

Final Exam (40 pts)

This is an open book exam. You are allowed to consult only Sudkamp text during the exam.

1 Regular Language Specifications (4 + 6 + 4)

Construct (1) an NFA- λ , (2) a minimal DFA, and (3) a regular grammar for the language of the regular expression $a^*b \cup c^+$ over the alphabet $\{a, b, c\}$.

2 Language Operations and Type (3 + 4)

Consider the language $\mathcal{B} = \{a^i b^j \mid i = j\}$.

Is $\overline{\mathcal{B}} = \{a, b\}^* - \mathcal{B}$, the complement of \mathcal{B} , regular? Justify your answer.

Give a context-free grammar for $\overline{\mathcal{B}}^*$, the Kleene closure of the complement of \mathcal{B} .

3 Pumping Lemma (6 + 4)

Consider the language $\mathcal{P} = \{w c w \mid w \in \{a, b\}^*\}$.

Prove that \mathcal{P} is not regular using pumping lemma.

Is $\mathcal{B} \cap \mathcal{P}$ regular? Justify your answer.

4 Closure Properties (3 + 3 + 3)

(CS466 Only) Let \mathcal{FS} denote the set of finite languages over $\{0, 1\}$, and \mathcal{CS} denote the set of non-regular context-free languages over $\{0, 1\}$. Prove or disprove the following claims:

1. The set \mathcal{CS} is closed under set-difference operation.
2. There exist $\mathcal{L} \in \mathcal{FS}$ such that $\mathcal{L}^+ \in \mathcal{FS}$.
3. There exist $\mathcal{L}_1, \mathcal{L}_2 \in \mathcal{CS}$ such that $\mathcal{L}_1 \cap \mathcal{L}_2 \in \mathcal{FS}$.

(CS666 Only) Let \mathcal{FKS} denote the set of languages over $\{0, 1\}$ that are either finite or whose complement is finite, and \mathcal{CS} denote the set of non-regular context-free languages over $\{0, 1\}$. Prove or disprove the following claims:

1. The set \mathcal{FKS} is closed under set-union operation.
2. The set \mathcal{FKS} is closed under Kleene-* operation.
3. There exist $\mathcal{L}_1, \mathcal{L}_2 \in \mathcal{CS}$ such that $\mathcal{L}_1 \cap \mathcal{L}_2 \in \mathcal{FKS}$.