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**Final Exam (40 pts)**

This is an open textbook and open notes exam.

## 1 Regular Language Specifications (5 + 5 + 5)

Construct (1) an NFA- $\lambda$ , (2) a minimal DFA, and (3) a regular grammar for the language corresponding to the regular expression  $ab^+ \cup c^*$  over the alphabet  $\{a, b, c\}$ .

## 2 Language classification (3 + 3)

Consider the context-free language  $\mathcal{B} = \{0^i1^j \mid i \neq j\}$ .

Is  $\mathcal{B}^*$ , the Kleene closure of  $\mathcal{B}$ , regular? Justify your answer.

Give a context-free grammar for  $\mathcal{B}^*$ .

## 3 Pumping Lemma (5 + 3 + 2)

Consider the language of bit strings  $\mathcal{P} = \{vw \mid w \in \{0,1\}^* \text{ and } w = \text{reverse}(v)\}$ .

Prove that  $\mathcal{P}$  is not regular using pumping lemma.

What is  $\mathcal{B} \cap \mathcal{P}$ ?

Is  $\mathcal{B} \cap \mathcal{P}$  regular? Justify your answer.

## 4 Closure Properties (3 + 3 + 3)

Let  $\mathcal{FS}$  denote the set of finite languages over  $\{0,1\}$ , and  $\mathcal{CS}$  denote the set of non-regular context-free languages over  $\{0,1\}$ . Prove or disprove the following claims:

1. The set  $\mathcal{CS}$  is closed under set-difference operation.
2. There exists  $\mathcal{L} \in \mathcal{FS}$  such that  $\mathcal{L}^* \in \mathcal{FS}$ .
3. There exist  $\mathcal{L}_1, \mathcal{L}_2 \in \mathcal{CS}$  such that  $\mathcal{L}_1 \cap \mathcal{L}_2 \in \mathcal{FS}$ .