APPENDIX E

Table of Useful Fourier Transforms

In Table E-1 of Fourier transform pairs, we define

$$u(\xi) = \begin{cases} 1 & \xi > 0 \\ 0 & \xi < 0 \end{cases} \tag{E-1}$$

$$\text{rect}\,(\xi) = \begin{cases} 1 & |\xi| < \frac{1}{2} \\ 0 & |\xi| > \frac{1}{2} \end{cases} \tag{E-2}$$

$$\text{Sa}(\xi) = \frac{\sin(\xi)}{\xi} \tag{E-3}$$

$$\text{tri}\,(\xi) = \begin{cases} 1 - |\xi| & |\xi| < 1 \\ 0 & |\xi| > 1 \end{cases} \tag{E-4}$$

$$x(t) \leftrightarrow X(\omega) \tag{E-5}$$

and let α, τ, σ, ω_0, and W be real constants.

TABLE E-1

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$	Notes
1	$\alpha\delta(t)$	α	
2	$\alpha/2\pi$	$\alpha\delta(\omega)$	
3	$u(t)$	$\pi\delta(\omega)+(1/j\omega)$	
4	$\frac{1}{2}\delta(t)-\frac{1}{j2\pi t}$	$u(\omega)$	
5	$\text{rect}(t/\tau)$	$\tau\,\text{Sa}(\omega\tau/2)$	$\tau>0$
6	$(W/\pi)\text{Sa}(Wt)$	$\text{rect}\,(\omega/2W)$	$W>0$
7	$\text{tri}\,(t/\tau)$	$\tau\,\text{Sa}^2(\omega\tau/2)$	$\tau>0$
8	$(W/\pi)\text{Sa}^2(Wt)$	$\text{tri}\,(\omega/2W)$	$W>0$
9	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
10	$\delta(t-\tau)$	$e^{-j\omega\tau}$	
11	$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
12	$\sin(\omega_0 t)$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	
13	$u(t)\cos(\omega_0 t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$u(t)\sin(\omega_0 t)$	$-j\frac{\pi}{2}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$u(t)e^{-\alpha t}$	$\frac{1}{\alpha+j\omega}$	$\alpha>0$
16	$u(t)te^{-\alpha t}$	$\frac{1}{(\alpha+j\omega)^2}$	$\alpha>0$
17	$u(t)t^2e^{-\alpha t}$	$\frac{2}{(\alpha+j\omega)^3}$	$\alpha>0$
18	$u(t)t^3e^{-\alpha t}$	$\frac{6}{(\alpha+j\omega)^4}$	$\alpha>0$
19	$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2+\omega^2}$	$\alpha>0$
20	$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	$\sigma>0$