

# Geometric Correction of Satellite Images Using Composite Transformation Functions\*

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**Abstract:** Geometric distortion in a satellite image could be due to many local factors such as sensor nonlinearity, atmospheric turbulence, and scene elevation. In this paper, a transformation function that is composed of many local transformation functions is proposed for image geometric correction. Each local function represents distortion in a small area in an image. The composite transformation function obtained in this manner can represent high degree distortions in an image and has the ability to describe the position and the amount of the distortion. Distortion at a point in an image is modeled by a polynomial and the parameters of the polynomial are determined by weighted least-squares. Distortion in an image is corrected relying more on local information than on global data to ensure that a local distortion is not spread all over the image.

## 1. INTRODUCTION

Image distortion is a relative quantity and can be measured only with respect to a given reference image. The reference image could be a generated one or a natural one taken under specific conditions. The image to be corrected, which we will refer to as the sensed image, is assumed to be taken under conditions different from those of the reference image. The objective in image geometric correction is to transform the sensed image so that it would overlay with the reference image.

If the conditions under which the images were obtained are known, then determination of the right transformation function that would transform the sensed image to overlay with the reference image is straightforward. However, most of the times the image acquisition parameters are not known and the transformation function should be determined solely from the content of the images.

The image acquisition parameters are: 1) Parameters corresponding to the platform such as the position, attitude, view angle, and speed of the sensor at the time an image is obtained. 2) Parameters corresponding to the sensor such as the lens nonlinearity and mirror velocity nonlinearity (for sensors with oscillating scanning mirrors). 3) Parameters corresponding to the scene such as earth curvature, earth rotation, and scene elevation.

Image geometric correction has many applications in map construction [Colvocoresses '86] and image enhancement [Simard 83]. The major application of image geometric correction, however, is in image registration where an image should be geometrically transformed to overlay with another image. Image registration itself is a general problem and arises in many applications such as:

- 1) Depth perception: By registering stereo images, it is possible to obtain depth information about the underlying scene [Grimson '81].
- 2) Dynamic scene analysis: By registering a sequence of images from a dynamic scene it is possible to analyze the scene [Ullman '79].
- 3) Change detection: By registering images of the same scene taken at different times and by subtracting corresponding feature values in the images it is possible to determine where and how much a change has taken place in the scene between the times the images were obtained [Price '77].
- 4) Scene classification: By registering multi-sensor images, it is possible to use multi-sensor data to classify the underlying scene [Robertson *et al.* '73].

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In the following, a technique for image geometric correction is described. In Section 2, we review the past works in this area, and in Section 3, we discuss the problems that are involved in implementing a geometric correction system. Then in Section 4, we propose a locally sensitive transformation function for correction of locally distorted images. It is assumed that the control points have been selected from the images (either manually or by some automatic procedure) and correspondence has been established between them. This paper uses the correspondence between the control points to determine the transformation between the images.

## 2. PAST WORKS IN IMAGE GEOMETRIC CORRECTION

Some techniques in the past have been successful in geometrically correcting satellite images. In satellite images, since it is possible to determine the position, orientation, and speed of a satellite at any given time, and since the characteristics of the sensor, the earth curvature and earth rotation are known too, it is possible to formulate a transformation function that would transform one image into another. Also, in satellite images, the distance of the camera to the scene is much larger than the elevation of the scene and distortion due to scene elevation is usually small and sometimes not included in the transformation function [Legeckis and Pritchard '76].

Assuming that second and higher order distortion in an image is negligible, Horn and Woodham [79] showed that an affine transformation can correct an image of the earth's surface if the image was a plane tangent to the earth's surface at the nadir point. Recently Salamowicz [86] reformulated the image correction problem to include distortion due to scene elevation. This technique, however, requires elevation at some points in the scene.

In most cases, parameters of the image acquisition system are not known and it is required to correct the images with information solely from the images. In such a case, the usual procedure is to select a number of feature points from the sensed and reference images, determine correspondence between the feature points, and use the feature correspondences to determine a transformation function that would transform the sensed image to the reference image. The feature points that are invariant under geometric transformations are line intersections and line end points [Kanal et al. '81], [Stockman et al. '82]. If geometric distortion in an image can be assumed locally linear, then centers of gravity of closed boundary regions can also be used as feature points [Goshtasby et al. '86].

Determination of correspondence between feature points in two images that have geometric distortions is a very difficult problem and there are only solutions to some restricted cases. For images having translational, rotational and scaling differences [Zahn '74] determined correspondence between the feature points by matching minimum spanning trees of the points. Goshtasby and Stockman [85] determined point correspondences by matching convex hull edges of the two sets of points. [Baird '85] transformed the point correspondence problem into a set of linear inequalities and by using the Soviet ellipsoid algorithm solved the inequalities. Wang et al. [83] solved the point correspondence problem by a relaxation process.

Another difficulty in image geometric correction is the selection of the right transformation function. In satellite images, since the distance of the camera to the scene is much larger than the elevation of the scene, and because usually view angle difference between the images is small, geometric distortion between images due to scene elevation is small and sometimes not included in the formulas. Affine transformation has been found satisfactory in many cases to correct small to moderate size images with accuracy of up to one pixel [Steiner and Kirby '77], [Watson et al. '82], [Kulkarni et al. '81].

For correction of images having distortions of order two or higher, polynomials of degree two and three have been used [Van Wie and Stein '77], [Nack '77], [Markarian et al. '73]. Leckie [80] tested polynomials of degree two and three with different numbers of terms and determined the polynomial that best overlaid two given images. In many studies local geometric distortion in the images is assumed to be linear and template matching has been used to locate corresponding feature points in the images.

Geometric distortion in an image is usually due to a combination of many factors. Some of these factors have only local effects and vary from point to point such as the scene elevation and sensor nonlinearity. When a locally varying distortion is present in an image usually a global transformation function cannot correct the image.

In the literature, there has been attempts to overlay images with local geometric distortions. [Bajcsy and Broit '82] modeled the sensed image by an elastic sheet, and then by applying external forces to the sheet, deformed it to overlay with the reference image. In this model, the images were segmented and closed-boundary regions were extracted, then correspondence was established between points on corresponding regions. The forces that deformed the sensed image were obtained by requiring corresponding points in the images overlay as closely as possible.

In a similar study, [Burr '81] deformed one image to overlay with the other image using a dynamic cooperative model. Initially, correspondence was established between some points in the images by template matching and then one image was iteratively deformed by reducing the stiffness of the elastic material of the image using similarity of  $7 \times 7$  windows in the images. The process was continued until a satisfactory match was obtained between the images. Template matching has always been a tool to determine correspondence between points in two images having small rotation, scaling, or nonlinear distortions. In [Mori *et al.* '73], [Keating *et al.* '75], and [Panton '78] template matching was used to overlay entire images point by point.

The above mentioned image correction techniques have been successful on restricted geometric distortions. A technique that is able to correct geometric distortions of any type is yet to be invented. There are many difficulties in formulating such a technique. Next we discuss some of these difficulties.

### 3. DIFFICULTIES IN IMAGE GEOMETRIC CORRECTION

The main difficulties in image geometric correction are: 1) selection of the feature points from the images, 2) determination of the correspondence between the feature points, and 3) selection of the right transformation function that can represent geometric distortion between the images. For images with negligible local distortion and negligible rotation or scaling distortion, window centers located at high variance areas of the images can be used as the feature points and template matching can be used to establish correspondence between them [Davis and Kenue '78]. For other images, window centers won't be reliable and features invariant to rotation, scaling, and nonlinear distortion such as line intersections and line end points should be used. There is no technique known that can determine correspondence between feature points when the images have nonlinear geometric distortions.

Selection of the right transformation function is another problem in image geometric correction. Usually, the form of the distortion is not known and it is necessary to make an approximation to the right transformation function. Global transformation functions are generally not appropriate for geometric correction. This is because, even though the distortion in an image may be definable by a global function, the form of the function is not known.

Problems related to feature selection and feature correspondence can be alleviated if they are assisted by hand. Selection of the right transformation function, however, is not that simple and requires further consideration. In this paper, we design a procedure that would use different transformation functions to correct geometric distortions in different areas of an image. The whole image is, therefore, corrected piece by piece using local transformation functions. The local transformation functions when combined, will provide the global transformation function. No matter how complicated the geometric distortion in an image, if local neighborhoods are taken small enough, by piecewise linear or piecewise quadratic functions it is possible to correct an image with a reasonably good accuracy. Next, the proposed technique to image geometric correction is described.

### 4. DETERMINATION OF THE TRANSFORMATION FUNCTION

Suppose  $N$  feature points from each image are given. Let  $(x_i, y_i)$  and  $(X_i, Y_i)$   $i=1, N$ , be the feature points in the reference and sensed images, respectively. Suppose the correspondence between the feature points is also known. The objective is to determine two transformation functions  $X=f(x, y)$  and  $Y=g(x, y)$  that would transform the sensed image to overlay with the reference image. It is obvious that if  $f$  and  $g$  truly represent the geometric difference between the images, then by substituting the coordinates of a feature point in the reference image to  $f$  and  $g$  we obtain the coordinates of the corresponding feature point in the sensed image:

$$\begin{cases} X_i = f(x_i, y_i) \\ Y_i = g(x_i, y_i) \end{cases} \quad i=1, N$$

If the form of  $f$  and  $g$  are known then by solving the above system of equations we can determine the parameters of  $f$  and  $g$ . However, the form of  $f$  and  $g$  are not known and there is a need to approximate them.

The problem of finding  $f$  and  $g$  can be redefined as follows. Given two sets of 3D points  $(x_i, y_i, X_i)$ ,  $(x_i, y_i, Y_i)$   $i=1, N$  determine two functions  $X=f(x, y)$  and  $Y=g(x, y)$  that would fit to the two sets of

points, respectively. If the objective was to determine  $f$  and  $g$  as global functions then we could represent  $f$  and  $g$  by appropriate polynomials and determine the parameters of the polynomials by the least-squares approach [Bernstein '76] [Van Wie and Stein '77]. The problem with using a global transformation function and least-squares is that a local geometric distortion is averaged uniformly all over the image. This is not a desirable property, and we want to find a transformation function that is locally sensitive and uses local image differences to determine local transformation functions. In the following, we will show how to obtain  $f$  from coordinates of control points in the reference image  $(x_i, y_i)$ ,  $i=1, N$  and the  $X$ -component of the corresponding control points in the sensed image  $X_i$ ,  $i=1, N$ . Function  $g$  can be obtained in the same manner.

Consider defining a different function for each point in the reference image. If we use the least-squares technique to determine the parameters of the function, we would obtain the same function for every point in the image. This would make the transformation function global. To make the technique local, we propose determining the parameters of the transformation function by the weighted least-squares where the weights being the inverse distances of the control points to the point in the reference image where the value of the function is to be estimated. More specifically, suppose

$$f(x, y) = \sum_{j=0}^M \sum_{k=0}^j a_{jk} x^k y^{j-k} \quad (1)$$

is a polynomial of order  $M$  representing the local geometric difference between the images at point  $(x, y)$  in the reference image, then the  $X$ -component of the weighted sum of squared errors when mapping the control points in the sensed image to the corresponding control points in the reference image is

$$E(x, y) = \sum_{i=1}^N \left[ f(x_i, y_i) - X_i \right]^2 W_i(x, y) \quad (2)$$

where  $W_i(x, y) = \frac{1}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + \delta}}$  and parameter  $\delta \neq 0$  is used in this formula to avoid weights of infinity at the control points. The smaller the value of  $\delta$ , the larger the influence of the nearby control points on point  $(x, y)$ .

Parameters  $a_{jk}$  of (1) may be determined by minimizing (2). To do that, we determine the partial derivatives of (2) with respect to parameters  $a_{jk}$ , set them equal to zero, and solve the obtained system of equations:

$$\sum_{j=0}^M \sum_{k=0}^j a_{jk} \left[ \sum_{i=1}^N W_i(x, y) x_i^l y_i^{m-l} x_i^k y_i^{j-k} \right] = \sum_{i=1}^N W_i(x, y) X_i x_i^l y_i^{m-l} \quad (3)$$

for  $l=0, M$ ,  $m=0, l$ .

For each point  $(x, y)$  in the reference image, we obtain system (3) which should be solved to determine the  $X$ -component transformation function. Solution of system (3) can be avoided if orthogonal polynomials are used [Forsythe, '57]. However, about the same computation time is required to prepare the orthogonal polynomials. Observe that since the transformation functions are defined locally in a one pixel area, they need not be high order and a linear or a quadratic functions can correct a globally high order distortion. In this representation, a global distortion is approximated by many local functions.

In the following, the proposed composite transformation function is used to geometrically transform a Landsat Multi-Spectral Scanner image to overlay with a Landsat Thematic Mapper image. A computer program that finds the transformation function from a set of corresponding control points in the images is included in the Appendix.

## 5. RESULTS

To show the practicality of the proposed image geometric correction, we applied the procedure of Section 4 to a Landsat Thematic Mapper (TM) image and transformed it to overlay with a Landsat Multi-Spectral Scanner (MSS) image. Fig. 1.a shows band-4 of a Landsat MSS image from an area over Crawford County, Michigan, acquired on 17 June 1980. Fig. 1.b shows band-4 of a Landsat TM image acquired from about the same area on 18 October, 1982. The MSS and the TM images were assumed to be the reference and sensed images, respectively. The images were segmented using the segmentation technique of

Goshtasby and Page [Goshtasby and Page '83]. The closed-boundary regions were extracted from the images as shown in Fig. 2, and centers of gravity of the regions were used as corresponding control points in the images.

Using weighted least-squares with linear polynomials, the images were registered as shown in Fig. 3. The root-mean-squared-error in this process was 0.4 pixels at the control points. The obtained error is not all from the transformation function but part of the error comes from the extracted control points. The errors in the control points are due to the fact that the images are from different sensors and are at different resolutions and a perfect segmentation is impossible to obtain.

The proposed technique requires solution of a linear system of equations at every point in the image. When linear transformation functions are used to represent local geometric distortion between images, it is required to solve a system of six equations at every point in the reference image. If the reference image has  $Q$  pixels, the proposed technique requires in order of  $Q$  operations to determine a composite transformation function. If global transformation with least-squares was used, there would have been a need to solve only one system of twelve equations to determine the parameters of the transformation function. Therefore, computationally, determination of a composite transformation function is much slower than the determination of a single global transformation function. To speed up the proposed technique, if the amount of distortion is not large, instead of fitting a new mapping function to each point in the image, we may fit a new function to each block of  $5 \times 5$  or  $10 \times 10$  pixels. This would reduce the computation time considerably. The technique may be speeded up also by implementing it on a parallel machine. The Appendix shows implementation of the proposed composite transformation function on a Sequent Parallel Computer.

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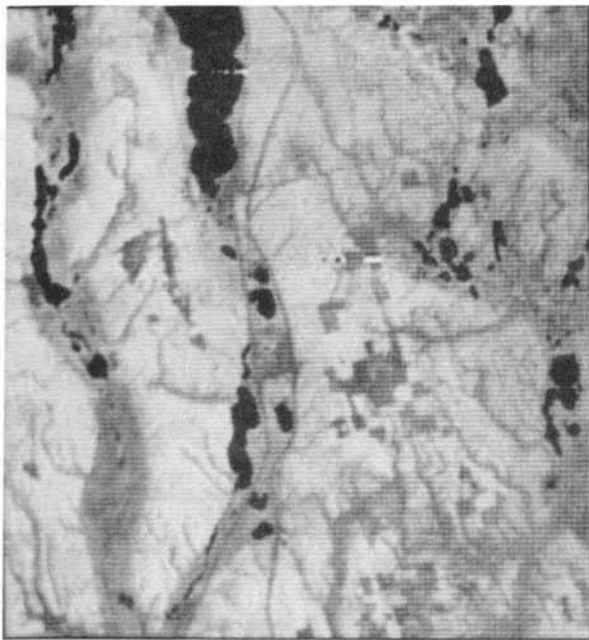
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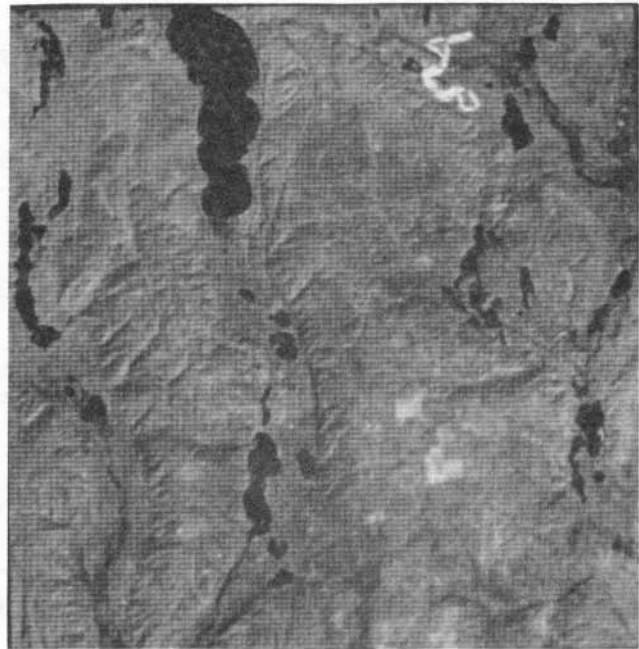
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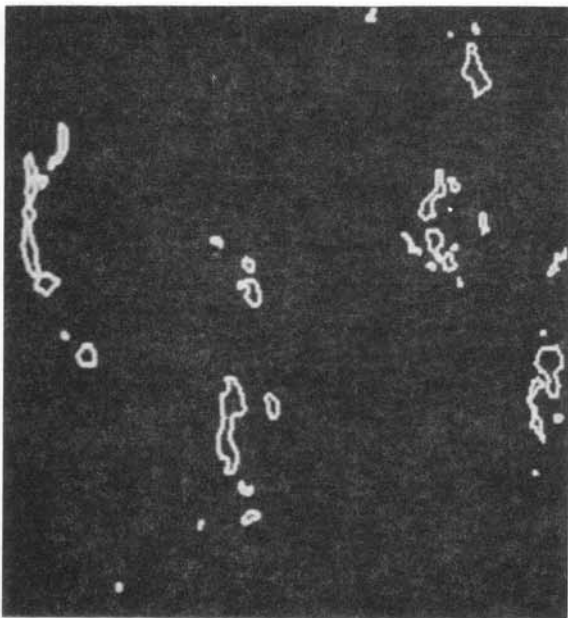


(a)

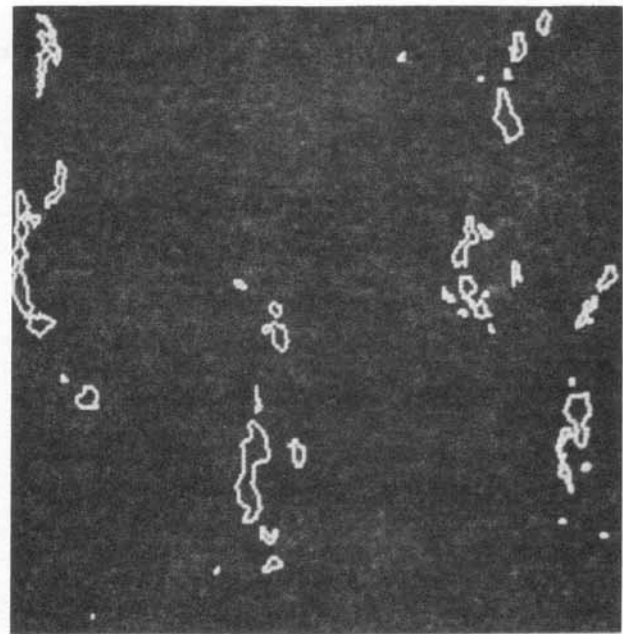


(b)

**Fig. 1.** (a) Landsat Multi-Spectral Scanner image used as the reference image. (b) Landsat Thematic Mapper image used as the sensed image.



(a)



(b)

**Fig. 2.** (a) and (b) are, correspondingly, closed-boundary regions extracted from images of Figs 1.a and 1.b.



```

denum:=0.0;
for i:=1 to n do
denum:=-denum+h(mm,xx[i],yy[i]);
alpha:=-n/denum;
end else
begin
num:=0.0; denum:=0.0;
for i:=1 to n do
begin
pk:=p(kk,xx[i],yy[i]);
num:=num+pk*h(mm,xx[i],yy[i]);
denum:=-denum+pk*pk;
end;
alpha:=-num*alpha(kk,kk)/denum;
end;
end;
end;

(***** p *****)
function p;
var i: integer; { This function determines the polynomial }
    pl: real; { value at point (x,y). }
begin
pl:=0.0;
for i:=1 to k-1 do
pl:=pl+C[i,k]*p(i,x,y);
p:=pl+C[k,k]*h(k,x,y);
end;

(***** a *****)
function a(k: integer; W: vctr): real;
var i: integer; { This function determines parameters of the }
    num,denum,pk: real; { mapping function. }
begin
num:=0.0; denum:=0.0;
for i:=1 to n do
begin
pk:=p(k,xx[i],yy[i]);
num:=num+W[i]*G[i]*pk;
denum:=-denum+W[i]*pk*pk;
end;
a:=-num/denum;
end;

(***** f *****)
function f(t: integer; i,j: integer): real;
var k: integer; { This function determines the value of the }
    x,y,fl: real; { mapping function with t parameters }
    W: vctr;
begin
{ fitting at point (x,y). }
x:=i; y:=j;
for k:=1 to n do
W[k]:=1.0/sqrt((x-xx[i])*(x-xx[i])+(y-yy[i])*(y-yy[i])+delta);
fl:=0.0;
for k:=1 to t do fl:=fl+a(k,W)*p(k,x,y);
f:=fl;
end;

```

```

(***** findmap *****)
{$E+}
procedure findmap;
{$E-}
var i,j,k,kmax: longint; { This procedure does most }
begin { of the computations in }
    kmax:=dx*dy; { parallel. }
    k:=m_next;
    while k<=kmax do
    begin
i:=(k div dy)+1;
if (k mod dy)=0 then i:=i-1;
j:=(k-(i-1)*dy);
image[i].col[j]:=round(f(m,i,j));
k:=m_next;
end;
end;

(***** main *****)
begin
reset(ptfile,'ptfile');
rewrite(outfile,'outfile');
readln(ptfile,n);
for i:=1 to n do readln(ptfile,xx[i],yy[i],G[i]);
writeln(' Enter number of required terms in th',
'e orthogonal polynomial. ');
readln(m);
writeln(' Enter dimensions of output image. ');
readln(dx,dy);
for i:=1 to dx do new(image[i]);
writeln(' Enter delta and number of processors. ');
readln(delta,nprocs);
writeln(outfile,dx:6,dy:6);
writeln(outfile,' Function determined by weighted',
' least-squares with ',
' orthogonal polynomials. ');
for i:=1 to m do for j:=1 to i do
C[j,i]:=alpha(j,i);
m_set_procs(nprocs);
m_fork(findmap); { Do not remove this statement }
{ but replace it with 'findmap;' }
for i:=1 to dx do { for sequential processing. }
begin
for j:=1 to dy do
begin
write(outfile,image[i].col[j]:4);
if (j mod 40)=0 then writeln(outfile);
end;
if (j mod 40)>0 then writeln(outfile);
end;
end.

```