

FINAL EXAM

Two hours

16th August, 1995

Note : Please show all steps to earn partial credits

- (a) Find an analytical expression for
- $h(n)$
- by using the analytical Convolution method :

$$h(n) = \left(\frac{1}{3}\right)^n u(n) \star 2^n u(-n-1)$$

(3.5 pts.)

- (b) Verify your results of part-a by performing the same convolution using the
- Z
- transform approach. You must specify all the appropriate ROCs. Does
- $h(n)$
- correspond to a stable system? Is it causal? Justify your answers. (3.5 pts.)

2. The following continuous-time periodic signal,

$$x(t) = 10 \cos(300\pi t)$$

is sampled with the sampling frequency of $f_s = 500\text{Hz}$.

- a. Find the discrete sequence $x(n)$ and find its period. (0.5 pts.)
- b. Find and sketch the DFT series (magnitudes only) of the discrete signal $x(n)$. (2.5 pts.)

3. A continuous-time signal is given by,

$$x(t) = 2 \cos(400\pi t) + 5 \sin(600\pi t) + 5 \sin(1600\pi t) + 2.1 \cos(2600\pi t).$$

The sampling frequency is 3000Hz. You need to design a nonrecursive digital filter with 21 coefficients such that the two mid-frequencies are passed and the other two (higher and lower) are suppressed.

- (a) Find a general expression for the coefficients of the filter that meets the requirements. Make sure that the filter is practically realizable (i.e., causal). Note : you have to select appropriate cut-off frequencies. (3 pts.)
- (b) What are the advantages and disadvantages of (i) increasing filter length and (ii) using windows. (1 pts.)

4. What are the relationships between (i)
- z
- Transform and Fourier Transform, (ii) Fourier Transform and DFT and (iii) DFT and DFS? NOTE : You should not need more than 2-3 lines in each case to explain. (2 pts.)

5. A discrete-time system is described by the difference equation.

$$y(n) = 0.5y(n-1) - 0.2x(n-1)$$

For an input sequence $x(n) = u(n-1) + (2/3)^n u(n)$ and initial condition $y(-1) = 1.0$, find the steady-state solution of this difference equation by using the z -Transform approach. (6 pts.)