
First Midterm (30 pts)

This is an open book exam. You are allowed to consult only Sudkamp text during the exam.

1 Recursive Definition (5 pts)

Give a recursive definition for the following set of grid points using the successor function s :

$$\{(m, n) \in \mathcal{N} \times \mathcal{N} \mid (m + n) \text{ is even}\}.$$

2 Regular Sets (4 pts)

Give an example of an infinite regular set whose complement is finite.

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3 Regular Grammar (4 pts)

Give a regular grammar for the set of strings over $\{a, b, c\}$ in which the total number of b 's and c 's is even.

4 Regular Expressions (5 pts)

Give a regular expression for the language \mathcal{L} over $\{0, 1\}$ whose strings do not contain the substring 010.

5 Context-Free Grammars (5 pts)

Give a context-free grammar for the following language. (Assume $i, j \in \mathcal{N}$.)

$$\{a^{i+j}b^j \mid i \geq j\}$$

6 Mathematical Induction (6 + 1 pts)

Prove that the strings generated by the following grammar contain equal number of as and bs .

$$S \rightarrow aSb \mid SS \mid \lambda$$

(Clearly state the quantity you are inducting on, the basis case, the induction hypothesis and the induction step.)

Does the grammar generate all strings with equal number of as and bs ? Justify your answer.