<u>**EE 322/522 Linear Systems II**</u>

<u>**Computer Project 2: Convolution and Simulation of Linear Systems**</u>

Due Date: May 5, 2009

1. Use convolution sum to find the response of an LTI system. Let the input signal be given by:

$$x(t) = 2*(u(t-0.05)-u(t-0.64)),$$

where, $u(t)$ denotes a unit step function. The Impulse Response of the LTI system is given by,

$$h(t) = e^{-4t}(u(t)-u(t-0.8)),$$ (an FIR filter)

Both the input signal and the impulse response are to be sampled with a sampling interval of T=0.01s.

(a) INPUT GENERATION:

The input is sampled from t = 0 and exactly 64 samples are collected. *Hand-calculate* the number of zeros at the beginning of the input and the number of ones. Then use the MATLAB commands "ones" and "zeros" to create the input sequence, x(n). Note that, N1 = 64.

Examples:

```
>> v1 = zeros(1,4);  % Creates a row vector with 4 zeros, i.e., v1 = [0 0 0 0]

>> v2 = ones(1,5);   % Creates a row vector with 5 ones. i.e., v2 = [1 1 1 1 1]

>> v3 = [v1 v2];     % Concatenates v1 and v2 to form a larger row vector v3.
                     % In this case, v3 = [0 0 0 0 1 1 1 1 1].

>> v3 = [zeros(1,4) ones(1,5)] ;   % Is equivalent to the previous three commands.

>> length(v3);       % Will return the length of the vector v3.
                     % In this case, the result is 9.
```

- The nth member of an array can be addressed by, x(n). For example, if x is formed as,

```
>> x = [-1 3 -5 4 2 4];
```

then, x(1) =-1, x(2)=3, and so on. The $m-$th through $n-$th members of a sequence can be addressed by x(m:n). In the above example, x(1:3) = [-1 3 -5] and x(4:6) = [4 2 4].

(b) IMPULSE RESPONSE GENERATION

Generate the finite length impulse response sequence h(n).

- What are the start and end times for h?
- What is the length (N2) of h?

Example:

- To generate a function $f(t) = 3e^{-10t}$ for the time-duration 1s to 10s in increments of 0.1s, first define the desired time-vector,

```
>> t=1:0.1:10;     % Generates a time base from 1 to 10, in increments of 0.1.
                   % In this case, t=[1 1.1 1.2 1.3 .... 9.9 10].
```

Then, generate the signal $f(t)$ as follows,

```
>>  f=3*exp(-10*t);   % Will return a sequence of the corresponding function
                      % values, i.e., [3e^-10  3e^-11 ··· 3e^-100]
```

- To plot the function sequence f with the desired time-scale, use,

```
>> stem(t,f,'fill')          % NOTE: t and f MUST have the same lengths.
```

(c) Use the MATLAB command "conv" to obtain the output, y(n). Use,

```
>> y = conv(h,x);
```

- Make sure that the length of y is as expected, i.e., N = N1 + N2 - 1.

You can verify this by,

```
>> N1 = length(x)      % The variable N1 is assigned the length of x
>> N2 = length(h)      % The variable N2 is assigned the length of h
>> N = length(y)       % The variable N is assigned the length of y
```

Then, generate the plots of x(1:N1), h(1:N2), y(1:N). **[3 plots]**

Note: You must first generate the longer time-scale required for plotting y. Use,

```
>> t=0:0.01:0.01*(N-1);    % Creates the maximum time-scale needed

>> stem(t(1:N1),x,'filled')        %  Plots the input. Uses only the first N1 of t
>> stem(t(1:N2),h,'filled')        %  Plots the impulse response. Lengths = N2
>> stem(t,y,'filled')              %  Plots the output. Both t and y have same lengths
```

2. An LTI discrete-time system is modeled by the following difference equation :

$$y(n) = 1.8\,y(n\text{-}1) - 0.81\,y(n-2) + 0.01\,x(n) + 0.01\,x(n-2).$$

The initial conditions are: $y(\text{-}1) = y(\text{-}2) = 0.$

(a) Find the system response to the input signal,

$x_1(t) = 30 + 4t + 10\sin(5\pi t + \frac{\pi}{4})$ for t from 0.0 to 20.0 seconds.

The sampling interval is 0.1s. Plot the input sequence $x_1(n)$ and the output sequence $y_1(n)$. **[2 plots]**

HINT: Use the Matlab command "filter" to obtain the output.

Example: The "filter" command is used as follows: (also try "help filter" in matlab)

```
>> y=filter(b,a,x);    % will produce the output sequence due to the input sequence x.
                       % a and b should be specified as,

>> b =  [b(1) b(2) ...];     %
>> a =  [1 a(2) a(3) ..];    % Note: a(1) must be 1.
```

- For example, if $y(n) = 2x(n) - x(n\text{-}1) + 3y(n\text{-}1) - 4y(n-2),$ then use,

```
>> b = [2 -1];
>> a = [1 -3 4];
>> y=filter(b,a,x);
```

(b) Find the system response to the input signal :

$x_2(t) = 30 + 4t$ (for t from 0.0 to 20.0 seconds)

with a sampling interval 0.1 second.

Note: The only difference between the signal $x_1(t)$ and $x_2(t)$ is that the "sine" function has been removed.

i. Plot the input sequence $x_2(n)$ and the output sequence $y_2(n)$. **[2 plots]**

ii. Compare the plots of $x_1(n)$ with $x_2(n)$. Explain the difference between these two sequences?

iii. Compare the output sequences obtained in part-(a) with the one in part-(b). Explain why they look the same!