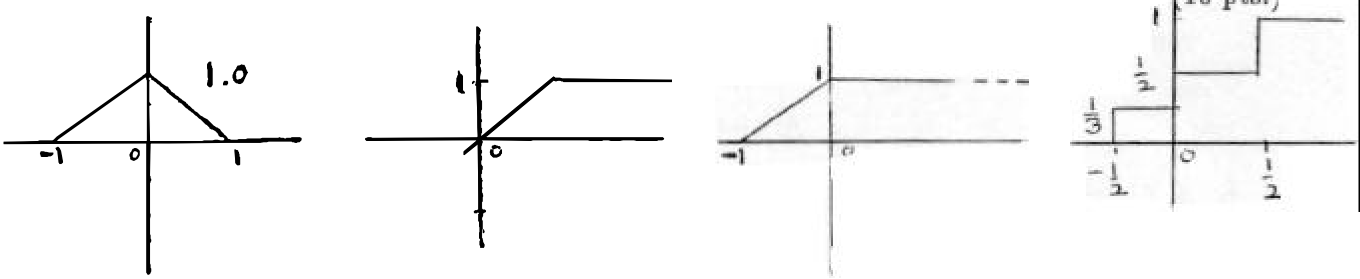
# Final Exam

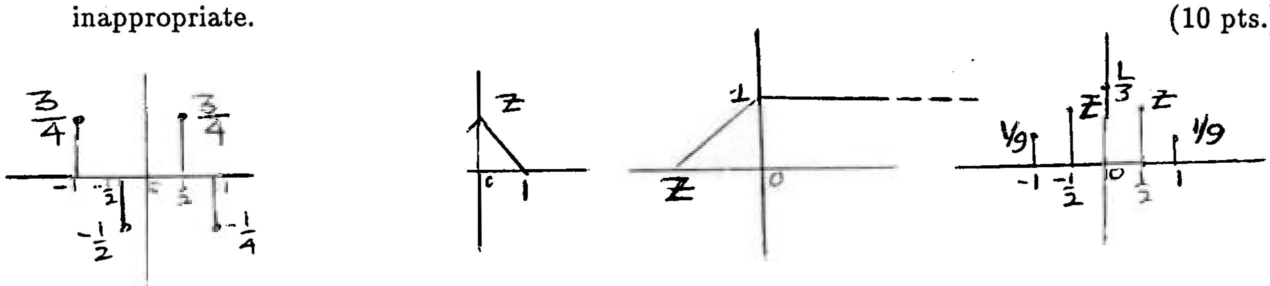
2 hours 4*th* December, 1991

The exam will be graded out of 200 points, but you may attempt more to earn extra points.

1. Which of the following functions are proper candidates for probability distribution (cumulative) functions? Provide one line explanation *if* you consider any function as inappropriate. Assume that the x-axes are properly scaled. (10 pts.)




2. The following are candidates for probability density/mass functions. Select the correct candidates and find $z$ for those cases. Provide one line explanation *if* you consider any function as inappropriate. (10 pts.)




3. This problem is based on problems 1 and 2.
   a. Find the probability density/mass functions of the *appropriate* distribution functions of problem 1.
   b. Find the probability distribution functions of the *appropriate* density/mass functions of problem 2. (15 pts.)

4. A coffee machine takes a quarter to pour out a cup of coffee. But due to equipment malfunctioning, it gobbles up the quarter but gives no coffee *randomly* one out of six times (sounds familiar!). Someone desperate for caffeine ended up spending one full dollar to get a single cup of coffee! What is the probability of such an unfortunate foul-up to occur in that vending machine? (25 pts.)

5. A zero-mean WSS Gaussian random process has the following autocorrelation function :

$$R_{XX}(\tau) = 0.5(3 - |\tau|) \quad \text{for, } |\tau| \leq 5.$$

a. Is this process mean-ergodic?

b. Is this process autocorrelation-ergodic? (50 pts.

6. Consider the complex exponential random process

$$X(t) = \alpha e^{j(\Omega t+\Phi)},$$

where $\alpha$ is a constant, $\Phi$ is uniformly distributed between $-\pi$ and $\pi$ and $\Omega$ has an exponential density function,

$$f_\Omega(\omega) = e^{-\omega}u(\omega).$$

a. Find the mean and the autocorrelation function of $X(t)$. Assume that the two random variables are independent, *i.e*,

$$f_{\Omega,\Phi}(\omega,\phi) = f_\Omega(\omega)f_\Phi(\phi)$$

b. Is $X(t)$ a WSS process? Discuss its ergodicity properties. (60 pts.)

**SUGGESTION** : Don't worry about the $j$! Just treat it as a constant.

7. Assume that the input $X(t)$ to a linear time-invariant system is a zero-mean white Gaussian random process with the following Power Spectral Density (PSD),

$$S_{XX}(f) = 10.$$

The impulse response of the LTIVC system is given by,

$$h(t) = \begin{cases} 3e^{-2t} & t \geq 0; \\ 0, & \text{elsewhere.} \end{cases}$$

Let $Y(t)$ be the output of the system.

a. Find $S_{YY}(f)$, the PSD at the output.

b. Find $E[Y(t)]$ and $E[Y^2(t)]$.

c. If the input is WSS, what about the output?

d. Find and plot $R_{YY}(\tau)$, the autocorrelation function of the output.

e. Discuss the ergodicity properties of the output. (70 pts.

**Formulae :**

1. $\int e^{at}dt = \frac{1}{a}e^{at}$
2. $2\pi \int \frac{1}{1+(2\pi f)^2}df = \tan^{-1}(2\pi f)$
3. Fourier Transform of $e^{-at}u(t)$ is $\frac{1}{a+j2\pi f}$ for $a > 0$.
4. Fourier Transform of $e^{-a|t|}$ is $\frac{2a}{a^2+(2\pi f)^2}$ for $a > 0$.

**TOTAL POINTS 240**

**EE 761** **Winter 2005**

**FINAL** (Open Text)

2 hours 15*th* March, 2005

The exam will be graded out of 45 points, but you may attempt more to earn **BONUS** points.

1. In a Power grid 6 Transformers are in simultaneous operation. From previous experience it is known that the probability that a Transformer will malfunction before 30 days of continuous operation 2%. What are the probabilities that, before 30 days have elapsed,
   a. All 6 Transformers malfunctioned once.
   b. None of the 6 Transformers malfunctioned even once.
   c. Only 3 of the 6 malfunctioned.
   d. 2 or more of the 6 malfunctioned. (7 pts.)

2. $\mathbf{X}$ and $\mathbf{Y}$ are independent zero mean Gaussian Random variables with variance=4 in both cases. Let,

$$\begin{aligned} \mathbf{Z} &= 2(\mathbf{X}+\mathbf{Y}) \qquad \text{and} \\ \mathbf{W} &= (2\mathbf{X}-\mathbf{Y}) \end{aligned}$$

   a. Find the mean and variances of the random variables $\mathbf{Z}$ and $\mathbf{W}$.
   b. Find the Joint Probability Density Function (pdf) $f_{Z,W}(z,w)$.
   c. Find the marginal pdf $f_Z(z)$.
   d. Find the conditional pdf $f_{W|Z}(w|z)$.
   e. Are $\mathbf{Z}$ and $\mathbf{W}$ independent? (11 pts.)

3. Let $X(1)$, $X(2)$, $X(3)$, . . . $X(n)$, . . . be a sequence of zero-mean and independent random variables. Define another random process sequence,

$$Z(n) = \sum_{k=1}^{n} X(k), \qquad n = 1,\ 2,\ 3,\ \ldots$$

   a. Show that $Z(n)$ is a Markov sequence.
   b. Show that $Z(n)$ is Martingale.
   c. Would $Z(n)$ be Martingale if the sequence $X(k)$ had not been zero mean? Explain. **[Hint: Assume, $E[X(k)] = 1$, and redo part-b**
   d. Find the mean and variance of $Z(n)$. (8 pts.)

4. Find the mean and the auto-covariance of the random process defined as $\mathbf{X}(t) = b\cos(wt+\phi)$, where, the random variables $b$ and $\phi$ are independent. $\phi$ is uniformly distributed between $-\pi$ and $\pi$ and $b$ is Poisson distributed with $\lambda = 5$, *i.e.,*

$$Pr[b=k] = \frac{\lambda^k}{k!}e^{-\lambda}; \qquad k = 0, 1, 2, \ldots.$$

   Is this Process WSS? Justify your answer. (9 pts.)

**Note :** You may make use of the fact that both the mean and variance of a Poisson Distributed Random variable are $= \lambda$.

5. A zero-mean WSS Gaussian random process has the following autocorrelation function :

$$R_{XX}(\tau) \;=\; 2\cos(10\pi\tau) \qquad \text{for, } |\tau| \leq \frac{1}{20}.$$

and zero outside this range of $\tau$.

   a. Is this process mean-ergodic?

   b. Is this process autocorrelation-ergodic? (8 pts.)

6. Zero mean white noise is passed through a low pass filter having the transfer function $H(j\omega)$ shown below. Find the power spectral density, $S_{YY}(\omega)$ and the autocorrelation function, $R_{YY}(\tau)$ of the filter output. Determine the values of $\tau$ for which the output is uncorrelated. (7 pts.)

**Total Points = 50**