution factor, $q_i^{(k)}(b^*)$. This measure also shows, if by assigning label b^* to a_i

the largest label probability for all objects increase, then $q_i^{(k)}(b^*)=1.0$, while if the largest label probability of all objects decrease, then $q_i^{(k)}(b^*)=-1.0$. This measure well characterizes the neighbor contribution factor for object a_i having the undefined region label.

In the above, we used the distance between regions as the world knowledge to estimate the neighbor contribution factors. Other informations such as relative sizes of the regions, relative positions of the regions, etc., could as well be used for this purpose.

For images with translational, rotational, and scaling differences we should select a knowledge matrix which makes the neighbor contribution factors, $q_i^{(k)}(b_j)$, invariant with respect to translation, rotation, and scaling of the images. For example, objects' relative positions (above, below, to the right, to the left) are not invariant under rotation, but inter-object distances are.

5. CONCLUSION

Image matching was approached by a probabilistic relaxation labeling process. Initial label probabilities were determined from local information in the images, and neighbor contribution factors were obtained from global information in the images. The process can be applied to matching of images with translational, rotational, and scaling differences.

Depending on noise and ambiguity in the images, the process takes different numbers of iterations to converge. A strategy to speed-up the process has been formulated and is reported in reference 3. To save more computation time, the process can be stopped when the largest probability of every object passes a threshold value.

This technique for image matching does not use signal-based information in the images such as intensity or color and therefore can be used to match images with some signal differences. Also, since the technique uses both local and global information in the images, errors made locally in extracting the features are compensated by global information in the images.

The results of the above described process in matching of real images is fully given in reference 3.

6. REFERENCES

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