EE-322 KEYS TO SAMPLE FINAL SUMMER-95

1 (a) ANALYTICAL

(a.) $h(n) = \sum_{K=-\infty}^{\infty} \left(\frac{1}{3}\right)^K u(K)\, 2^{n-K} u(-(n-K)-1)$

$= 2^n \sum_{K=-\infty}^{\infty} \left(\frac{1}{6}\right)^K u(K)\, u(-n+K-1)$

$u(K) = \begin{cases} 1 & ; K \geq 0 \\ 0 & K < 0 \end{cases}$

$u(-n+K-1) = \begin{cases} 1 & K \geq n+1 \\ 0 & EW \end{cases}$

TWO CASES (I) $\boxed{n \geq 0}$ $h(n) = 2^n \sum_{K=n+1}^{\infty} \left(\frac{1}{6}\right)^n$

$= \frac{2^n \left(\frac{1}{6}\right)^{n+1}}{\left(1-\frac{1}{6}\right)} = \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)^n$

(II) $\boxed{n \leq -1}$ $h(n) = 2^n \sum_{K=0}^{\infty} \left(\frac{1}{6}\right)^K = 2^n \frac{1}{1-\frac{1}{6}} = \left(\frac{6}{5}\right) 2^n$

$\therefore h(n) = \frac{1}{5}\left(\frac{1}{3}\right)^n u(n) + \left(\frac{6}{5}\right) 2^n u(-n-1)$

(b) USING Z-TRANSFORM

$H(z) = \left(\frac{z}{z-\frac{1}{3}}\right)\left(-\frac{z}{z-2}\right); \frac{1}{3} \leq |z| \leq 2$

$H(z) = -\frac{z^2}{(z-\frac{1}{3})(z-2)} \Rightarrow \frac{H(z)}{z} = \frac{-z}{(z-\frac{1}{3})(z-2)} = \frac{A_1}{z-\frac{1}{3}} + \frac{A_2}{z-2}$

$A_1 = \frac{H(z)}{z}(z-\tfrac{1}{3})\Big|_{z=\frac{1}{3}} = \frac{-\frac{1}{3}}{\frac{1}{3}-2} = \frac{1}{5}$

$A_2 = \frac{H(z)}{z}(z-2)\Big|_{z=2} = \frac{-2}{(2-\frac{1}{3})} = -6/5$

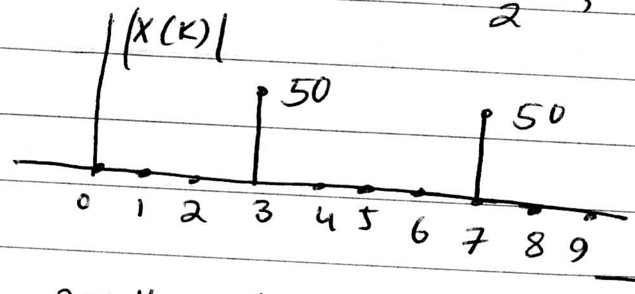
$\therefore H(z) = \frac{\frac{1}{5}z}{z-\frac{1}{3}} - \frac{\frac{6}{5}z}{z-2}$

$\therefore h(n) = \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)^n u(n) + \left(\frac{6}{5}\right)(2)^n u(-n-1)$

2. (a) $x(n) = x(t)\big|_{t=nT_s} = x(t)=\big|_{t=\frac{n}{300}} = 10\cos\frac{3\pi}{5}n = 10\cos\frac{2\pi}{10}3n$

∴ PERIOD, $N=10$

(b) $|X(K)| = X(N-K) = \frac{NA}{2}$, $K=3$, & $N=10$, $A=10$



3. $f_1 = 200\,Hz$, $f_2 = 300\,Hz$, $f_3 = 800\,Hz$, $f_4 = 1300\,Hz$

since $f_s = 3000\,Hz$

$\theta_1 = 2\pi\frac{200}{3000} = \frac{2\pi}{15}$, $\theta_2 = \frac{\pi}{5}$, $\theta_3 = \frac{8\pi}{15}$, $\theta_4 = \frac{13}{15}\pi$

NEED

$\frac{8\pi}{15} \le \theta_u < \frac{13}{15}\pi$ & $\frac{2\pi}{15} \le \theta_L \le \frac{\pi}{5}$

USE $\theta_u = \frac{10\pi}{15} = \frac{2}{3}\pi$ & $\theta_L = \frac{\pi}{6}$.

(a) $h(n) = \frac{2K\cos n\theta_0 \sin\theta_c n}{n\pi}$

WHERE $\theta_0 = \frac{\theta_u + \theta_l}{2} = \frac{\frac{\pi}{6}+\frac{4\pi}{6}}{2} = \frac{5\pi}{12}$

& $\theta_c = \frac{\theta_u - \theta_l}{2} = \frac{\frac{4\pi}{6}-\frac{\pi}{6}}{2} = \frac{\pi}{4}$

∴ $h(n) = \frac{2K}{n\pi}\cos\frac{5n\pi}{12}\sin\frac{n\pi}{4}$, $n = 0, \pm1, \pm2, \ldots$

REACIZABLE WITH $I=10$,

$h_{BP}(n) = \frac{2K}{(n-10)\pi}\cos\frac{5(n-10)\pi}{12}\sin\frac{(n-10)\pi}{4}$

$n = 0, 1, 2, \ldots, 19, 20$

(b) READ BOOK/NOTES

4. READ BOOK/NOTES

5 $Y(z) = 0.5\left(Y(z)z^{-1} + y(-1)\right) - 0.2\,X(z)z^{-1}$; $y(-1) = 1$

WHERE, $X(z) = \dfrac{z^{-1}}{\cancel{z} - z^{-1}} + \dfrac{1}{1 - \frac{2}{3}z^{-1}} = \dfrac{1}{z-1} + \dfrac{z}{z - \frac{2}{3}}$

$$\therefore Y(z) - 0.5Y(z)z^{-1} = 0.5 - 0.2z^{-1}\left(\frac{\cancel{z} - \frac{2}{3} + z^2 - \cancel{z}}{(z-1)(z-\frac{2}{3})}\right)$$

$$Y(z)\left(1 - 0.5z^{-1}\right) = 0.5 - \frac{0.2(z^2 - \frac{2}{3})}{z(z-1)(z-\frac{2}{3})}$$

$$\therefore Y(z) = \frac{0.5z}{z-0.5} - \frac{0.2(z^2 - \frac{2}{3})}{(z-\frac{1}{2})(z-1)(z-\frac{2}{3})}$$

$= \text{ZERO INPUT} + \text{ZERO-STATE}$

PERFORM PARTIAL FRACTION EXPANSION & FIND $y(n)$