

Direct measurement of ultrasonic dispersion using a broadband transmission technique

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Abstract

A broadband, through-transmission technique is described that measures directly the change in phase velocity with frequency without first measuring the absolute phase velocity at different frequencies. The method only requires the measurement of the specimen's thickness and recording of two pulses: one without and one with the specimen inserted between the transmitting and receiving transducers. From the two pulses, the dispersion as well as the attenuation can be determined using Fourier analysis without phase unwrapping. Experimental results of two specimens show an excellent agreement between the measured dispersion and the dispersion determined from the measured attenuation using a time causal model. This agreement verifies the model as well as demonstrates the accuracy and precision of the proposed method. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Accurate measurements of the acoustic attenuation and dispersion have important applications in theoretical acoustics, non-destructive testing, and ultrasound tissue characterization. The attenuation of a layer of immiscible material can be measured quite accurately using a broadband, through transmission technique [1]. Two broadband transducers, one for transmitting and one for receiving the ultrasound pulses, are placed in a water tank and aligned properly. Two pulses are recorded: one without and one with the specimen inserted between the two transducers. From the amplitude spectra of the two pulses, the attenuation can be determined over the frequency range of the recorded pulses. For a wide variety of materials including soft tissues, the attenuation increases with frequency according to a power-law relation: $\alpha = \alpha_0 |\omega|^y$, where α_0 and y are two material-dependent parameters [2]. Dispersion refers to the fact that the phase velocity of a propagating wave may also change with frequency. Comparing with the attenuation, the magnitude of the dispersion is very small (often less than 1%), its frequency dependency is

less certain, and its measurement is generally considered more difficult.

In earlier experiments for determining the dispersion, narrowband pulses were used to measure the phase velocity at a number of discrete frequencies [3]. In 1978, Sachse and Pao [4] proposed to use the broadband, through transmission technique to measure the dispersion. Their method requires intimate contact between the transducers and the specimen. More recently, Lee et al. [5] used the broadband, through transmission technique to measure both the attenuation and dispersion.

In the above measurements, the absolute phase velocity is first measured at different frequencies, and the dispersion is then determined as the change in phase velocity. Since the magnitude of dispersion is very small compared with the absolute phase velocity, a small uncertainty in the measurement of individual phase velocity can produce a large (relative) error in the measurement of dispersion. In this paper, we propose an alternative method that directly measures the change in phase velocity without first measuring the individual phase velocities. The main advantage of the proposed method is the improved accuracy and precision in determining the dispersion. In addition, the overall complexity of data acquisition and signal processing is reduced to the level as for the attenuation measurement.

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The paper is organized as follows. The principle of direct dispersion measurement is first presented. A time causal model proposed by Szabo [6] is then described, which allows one to determine the dispersion from the attenuation. Next, experimental results from two specimens, a layer of special rubber compound and a block of Plexiglas, are presented. The measured dispersion is compared with the dispersion predicted by Szabo's model, and a good agreement is demonstrated. Finally, a discussion is given.

2. Method

2.1. Principle of direct dispersion measurement

The experimental setup for the direct dispersion measurement is the same as the one used for attenuation measurement [1]. Two broadband transducers, separated by distance D , are placed in a water tank and aligned properly. For a specimen having a thickness L , two measurements are performed: one without and one with the specimen inserted between the two transducers. If we use $P_0(t)$, $P_w(t)$, and $P_s(t)$ to designate the transmitted pulse, the received pulse with the water path only, and the received pulse with the specimen inserted, respectively, the Fourier transforms of $P_w(t)$ and $P_s(t)$ can be found as [5]:

$$U_w(\omega) = A_w(\omega)e^{-j\hat{\phi}_w(\omega)} = U_0(\omega)e^{-(\alpha_w + j\beta_w)D} U_r(\omega) \quad (1)$$

and

$$U_s(\omega) = A_s(\omega)e^{-j\hat{\phi}_s(\omega)} \\ = U_0(\omega)e^{-(\alpha_w + j\beta_w)(D-L)} e^{-(\alpha + j\beta)L} T_1 T_2 U_r(\omega) \quad (2)$$

where $U_0(\omega)$, $U_w(\omega)$, $U_s(\omega)$ are the Fourier transforms of $P_0(t)$, $P_w(t)$, and $P_s(t)$, respectively. $U_r(\omega)$ is the spectral response of the receiving transducer. α_w and α are the attenuation, and β_w and β are the propagation constants, of the water and the specimen, respectively. T_1 and T_2 are the transmission coefficients at the two water-specimen interfaces.

In determining the phase angles $\hat{\phi}_w$ and $\hat{\phi}_s$ in Eqs. (1) and (2), the origin of time ($t=0$) is defined as the instant when $P_0(t)$ is launched. In practice, $P_w(t)$ and $P_s(t)$ are usually sampled after a certain time delay, and the origin of time for calculating the phase angles of $P_w(t)$ and $P_s(t)$ is the beginning of the sample window containing the recorded pulse. Consequently, the phase spectra, φ_w and φ_s , of the recorded pulses have an additional term:

$$\varphi_w(\omega) = \hat{\phi}_w(\omega) - \omega t_w \quad (3)$$

and

$$\varphi_s(\omega) = \hat{\phi}_s(\omega) - \omega t_s \quad (4)$$

where t_w and t_s are the time delay of the sample windows containing $P_w(t)$ and $P_s(t)$, respectively. From Eqs. (1)–(4), we can solve for β :

$$\beta = \frac{\varphi_s(\omega) - \varphi_w(\omega)}{L} + \beta_w - \frac{\omega(t_w - t_s)}{L} \quad (5)$$

where φ_w and φ_s are the phase spectra of the recorded pulses within the sample window. Using the relation $\beta = \omega/V(\omega)$, where V is the phase velocity, we have:

$$\frac{1}{V_p(\omega)} = \frac{\varphi_s(\omega) - \varphi_w(\omega)}{\omega L} + \frac{1}{V_w} - \frac{(t_w - t_s)}{L} \quad (6)$$

where V_p and V_w are the phase velocities of the specimen and the water, respectively. Finally, by neglecting the dispersion of water, we obtain the dispersion of the specimen:

$$\frac{1}{V_p(\omega_0)} - \frac{1}{V_p(\omega)} = \frac{\varphi_w(\omega_0) - \varphi_s(\omega_0)}{\omega L} - \frac{\varphi_w(\omega) - \varphi_s(\omega)}{\omega_0 L} \quad (7)$$

where ω_0 is a reference frequency at which $P_w(t)$ and $P_s(t)$ have significant energy.

An important feature of Eq. (7) is that the variables V_w , t_w and t_s are all cancelled. This cancellation has two important consequences. First of all, it simplifies the measurement procedure, and eliminates the uncertainties associated with the measurement of these variables. Secondly, since the actual values of t_w and t_s have no effects on the dispersion, one can arbitrarily shift the recorded pulse within the sample window prior to the phase calculation. Specifically, a procedure of circularly rotating the pulse to the left is applied so that the center of gravity of the pulse is moved to the beginning of the sample window. The purpose of this procedure is to remove the rapidly-changing component in the phase spectrum. As a result, one can use the original phases, not the unwrapped phases, to calculate the dispersion using Eq. (7), and the 2π ambiguity in phase unwrapping is avoided.

From the two recorded pulses $P_w(t)$ and $P_s(t)$, the attenuation of the specimen can also be determined [5]:

$$\alpha = b_0 + \frac{1}{L} \ln\left(\frac{A_w(\omega)}{A_s(\omega)}\right) \quad (8)$$

where the offset $b_0 = [\ln(T_1 T_2)]/L$, and A_w and A_s are the amplitude spectra of $P_w(t)$ and $P_s(t)$, respectively.

2.2. A time causal model for determining dispersion from attenuation

For attenuation having a power-law frequency dependency, Szabo [6] proposed a time causal model which can be used to determine the dispersion from the attenu-

ation:

$$\frac{1}{V_p(\omega_0)} - \frac{1}{V_p(\omega)} = \frac{2\alpha_0}{\pi} \ln \frac{\omega}{\omega_0} \quad \text{for } y=1, \quad (9)$$

$$\frac{1}{V_p(\omega_0)} - \frac{1}{V_p(\omega)} = -\alpha_0 \tan(y\pi/2) \times (\omega^{y-1} - \omega_0^{y-1}) \quad \text{for } y>1. \quad (10)$$

These two equations will be used to predict the dispersion from the measured attenuation. The predicted dispersion will then be compared with the directly measured dispersion.

3. Experiment and results

Two specimens are used in this study. The first one is a layer of a special rubber compound (No. 35084, BFGoodrich Company, Brecksville, OH) that has a thickness (L) of 1.28 cm. The second one is a cylindrical

block of Plexiglas that has a thickness (L) of 4 cm. The transmitting transducer used in this study is Panametrics V309 (5 MHz center frequency, 1.27 cm aperture, 8.9 cm focal length), and the receiving transducer is Panametrics V384 (3.5 MHz center frequency, 0.635 cm aperture, non-focusing). The distance (D) between the two transducers is 18 cm. The pulser/receiver is Panametrics 5052PR. The amplified pulse is digitized by a SONY/TEK 390AD programmable digitizer (10-bit resolution, 60 MHz sampling frequency). The size of the sample window is 512 (samples).

Fig. 1 shows the phase spectra (top two curves) of the properly shifted $P_w(t)$ and $P_s(t)$ from the rubber specimen. The bottom curve shows the phase difference which is used in Eq. (7) to calculate the dispersion. Fig. 2 summarizes the results from the rubber specimen ($L=1.28$ cm). The top plot shows the measured attenuation (small circles) by using Eq. (8). Due to the close impedance match between the specimen and the water,

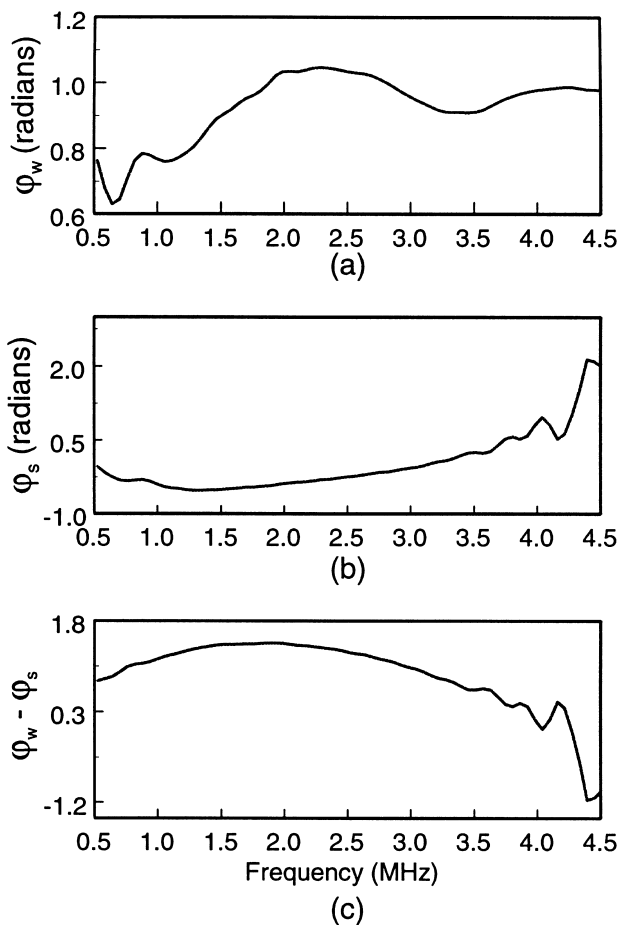


Fig. 1. Phase measurement from the rubber specimen. (a) Phase spectrum of the water-path pulse (P_w) after a proper shift to minimize the phase variation with frequency. (b) Phase spectrum of the properly shifted pulse (P_s) passing through the specimen. (c) Difference between the two phase spectra shown in (a) and (b).

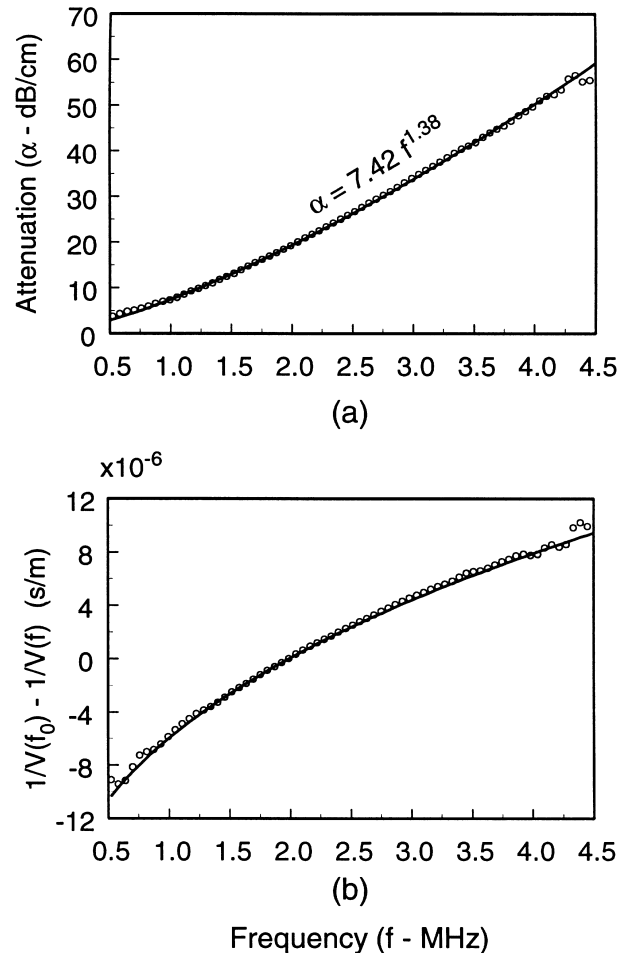


Fig. 2. Attenuation and dispersion of the rubber specimen. (a) Measured attenuation (small circles) and the least-squares fitted curve (solid line). (b) Measured dispersion (small circles) and the model-predicted dispersion (solid line) using the measured attenuation parameters $y=1.38$ and $\alpha_0=7.42$ ($\text{dB cm}^{-1} \text{MHz}^{-1.38}$). The reference frequency $f_0=2$ MHz.

b_0 in Eq. (8) is almost zero. The two parameters obtained using least-squares fitting (solid curve) are $\gamma = 1.38$, and $\alpha_0 = 7.42$ ($\text{dB cm}^{-1} \text{MHz}^{-1.38}$). The bottom plot of Fig. 2 shows the directly measured dispersion (small circles) by using Eq. (7) and the model-predicted dispersion (solid line) by using Eq. (10) with $\gamma = 1.38$ and $\alpha_0 = 7.42$ ($\text{dB cm}^{-1} \text{MHz}^{-1.38}$). In the dispersion calculation, a reference frequency of 2 MHz is used. Fig. 3 summarizes the results from the Plexiglas ($L = 4$ cm). The attenuation of this specimen (top plot) shows a linear frequency dependency ($\gamma = 1$) with a slope (α_0) of 0.94 ($\text{dB cm}^{-1} \text{MHz}^{-1}$). Due to the large impedance mismatch, the offset $b_0 = -1.29$ (dB cm^{-1}) (not shown in the figure). The bottom plot of Fig. 3 shows the measured (small circles) dispersion by using Eq. (7) and the model-predicted dispersion (solid line) by using Eq. (9) with $\alpha_0 = 0.94$. Again, the reference frequency is 2 MHz.

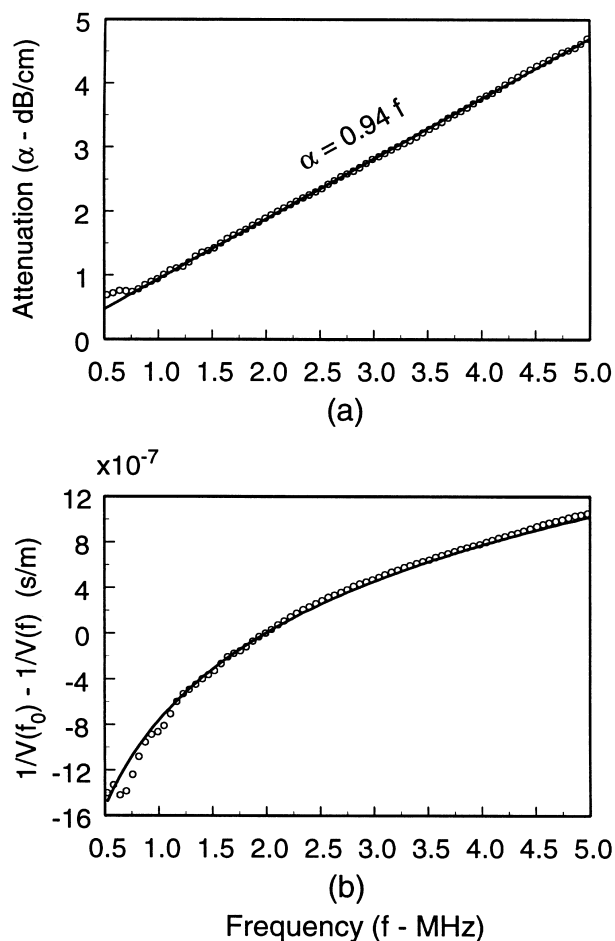


Fig. 3. Attenuation and dispersion of the Plexiglas specimen. (a) Measured attenuation (small circles) and the least-squares fitted curve (solid line). (b) Measured dispersion (small circles) and the model-predicted dispersion (solid line) using the measured attenuation parameters $\gamma = 1$ and $\alpha_0 = 0.94$ ($\text{dB cm}^{-1} \text{MHz}^{-1}$). The reference frequency $f_0 = 2$ MHz.

4. Conclusion and discussion

In summary, a simple method for directly measuring the dispersion is described. The method only requires the recording of two pulses: one without and one with the specimen inserted between the transmitting and receiving transducers. The only other variable to be measured is the thickness of the specimen. From the two recorded pulses, one can determine the dispersion as well as the attenuation of the specimen using a personal computer. In addition to the simplicity in data acquisition, the proposed method eliminates the need for phase unwrapping which often produces uncertainties in phase calculation due to the erratic behavior of the phase spectrum near the zero frequency. Finally, as indicated by Fig. 3, the proposed method is very sensitive and can measure very small dispersion (0.1–1%) reliably.

The two specimens used in this study show significantly different attenuation: the rubber material is highly attenuative and its attenuation has a non-linear frequency dependency while the Plexiglas shows a moderate attenuation that increases linearly with frequency. Results from this study provide, for the first time, a direct experimental verification of the time causal model for both cases of $\gamma = 1$ and $\gamma > 1$. Besides the time causal model, several other models have been proposed for determining the dispersion from the attenuation [7,8]. Due to the difficulty in the previous dispersion measurement, the relative accuracy of these models has not been compared directly and experimentally. With the improved accuracy and precision, the proposed method will be suitable for such a comparative study.

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