EQUIVALENT STATIC LOADS FOR RANDOM VIBRATION
Revision B

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Introduction

A particular engineering design problem is to determine the equivalent static load for equipment subjected to base excitation random vibration. The goal is to determine peak values for

1. Relative Displacement
2. Absolute Acceleration
3. Transmitted Force

Model

The first step is to determine the acceleration response of the equipment or structural component. As a first approximation, model the component as a single-degree-of-freedom system.

Consider the single-degree-of-freedom system subjected to base excitation shown in Figure 1.

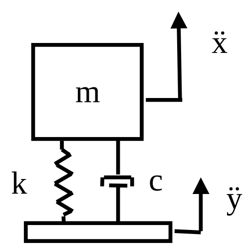

Figure 1. Single-degree-of-freedom System

The variables are

m = mass
c = viscous damping coefficient
k = stiffness
x = absolute displacement of the mass
y = base input displacement

Furthermore, the relative displacement z is

$$z = x - y \tag{1}$$

The natural frequency of the system f_n is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{2}$$

Acceleration Response

The Miles equation is a simplified method of calculating the response of a single-degree-of-freedom system to a random vibration base input, where the input is in the form of a power spectral density.

The overall acceleration response $\ddot{x}_{GRMS}$ is

$$\ddot{x}_{GRMS} = \sqrt{\frac{\pi}{2} P f_n \left(\frac{1}{2\xi}\right)} \tag{3}$$

where

P	= the power spectral density level at the natural frequency
f_n	= the natural frequency
ξ	= the damping ratio

Note that the damping is often represented in terms of the quality factor Q.

$$Q = \frac{1}{2\xi} \tag{4}$$

Equation (3), or an equivalent form, is given in numerous references, including those listed in Table 1.

Table 1. Miles Equation References			
Reference	Author	Equation	Page
1	Himelblau	(10.3)	246
2	Fackler	(4-7)	76
3	Steinberg	(8-36)	225
4	Luhrs	-	59
5	Mil-Std-810E	-	516.4-3
6	Caruso	(1)	28

Furthermore, the Miles equation is an approximate formula that assumes a flat power spectral density from zero to infinity Hz. As a rule-of-thumb, it may be used if the power spectral density is flat over at least two octaves centered at the natural frequency.

An alternate response equation that allows for a shaped power spectral density input is given in Appendix A.

Relative Displacement

The overall relative displacement response z_{RMS} is

$$Z_{RMS} = \left(\frac{1}{2\pi \, f_n} \right)^2 \ddot{x}_{GRMS} \tag{5}$$

where $\ddot{x}_{GRMS}$ is calculated from the Miles equation (3).

The RMS force in the spring is thus

$$F_{RMS} = k \, Z_{RMS} \tag{6}$$

The resulting axial or bending stress can then be calculated from the force in equation (6).

Probability Values

A discussion of the probability values associated with random vibration is given in Appendix B.

RMS and Standard Deviation

Note that the RMS value is equal to the 1σ value assuming a zero mean. The 1σ value is the standard deviation.

A 3σ value is thus three times the RMS value.

Peak Acceleration

There is no exact peak acceleration value for random vibration.

An instantaneous peak value of 3σ is often taken as the peak equivalent static acceleration. A higher or lower value may be appropriate for given situation.

Some sample guidelines are for peak acceleration are given in Table 2.

Table 2. Sample Design Guidelines for Peak Acceleration or Force				
Reference	Author	Design Equation	Page	Qualifying Statements
1	Himelblau	3σ	246	3σ is arbitrary
2	Fackler	3σ	76	3σ is the usual assumption for the equivalent peak sinusoidal level.
4	Luhrs	3σ	59	Theoretically, any large acceleration may occur.
7	NASA	3σ for STS Payloads 2σ for ELV Payloads	2.4-3	Minimum Probability Level Requirements
8	McDonnell Douglas	4σ	4-16	Equivalent Static Load

Furthermore, some references are concerned with fatigue rather than peak acceleration, as shown in Table 3.

Table 3. Design Guidelines for Fatigue based on Miner's Cumulative Damage Index		
Reference	Author	Page
3	Steinberg	229
6	Caruso	29

Note that the Miner's Index considers the number of stress cycles at the 1σ, 2σ, and 3σ levels.

Special Case

Consider a system that has a natural frequency that is much higher than the maximum base input frequency. An example would be a very stiff bar that was subjected to a low frequency base excitation in the bar's longitudinal axis.

This case is beyond the scope of Miles equation, since the Miles equation takes the input power spectral density at the natural frequency.

The response of this special case system is simply equal to the base input.

Conclusion

The average design level is 3σ, per Table 2. Again, this value is three times the RMS value, assuming a zero mean.

References

1. H. Himelblau et al, Guidelines for Dynamic Environmental Criteria (Preliminary Draft), Jet Propulsion Laboratory, California Institute of Technology, 1997.

2. W. Fackler, Equivalence Techniques for Vibration Testing, SVM-9, The Shock and Vibration Information Center, Naval Research Laboratory, United States Department of Defense, Washington D.C., 1972.

3. Dave Steinberg, Vibration Analysis for Electronic Equipment, Wiley-Interscience, New York, 1988.

4. H. Luhrs, Random Vibration Effects on Piece Part Applications, Proceedings of the Institute of Environmental Sciences, Los Angeles, California, 1982.

5. MIL-STD-810E, "Environmental Test Methods and Engineering Guidelines," United States Department of Defense, Washington D.C., July 1989.

6. H. Caruso and E. Szymkowiak, A Clarification of the Shock/Vibration Equivalence in Mil-Std-180D/E, Journal of Environmental Sciences, 1989.

7. General Environmental Verification Specification for STS & ELV Payloads, Subsystems, and Components, NASA Goddard Space Flight Center, 1996.

8. Vibration, Shock, and Acoustics; McDonnell Douglas Astronautics Company, Western Division, 1971.

9. W. Thomson, Theory of Vibration with Applications, Second Edition, Prentice-Hall, New Jersey, 1981.

APPENDIX A

Acceleration Response

The acceleration response $\ddot{x}_{GRMS}$ of the a single-degree-of-freedom system to a base input power spectral density is

$$\ddot{x}_{GRMS}(f_n, \xi) = \sqrt{\sum_{i=1}^{N} \left\{ \frac{\left\{1+(2\xi\rho_i)^2\right\}}{\left[1-\rho_i^2\right]^2+\left[2\xi\rho_i\right]^2} \right\} \hat{Y}_{APSD}(f_i)\,\Delta f_i} \quad , \quad \rho_i = f_i / f_n$$

(A-1)

where

ξ is the damping ratio

f_n is the natural frequency

$\hat{Y}_{APSD}(f_i)$ is the base input acceleration power spectral density at frequency f_i

The corresponding relative displacement is

$$z_{RMS} = \left(\frac{1}{2\pi f_n}\right)^2 \ddot{x}_{GRMS}$$

(A-2)

Equation (A-1) allows for a shaped base input power spectral density, defined over a finite frequency domain. It is thus less restrictive than the Miles equation.

Equation (A-1) is derived in

T. Irvine, An Introduction to the Vibration Response Spectrum, Vibrationdata.com Publications, 2000.

APPENDIX B

<u>Normal Probability Values</u>

Note that the RMS value is equal to the 1σ value assuming a zero mean. The 1σ value is the standard deviation.

Consider a broadband random vibration time history x(t), which has a normal distribution.

The amplitude x(t) cannot be calculated for a given time. Nevertheless, the probability that x(t) is inside or outside of certain limits can be expressed in terms of statistical theory.

The probability values for the amplitude are given in Tables B-1 and B-2 for selected levels in terms of the standard deviation or σ value.

Table B-1.
Probability for a Random Signal with Normal Distribution and Zero Mean

Statement	Probability Ratio	Percent Probability
$-\sigma < x < +\sigma$	0.6827	68.27%
$-2\sigma < x < +2\sigma$	0.9545	95.45%
$-3\sigma < x < +3\sigma$	0.9973	99.73%
$-4\sigma < x < +4\sigma$	0.99994	99.994%

Table B-2.
Probability for a Random Signal with Normal Distribution and Zero Mean

Statement	Probability Ratio	Percent Probability
$\vert x \vert > \sigma$	0.3173	31.73%
$\vert x \vert > 2\sigma$	0.0455	4.55%
$\vert x \vert > 3\sigma$	0.0027	0.27%
$\vert x \vert > 4\sigma$	6e-005	0.006%

Rayleigh Distribution

The following section is based on Reference 9.

Consider the response of single-degree-of-freedom distribution to a broadband time history. The response is approximately a constant frequency oscillation with a slowly varying amplitude and phase.

The probability distribution of the instantaneous acceleration is the same as that for the broadband random function.

The absolute values of the response peaks, however, will have a Rayleigh distribution, as shown in Table B-3.

Table B-3.
Probability for Response Peaks with a Rayleigh Distribution

Statement	Percent Probability
$\lvert x \rvert > 0$	100%
$\lvert x \rvert > \sigma$	60.7%
$\lvert x \rvert > 2\sigma$	13.5%
$\lvert x \rvert > 3\sigma$	1.2%
$\lvert x \rvert > 4\sigma$	0.03%