EXAMINATION BOOK (1)

PLEASE USE BOTH SIDES OF PAPER

Name SOLUTIONS, MID-TERM-I

Course EE 322

Instructor

Date SUMMER-99

1. (a) $h(n) = a^n u(n-3)$

$= (1.001)^n u(n-3)$

$h(0) = 0 = h(1) = h(2)$

$h(3) = (1.001)^3$, $h(4) = (1.001)^4$, $h(5) = (1.001)^5 \ldots$ growing.

FOR STABILITY, WE NEED $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

However, In this case, $\sum_{n=3}^{\infty} (1.001)^n = \frac{1.001^3((1.001)^\infty - 1)}{1-1.001} = \infty$

$\therefore$ NOT STABLE.

FOR STABILITY, NEED $|a| < 1$

(b) (i) Causality

Input

@ $n=0$, $x(0)$

@ $n=1$ $x(1)$

$\vdots$

Output

$y(0) = 0 = (30)(0)x(1)$

$30\, x^2(2)$

NEED FUTURE INPUTS

$\therefore$ NOT CAUSAL

(ii) LINEARITY

INPUT

$x_1(n)$

$x_2(n)$

$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

OUTPUT

$y_1(n) = 30n^2 x_1^2(n+1)$

$y_2(n) = 30n^2 x_2^2(n+1)$

$y_3(n) = 30n^2 x_3^2(n+1)$

$= 30n^2 (a_1 x_1(n+1) + a_2 x_2(n+1))^2$

$= 30n^2 (a_1^2 x_1^2(n+1) + a_2^2 x_2^2(n+1) + 2a_1 a_2 x_1(n+1) x_2(n+1))$

$\neq a_1 y_1(n) + a_2 y_2(n)$

$\therefore$ SUPERPOSITION FAILS $\Rightarrow$ NOT LINEAR

(iii) STABILITY

IF $|x(n)| \leq M_1$ For all n

Then $y(n) \leq 30n^2 M_1^2 \Rightarrow$ CAN GROW TO ∞ IF $n \to \infty$. NOT STABLE

(2)

(IV) TIME INVARIANT

Input: $x_1(n) = x(n-n_1)$

Output: $y_1(n) = 30n^2 x_1^2(n)$

$= 30n^2 x^2(n-n_1)$

$\neq y(n-n_1)$

$\therefore$ NOT TIME-INVARIANT

1 (c)

$x(n) = n\,u(n-1)$

$x(n+1) = (n+1)\,u(n)$

$x^2(n+1) = (n+1)^2 u(n)$

$y(n) = 30n^2 x^2(n+1) = 30n^2(n+1)^2 u(n)$

$n=0;\quad y(0) = 0$

$n=1;\quad y(1) = 120 = 30(1)^2(1+1)^2(1)$

$n=2;\quad y(2) = 30(2)^2(3)^2(1) = 1080$

$n=3;\quad y(3) = 30(3)^2(4)^2(1) = 4320$

$n=4;\quad y(4) = 30(4)^2(5)^2(1) = 12000$

2. (a) (i) $f_1 = 100\sqrt{2}$; $f_2 = 200\,Hz$

$\frac{f_1}{f_s} = \frac{100\sqrt{2}}{250}$ = NOT POSSIBLE TO WRITE AS A RATIO OF INTEGERS - NOT PERIODIC

$(2\pi)\frac{f_2}{f_s} = 2\pi\left(\frac{200}{250}\right) = \left(\frac{4}{5}\right)2\pi \rightarrow$ PERIODIC, $N = 5$

(ii) YES, THERE WILL BE ALIASING, BECAUSE $f_2 = 200\,Hz$ AND HENCE THE MAXIMUM FREQUENCY OF $x(t)$ IS ALSO 200 Hz, BUT WE NEED $f_s > 2f_{max} = 400\,Hz$ TO AVOID ALIASING.

(b) (i) $2\pi\frac{f_1}{f_s} = 2\pi\frac{100\sqrt{2}}{500\sqrt{2}} = \frac{2\pi}{5}$ PERIOD = 5

$\frac{f_2}{f_s} = \frac{200}{500\sqrt{2}} = \frac{2}{5\sqrt{2}} \rightarrow$ NOT PERIODIC

(ii) IN THIS CASE $f_s = 500\sqrt{2} = 707.0\,Hz > \underset{\uparrow}{400\,Hz}$ ③

$2f_{max}$

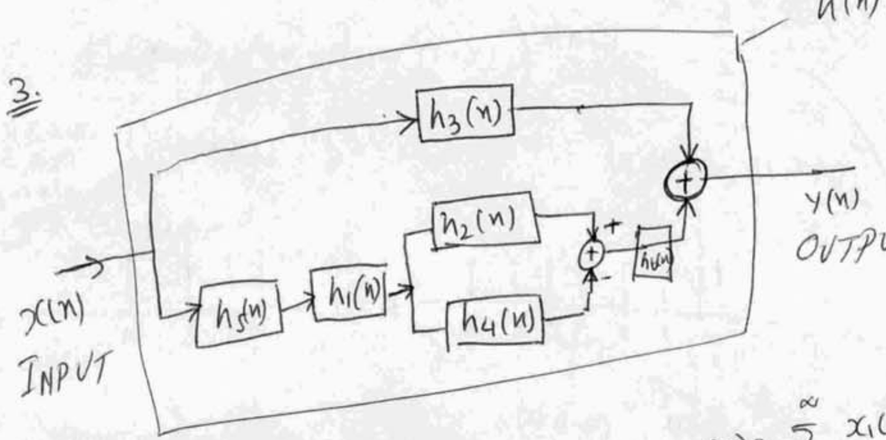
HENCE NO ALIASING.

ⓔ MINIMUM REQUIRED f_s FOR $x_1(t) \Rightarrow 200\sqrt{2}\,Hz$

$x_2(t) \Rightarrow 400\,Hz$

& $x(t) \Rightarrow 400\,Hz$ also

3.



$$y(n) = \sum_{K=-\infty}^{\infty} x_1(K)\,x_2(n-K)$$

4.

	K	-4	-3	-2	-1	0	1	2	3	4	
	$x_1(K)$			0	-1	1	2	1	0	0	
$n=-2$	$x_2(-2-K)$	0	0	2							
$n=-1$	$x_2(-1-K)$		1	0	2*						
$n=0$	$x_2(-K)$			1	0*	2*					
$n=1$	$x_2(-\cancel{K+1}+1)$				1*	0*	2				
$n=2$	$x_2(+2-K)$					1*	0*	2			
$n=3$	$x_2(3-K)$						1*	0*	2		
$n=4$	$x_2(4-K)$							1*	0	2	
$n=5$	$x_2(5-K)$								1	0	2

$y(-2) = 0$

$y(-1) = (-1)(2) = -2$

$y(0) = (-1)(0) + (2)(1) = 2$

$y(1) = (-1)(1) + (1)(0) + (2)(2) = 3$

$y(2) = (1)(1) + (2)(0) + (2)(1) = 3$

$y(3) = (2)(1) + (1)(0) = 2$

$y(4) = (1)(1) = 1$

$y(5) = 0$

4.(ii)

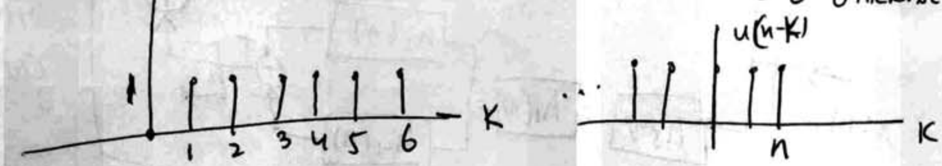
$$y(n) = \sum_{K=-\infty}^{\infty} x_1(K)\, x_2(n-K)$$

$$= \sum_{K=-\infty}^{\infty} 2\left(u(K-1) - u(K-7)\right) \left(\frac{1}{2}\right)^{(n-K)} u(n-K)$$

$$= \sum_{K=-\infty}^{\infty} (2)(2)^{K-n} \left(u(K-1) - u(K-7)\right) u(n-K)$$

$$= \sum_{K=-\infty}^{\infty} (2)^{K-n+1} \left(u(K-1) - u(K-7)\right) u(n-K)$$

$u(K-1) - u(K-7)$ & $u(n-K) = \begin{cases} 1 & n-K \geq 0 \text{ OR } K \leq n \\ 0 & \text{OTHERWISE} \end{cases}$



$$= \begin{cases} \sum_{K=1}^{n} (2)^{(K-n+1)} & ; 1 \leq n \leq 6 \\ 0 & n < 1 \\ \sum_{K=1}^{6} (2)^{K-n+1} & ; n \geq 6 \end{cases}$$

maximum at $n=6$

$$2^{2-n} \frac{(1-2^6)}{1-2} = (252)\, 2^{-n} \; ; \; n \geq 6$$

Tends to 0 as $n \to \infty$

5. (a) Homogeneous Equation; 5

$$y(n) - 0.2\, y(n-1) = 0$$

Let $y(n) = C_1 r^n$, BE THE SOLUTION OF THE HOMOGENEOUS EQ^n.

THEN IT MUST SATISFY IT,

i.e., $C_1 r^n - 0.2 C_1 r^{n-1} = 0 \Rightarrow \boxed{r - 0.2 = 0}$ CHARACTERISTIC EQUATION

$\therefore r = 0.2 \Rightarrow$ CHARACTERISTIC ROOT.

$\therefore y(n) = C_1 (0.2)^n$

GIVEN, AT $n = -1$, $y(n) = -1$ $\therefore -1 = \frac{C_1}{0.2} \Rightarrow C_1 = -0.2$

$\therefore y_h(n) = (-0.2)(0.2)^n ;\ n \geq -1$

SINCE $|r| = 0.2 < 1$, IT IS A STABLE SYSTEM.

(b) USING TABLE 3.1

$y_p(n) = C_2 + C_3 n$

USING THESE IN THE ORIGINAL DIFFERENCE EQUATION,

$$C_2 + C_3 n = 0.2(C_2 + C_3(n-1)) + 2(n+1) - 4(n)$$

GROUP TOGETHER ALL CONSTANT AND MULTIPLES OF "n"

$\therefore C_2 - 0.2 C_2 - 2 + 0.2 C_3 + n(C_3 - 0.2 C_3 - 2 + 4) = 0$

$\therefore$ TWO EQUATIONS, $0.8\, C_2 + 0.2 C_3 = 2$ ... (1)

& $0.8 C_3 = -2$ ... (2) $\Rightarrow C_3 = -2.5$

PLUG IN (2) $\therefore 0.8 C_2 = 2 + 0.5 = 2.5$

$\therefore C_2 = 2.5/0.8 = 3.125$

$\therefore y_p(n) = (3.125 - 2.5n)\, u(n)$

5(c) FOR TOTAL SOLUTION

6

$$y_{TOTAL}(n) = y_c(n) + y_p(n)$$

$y_p(n) \rightarrow$ from part (b)

$y_c(n)$: COMPLEMENTARY SOLUTION

BUT IN THIS CASE $y_c(n) = C_4(0.2)^n + A_0\delta(n)$

$N=1, L=1=N$

$\therefore y_{TOTAL}(n) = C_4(0.2)^n + A_0\delta(n) + 3.125 - 2.5n;\ n \geq 0$

GIVEN, $y(-1)=0$

NEED TWO DERIVED INITIAL CONDITIONS TO FIND, $y_{TOTAL}(n)$.

$x(n) = (n+1)u(n) \Rightarrow$ $x(-1)=0$, $x(0)=1$, $x(1)=2$

$$n=0,\quad y(0) = 0.2\,\overset{0}{\cancel{y(-1)}} + 2\,\underbrace{\overset{1}{\cancel{u(0)}}}_{x(0)} - 4(1)\,\underbrace{\overset{0}{\cancel{u(-1)}}}_{x(-1)}$$

$\therefore y(0)=2$

$$n=1,\quad y(1) = 0.2\,y(0) + 2x(1) - 4x(0)$$
$$= (0.2)(2) + (2)(2) - 4(1) = 0.4$$

$\therefore n=0;\ 2 = C_4(0.2)^0 + A_0 + 3.125 - (2.5)(0) = A_0 + C_4 + 3.125$

$n=1;\ 0.4 = C_4(0.2)^1 + A_0\overset{0}{\cancel{\delta(1)}} + 3.125 - (2.5)(1)$

$\therefore C_4 = \frac{1}{0.2}(0.4 - 3.125 + 2.5) = -1.125$

$A_0 = 0$

$\therefore y_{TOTAL}(n) = -1.125(0.2)^n + 3.125 - 2.5n;\ n \geq 0$